Roll No. | | | | | | | | | | | | Total No. of Pages : 03

Total No. of Questions : 09

B.Tech. (Sem.–2)

MATHEMATICS-II

Subject Code : BTAM-201-18

M.Code : 76254

Date of Examination : 23-01-23

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

Answer briefly :

1. a) Is this differential equation $\frac{d^2y}{dx^2} = \left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}$ linear ?

   b) Is this differential equation $(2x^2 + 3y^2 - 7)\, x\, dx - (3x^2 + 2y^2 - 8)\, y\, dy = 0$ exact ?

   c) Write the solution of the Clairaut's equation $y = px - a^2p/(p+1)$.

   d) Find the wronskian from $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$.

   e) Find complementary function of $\frac{\partial^2 z}{\partial t^2} - a^2\frac{\partial^2 z}{\partial x^2} = E \sin pt$

   f) Find particular integral of $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x$.

   g) Write one dimensional diffusion equation.

   h) Classify the equation $y^2u_{xx} - 2xyu_{xy} + x^2y_{yy} + 2u_x - 3u = 0$.

i) What is a boundary value problem?

j) Write Laplace equation in spherical coordinates.

SECTION-B

2. Solve :

a) $(1 + e^{x/y}) + e^{x/y}\left(1 - \frac{x}{y}\right)\frac{dy}{dx} = 0$

b) $x\frac{dy}{dx} + y \log y = xye^{x}$.

3. Solve :

a) $(D^2 - 4D + 3)\, y = 2xe^{3x} + \cos 2x$

b) Find the power series solution of the differential equation $(xD^2 + D - 1)\, y = 0$

4. Solve :

(a) $(z - y)p + (x - z)q = (y - x)$ (b) $z(xp - yq) = y^2 - x^2$

5. a) **Solve the PDE** $(4D^2 + 12DD' + 9D'^2)\, z = e^{3x-2y}$.

b) **Solve the PDE** $(D^2 - DD' + D' - 1)\, z = \cos(x + 2y)$.

SECTION-C

6. Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial y} + u$ by method of separation of variables. Given that $u = 6e^{-3x}$ when x=0.

7. Solve the BVP $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ using D' Alembert's technique subject to the conditions

$u(0, t) = u(5, t) = 0$, $u(x, 0) = 0$ and $\left.\frac{\partial u}{\partial t}\right)_{t=0} = 3\sin 2\pi x - 2\sin 5\pi x$.

8. Solve the BVP $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ using separation of variables method subject to the conditions $u(0, t) = 1$, $u(\pi, t) = 3$, $u(x, 0) = 1$ where $0 < x < \pi$, $t > 0$.

9. The bounding diameter of a semi-circular plate of radius a is kept at 0°C and the temperature along the semi-circular boundary is given by

$$u(a, \theta) = \begin{cases} 50\theta, & \text{when } 0 < \theta \leq \pi/2 \\ 50(\pi - \theta), & \text{when } \pi/2 < \theta < \pi \end{cases}$$

Estimate the steady state temperature in the plate using Laplace equation

$$r^2 \frac{\partial^2 u}{\partial r^2} + r \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0.$$

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.