Roll No. | | | | | | | | | | | | | **Total No. of Pages : 03**

Total No. of Questions : 09

B.Tech. FT (2018 & onwards) (Sem.–2)

MATHEMATICS-II

Subject Code : BTAM-206-18

M.Code : 76349

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

1. Answer the following :

a) Find Laplace transform of $e^{-t}\cos^3 t$.

b) Find Laplace Inverse Laplace transform of $\frac{1}{s^2+3s+2}$.

c) Explain Dirichlet's condition for expansion in terms of Fourier series.

d) Define Bessel's equations of order n.

e) Express $f(x) = x^2 + 3x - 5$ in terms of Lagendre function.

f) Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+3y=0$.

g) Solve $\frac{d^2y}{dx^2}+8\frac{dy}{dx}+16y=0$.

h) Solve the differential equation $\frac{dy}{dx}+y\cot x=\cos x$.

i) Explain, how can we classify a general second order linear partial differential equation into parabolic, elliptic and hyperbolic equations ?

j) Explain the concept of Half-range cosine series.

SECTION-B

2. a) Find the Fourier transform of $f(x) = e^{-x^2/2}, -\infty < x < \infty$.

b) Find the inverse Laplace transform of $\frac{1}{(s^2+1)(s+1)}$.

3. Find the Fourier series for $f(x)$ in the interval $(-\pi, \pi)$ when $f(x) = \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$.

4. Solve by method of Laplace transform $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 2, y(0) = 3, y'(0) = -5$.

5. a) Solve the differential equation by finding integrating factor

$$y dx + (y^2 - x)\, dy = 0$$

b) Solve the differential equation : $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = xe^x \sin x$

SECTION-C

6. By employing power series method, solve the differential equation : $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

7. a) If α and β are the roots of the equation $J_n(x) = 0$, then prove that

$$\int_0^1 x J_n(\alpha x) J_n(\beta x)\, dx = 0, \text{ if } \alpha \neq \beta$$

b) Prove that $P_n(-x) = (-1)^n P_n(-x)$, where n is any positive integer.

8. Solve one dimensional heat equation $\frac{u}{dt} = c^2 \frac{^2u}{dx^2}$ subject to $0 \leq x \leq L$ using separation of variable and Fourier series method.

Boundary conditions are : $u(0, t) = 0$, $u(L, t) = 0$

Initial conditions are : $u(x, 0) = \begin{cases} x & 0 \leq x \leq L/2 \\ L - x, & L/2 \leq x \leq L \end{cases}$, and $u(0, t) = 0$, $u(L, t) = 0$.

9. Using Laplace Transform Solve the partial differential equation

$x\frac{u}{x} \quad \frac{u}{t} \quad xt = 0$, $x > 0$, $t > 0$, $u(0, t) = 0$, $y(x, 0) = x$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.