Roll No. [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] **Total No. of Pages : 02**

Total No. of Questions : 09

B.Tech. (EE) (Sem.–1)

MATHEMATICS - I

Subject Code : BTAM-121B

M.Code : 76361

Date of Examination : 20-01-2023

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**

SECTION-A

1. Write briefly :

a) Evaluate $\int_0^\infty \sqrt{x}e^{-x^3}\,dx$

b) Prove that $\gamma(n) = (n-1)!$ if n is an integer.

c) Show that $|\cos b - \cos a| < |6 - a|$ using mean value theorem.

d) Find $\frac{df}{dt}$ at $t = 1$, where $f(x,y) = x\cos y - e^x \sin y$, $x = t^2 + 1$, $y = t^3 + t$.

e) Find the minimum value of the function $f(x, y) = 3x^2 + y^2 - x$.

f) Find the eigen values of the matrix : $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

g) Show that the matrices A and A^t have same set of eigen values.

h) Solve the following system of linear equations: $x + 3y = 9$, $6x - y = 3$.

i) Write the series expansion for $\cos x$.

j) Find the volume of the solid generated by revolving the finite region bounded by the curves $y = x^2 + 1$ and $y = 5$ about the line x = 3.

SECTION-B

2. Find the surface area of the solid generated by revolving the circle $x^2 + (y - h)^2 = a^2$, $b \geq a$ about the x-axis.

3. a) Evaluate $\int_0^{\infty} 2^{-16x^2} dx$ using gamma function.

   b) Evaluate $\lim_{x \to 0} x^x$.

4. Using Taylor's theorem, obtain the value of cos 31° correct to 4 decimal places.

5. Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$, for $p > 0$

SECTION-C

6. Find the shortest distance between the line $y = 10 - 2x$ and the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$.

7. Discuss the continuity of the function :

$$f(x, y) = \begin{cases} \frac{1}{1+e^{1/x}} + y^2 & if\ (x, y) \neq (0,0) \\ 0 & if\ (x, y) = (0,0) \end{cases}$$

8. State Cayley Hamilton's theorem and verify it for the matrix: $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$

9. Examine whether the matrix: $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable. If so, then diagonalize it.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.