Roll No. ⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚⬚   **Total No. of Pages: 02**

Total No. of Questions: 09

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B.Tech (Sem. – 1)

MATHEMATICS-I

Subject Code: BTAM- 104-18

M Code: 75362

Date of Examination : 11-01-2023

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Time: 3 Hrs.   **Max. Marks: 60**

INSTRUCTIONS TO CANDIDATES:

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each, carrying EIGHT marks each.**
3. **Attempt any FIVE questions from SECTION B & C, selecting atleast TWO questions from each of these SECTIONS B & C.**

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SECTION-A

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1. **Answer the following:**

    a) Can Rolle's theorem be applied to the function $f(x) = x(x + 3)^2, x \in [-3,1]$.

    b) Define *gamma* function.

    c) Evaluate $\lim_{x \to 0} \frac{x - \sin x}{\sin x(1 - \cos x)}$

    d) If $A + B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$ and $A - B = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$, then find values of A and B

    e) Find adjoint of $\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$

    f) Define the dimension of vector spaces.

    g) Give the statement of rank nullity theorem.

    h) Give any two properties of Eigen values.

    i) Define skew symmetric matrix with an example.

    j) Find sum and product of latent roots of the matrix $\begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix}$

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SECTION-B

2. a) Expand $f(x) = \tan^{-1} x$ by Maclaurin's theorem.

   b) Evaluate $\lim_{x \to 1} \frac{x^x - x}{-1 + x - \log x}$.

3. a) Evaluate the integral $\int_0^{\pi/2} \sqrt{\tan x}\, dx$ in terms of beta function.

   b) Find minima of $f(x, y) = 4x^2 + 9y^2 - 8x - 12y + 4$.

4. a) Prove that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^2$.

   b) Solve the equations $3x + y + 2z = 3, 2x - 3y - z = -3, x + 2y + z = 4$ using Cramer's rule.

5. a) Are the vectors $(1,1,1,3), (1,2,3,4), (2,3,4,9)$ linearly dependent.

   b) Find the rank of the matrix: $\begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$.

SECTION-C

6. Show that the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ satisfies the equation $A^3 - 6A^2 + 5A + 11I = 0$.

7. Let $T: R^3 \to R^2$ be the linear transformation defined by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} y + z \\ y - z \end{pmatrix}$, then find the matrix renresentation of T w.r.t. the ordered basis $X = \{(1,0,1), (1,1,0), (0,1,0)\}^T$ in R^3 and $Y = \{(1,0), (0,1)\}^T$ in R^2.

8. a) Is the matrix $\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix}$ orthogonal?

   b) Write the matrix $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrices.

9. Reduce the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to the diagonal form.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.