Roll No. | | | | | | | | | | | | | Total No. of Pages : 02

Total No. of Questions : 08

B.Tech. (CE/ME/ECE/EE) (2018 & Onward) (Sem.–1)

MATHEMATICS-I

Subject Code : BTAM-101-18

M.Code : 75353

Time : 2 Hrs. Max. Marks : 30

INSTRUCTIONS TO CANDIDATES :

1. **Attempt any FIVE question(s), each question carries 6 marks.**

1. a) Expand $f(x) = e^{\alpha \sin^{-1} x}$ in ascending powers of x upto x^4.

   b) Evaluate $\lim_{x \to 0} \dfrac{e^x - e^{-x} - 2\log(1+x)}{x \sin x}$.

2. a) Find the maximum value of $\sin^p x \cos^q x$.

   b) Find the volume of the solid generated by revolving the curve $xy^2 = 4\,(2 - x)$ about y-axis.

3. a) If $u(x, y) = \dfrac{x^2 + y^2}{x + y}$, then prove that $\left(\dfrac{\partial u}{\partial x} - \dfrac{\partial u}{\partial y}\right)^2 = 4\left(1 - \dfrac{\partial u}{\partial x} - \dfrac{\partial u}{\partial y}\right)$.

   b) Find the maximum and minimum values of $x^3 + 3xy^2 - 3y^2 + 4$.

4. a) Evaluate $\int_0^a \int_0^{\sqrt{a^2 - y^2}} (x^2 + y^2)\, dxdy$ after changing into polar coordinates.

   b) Evaluate $\iint_R (x + y)\, dxdy$ where R is the region bounded by $x = 0, x = 2, y = x, y = 2+x$.

5. a) Examine the convergence of the series $\sum_{n=1}^{\infty} \dfrac{1}{n^p}$ *when* $|\, p \,| \le 1$.

   b) Examine the series $1 + \dfrac{1}{2^2} + \dfrac{2^2}{3^3} + \dfrac{3^3}{4^4} + \ldots$ for convergence.

6. a) Examine $\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\ldots.$

b) Examine the series $\frac{x}{1+x}-\frac{x^2}{1+x^2}+\frac{x^3}{1+x^3}-\ldots,\ 0<x<1$ for convergence.

7. a) Determine whether the vectors $u = (1, 2, 3)$ and $v = (7, -4, 2)$ are linearly dependent?

b) Solve the system of linear equations $3x + y + 2z = 3$, $2x - 3y - z = -3$, $x + 2y + z = 4$.

8. Find the characteristic equation of the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and hence compute A^{-1}. Also express the matrix represented $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10\ I$.

Note: Any student found attempting answer sheet from any other person(s), using incriminating material or involved in any wrong activity reported by evaluator shall be treated under UMC provisions.

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