Roll No. | | | | | | | | | | | | Total No. of Pages : 03

Total No. of Questions : 09

B.Tech (Food Technology) (2018 & Onwards) (Sem.–1)

MATHEMATICS-I

Subject Code : BTAM-106-18

M.Code : 75368

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions EACH from SECTION - B & C.**

SECTION-A

1. Answer briefly :

a) Define rank of a matrix.

b) For any nonsingular matrix $A = (a_{ij})$ of order n, show that $|Adj\ (A)| = |A|^{n-1}$

c) Determine the values of k for which the system of equations

$$x - ky + z = 0,\ kx + 3y - kz = 0,\ 3x + y - z = 0$$

has a nontrivial solution.

d) Define orthogonal matrices.

e) Is the following matrix diagonalizable? Give reason to your answer.

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

f) Find the length of the following curve

$$r\,(t) = a\cos^3 t\ \mathrm{i} + a\sin^3 t\ \mathrm{j},\ 0 \le t \le \pi/2$$

g) Find gradient of the scalar field $f\,(x, y, z) = x^2y^2 + xy^2 - z^2$ at (3, 1, 1)

h) Define curl of a vector field.

i) Find the length of the arc $r(t) = 3\cos t\, \mathrm{i} + 3\sin t\, \mathrm{j},\ 0 \le t \le \pi$

j) Evaluate $\int_C x^2 y\, ds$, where C is the curve defined by $x = \cos t,\ y = \sin t,\ 0 \le t \le \pi/2$.

SECTION-B

2. a) Show that :

$$\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2$$

b) Solve the following system of equations

$$x - y + 3z = 3,\ 2x + 3y + z = 2,\ 3x + 2y + 4z = 5$$

3. a) Use Gauss Jordan method to find the inverse of the matrix

$$\begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ 1 & 3 & 4 \end{bmatrix}$$

b) For what values of k the following set of vectors form a basis in $\mathbb{R}^3$.

$$\{(k, 1-k, k), (0, 3k-1, 2), (-k, 1, 0)\}.$$

4. Find all the eigen values and the corresponding eigenvectors of the following matrix.

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

5. a) The eigen values of 3 × 3 matrix A corresponding to the eigenvalues 1, 1, 3 are $[1, 0, -1]^t$, $[0, 1, -1]^t$, $[1, 1, 0]^t$ respectively. Find the matrix A.

b) Prove that eigen values of a skew-symmetric matrix are zero or purely imaginary.

SECTION-C

6. a) Find directional derivative of the function $f(x, y, z) = x^2y + 4xyz + z^2$ at a point (1, 2, 3) in the direction of $3i + 4j - 5k$.

   b) If $r = xi + yj + zk$ and $r = |r|$, show that div $(r/r^3) = 0$.

7. a) For the vector field $v = xyz(yz\, i + xz\, j + xy\, k)$ find a scalar function $f(x, y, z)$ such that $v = \nabla f$.

   b) Find the angle between the surface $z = x^2 + y^2$ and $z = 2x^2 - 3y^2$ at the point (2, 1, 5)

8. a) Show that $\int_C (yz + 1)dx + (z + xz + z^2)dy + (y + xy + 2yz)dz$ is independent of the path of integration from (1, 2, 2) to (2, 3, 4). Evaluate the integral.

   b) Evaluate the integral of $v = x^2\, i - 2y\, j + z^2\, k$ over the straight line path from (–1, 2, 3) to (2, 3, 5).

9. Find the work done by the force $F = (x^2 - y^2)\, i + (x^2 + y^2) j$ in moving a particle along a closed path C bounding the region $x^2 + y^2 \le 16$, $x^2 + y^2 \ge 4$, $x \ge 0$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.