

Roll No.

97663

BCA 1st Semester
Examination – December, 2022

MATHEMATICS

Paper : BCA-103

Time : Three Hours]

Maximum Marks : 80

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt five questions in all, selecting one question from each Unit. Question No. 1 is compulsory. All questions carry equal marks.

1. (a) If $A = \{2, 4, 6, 8\}$ and $B = \{6, 8, 10, 12\}$, write $A - B$ and $B - A$. 2

(b) Without expanding, prove that : 2

$$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$$

(c) If $f : R \rightarrow R$ is defined by $f(x) = 3x^2 - 8x + 1$, find $f(f(x))$. 2

(d) Evaluate : $\lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 15x}$. 2

(e) Find : $\frac{dy}{dx}$, if $y = 5x^3 + 8x^2 - 7x + 10$. 2

(f) Find : $\frac{dy}{dx}$, if $y = \cos x^4$. 2

(g) Evaluate : $\int \frac{dx}{4+x^2}$. 2

(h) Evaluate : $\int_{-1}^1 x^{99} dx$. 2

UNIT – I

2. (a) Prove that : $(A \cap B)' = A' \cup B'$. 8

(b) There are 210 members in a Club, 100 of them drink Tea and 65 drink Tea but not Coffee. Find :

- (i) How many drink Coffee ?
(ii) How many drink Coffee but not Tea ? 8

3. (a) Prove that : 8

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$$

(b) Find the inverse of the matrix : 8

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

UNIT – II

4. (a) If R is a relation in $N \times N$, defined by $(a, b) R (c, d)$ if and only if $a + d = b + c$, show that R is an equivalence relation. 8

(b) Find the domain and range of the following functions : 8

(i) $y = \frac{x^2 - 1}{x - 1}, x \neq 1$

(ii) $y = \sqrt{9 - x^2}$

5. (a) Evaluate : 8

(i) $\lim_{x \rightarrow 0} \frac{\tan 3x - 2x}{3x - \sin^2 x}$

(ii) $\lim_{x \rightarrow 0} \frac{x^3 \cot x}{1 - \cos x}$

(b) Discuss the continuity of the function : 8

$$f(x) = \begin{cases} \frac{3-x}{2}, & \frac{1}{2} \leq x < 1 \\ \frac{1}{2}, & x = 1 \\ \frac{3+x}{2}, & 1 < x \leq 2 \end{cases}, \text{ at } x = 1.$$

UNIT - III

6. Differentiate the following functions w.r.t. x : $4 \times 4 = 16$

(i) $(x^4 + x)(5x^3 + 6x)$

(ii) $\frac{x^4 + 1}{x^2 + 1}$

(iii) $\frac{\sin x + \cos x}{\sin x - \cos x}$

(iv) $(\sin^{-1} x)^2$

7. Differentiate the following functions w.r.t. to x : $4 \times 4 = 16$

(i) $\tan^{-1}\left(\frac{\sin x}{1 + \cos x}\right)$

(ii) $\log(x + \sqrt{x^2 - a^2})$

(iii) $(\sin x)^x$

(iv) Differentiate $\sin x^3$ w.r.t. x^3

UNIT - IV

8. Evaluate : $4 \times 4 = 16$

(i) $\int \frac{x^4}{x+1} dx$

(ii) $\int \sqrt{1 + \sin 2x} dx$

(iii) $\int \frac{1}{x(1 + \log x)^2} dx$

(iv) $\int \frac{dx}{1 - 6x - 9x^2}$

9. Evaluate : $4 \times 4 = 16$

(i) $\int x^2 \cos x dx$

(ii) $\int \sqrt{x^2 - 4x} \cdot 2 dx$

(iii) $\int \frac{x}{(x+2)(3-2x)} dx$

(iv) $\int_0^{\pi/2} \log(\tan x) dx$