

Time: 3 hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

- 1.a) Find $L^{-1}\left(\frac{1}{(s-2)^2}\right)$. [2]
- b) Define Unit step function and find its Laplace transform. [3]
- c) Evaluate $\Gamma\left(-\frac{3}{2}\right)$. [2]
- d) Evaluate $\int_0^1 x^5(1-x)^6 dx$ [3]
- e) Using triple integral, find the volume of a rectangular box whose length is 6 ft, breadth is 5 ft and height is 4 ft. [2]
- f) Evaluate $\int_0^2 \int_0^x (x+y^2) dy dx$ [3]
- g) Define solenoidal vector. [2]
- h) Prove that \vec{r} is an irrotational where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ [3]
- i) State stokes theorem. [2]
- j) Evaluate $\iiint_V \text{div} \vec{f} dx dy dz$ where V is the volume of the sphere whose radius is 'a' units and $\vec{f} = x\vec{i} + y\vec{j} + z\vec{k}$. [3]

PART-B

(50 Marks)

- 2.a) Find the Laplace transform of $(\sin t + \cos t)^2$
- b) Find the inverse Laplace transform of $\frac{1}{(s^2+1)(s+1)}$. [5+5]
- OR**
3. Solve $y'' + 2y' + 5y = e^{-t}$, $y(0) = 1$, $y'(0) = 1$ using Laplace transform. [10]
- 4.a) Evaluate $\int_0^{\infty} e^{-x/3} x^3 dx$.
- b) Evaluate $\int_0^1 \frac{xdx}{\sqrt{1-x^4}}$. [5+5]

OR

5.a) Evaluate $\int_0^{\infty} e^{-x^3} x^7 dx$.

b) Evaluate $\int_0^1 \frac{x^2 dx}{\sqrt{1-x^4}}$. [5+5]

6.a) Evaluate $\int_0^2 \int_{\sqrt{2x-x^2}}^{\sqrt{2x-x^2}} (x^2 + y^2) dx dy$ by changing to polar coordinates.

b) Evaluate $\iint_R y dx dy$ where R is the region bounded by the parabola $y^2 = 4x$ and $x^2 = 4y$. [5+5]

OR

7.a) Evaluate $\iiint xy^2 z dx dy dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

b) Evaluate $\int_0^{ax} \int_0^{x+y} \int_0^x e^{x+y+z} dx dy dz$. [5+5]

8.a) Find the directional derivative to the surface $f(x,y,z) = xy^2z - 4$, at the point $(1, -1, 2)$ along $i+j+k$.

b) A butterfly is located at $(2, -1, 3)$ and desires to fly towards fragrance surface $f(x,y,z) = x^2 + yz^2$. Along which direction should it fly to get fragrance at the earliest? [5+5]

OR

9.a) Show that $\nabla^2 r^n = n(n+1)r^{n-2}$ where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $|\vec{r}|^2 = r$.

b) Prove that $\nabla \left(\frac{1}{r} \right) = -\frac{\vec{r}}{r^3}$ where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $|\vec{r}|^2 = r$. [5+5]

10. Verify Greens theorem for $\oint_C (y - \sin x) dx + \cos x dy$ where C is the triangle enclosed by the lines $y = 0, x = \frac{\pi}{2}$ and $x = 2x$. [10]

OR

11. Verify stokes theorem for a vector field defined by $\vec{F} = -y^3\vec{i} + x^3\vec{j}$ in the region $x^2 + y^2 \leq 1, z = 0$. [10]

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