(T) Roll No. .....

ID—4038

B.C.A. EXAMINATION, 2022

(First Semester)

MATHEMATICS

Code : BCA-103

Time : 3 Hours *Maximum Marks : 80*

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt any *Five* questions. All questions carry equal marks.

1. (a) Write the following in Roster form :

$$A = \{x : 4x - 3 < 6, x \in N\}$$

Unit II

4. (a) If R is a relation in N × N, defined by (a, b) R (c, d) if and only if $a + d = b + c$, show that R is an equivalence relation.

(b) If $f : N \to N$ be defined as :

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}; & \text{if } n \text{ is even} \end{cases}$$

for all $n \in N$, determine whether the function f is onto and one-one.

5. (a) If $\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$, find the value of k.

(b) If $f(x) = \begin{cases} \frac{|x-2|}{2-x}, & \text{when } x \neq 2 \\ -1; & \text{when } x = 2 \end{cases}$

show that f is discontinuous at $x = 2$. Also write the type of discontinuity.

Unit III

6. (a) If $y = \frac{x}{x+4}$, show that $x\frac{dy}{dx} = y(1-y)$.

(b) Differentiate w.r.t x :

$$\frac{\sqrt{x^2+1} - \sqrt{x^2-1}}{\sqrt{x^2+1} + \sqrt{x^2-1}}.$$

7. (a) Differentiate $\left(\frac{3+4x}{2-x}\right)^2$ w.r.t. x.

(b) Find $\frac{dy}{dx}$, if $y = x^x + (\tan x)^{\log x}$.

Unit IV

8. (a) Evaluate the following integral :

$$\int x^2 e^x \, dx.$$

(b) Evaluate $\int \frac{xe^x}{(x+1)^2} dx$.

9. (a) Evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$.

(b) Evaluate $\int_1^2 \frac{dx}{x(1+\log x)}$.