

**GUJARAT TECHNOLOGICAL UNIVERSITY****BE- SEMESTER-I & II(OLD)EXAMINATION – SUMMER 2022****Subject Code:110014****Date:02-08-2022****Subject Name:Calculus****Time:10:30 AM TO 01:30 PM****Total Marks:70****Instructions:**

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

- Q.1** (a) 1) Define Beta and Gamma function. 03
- 2) Evaluate  $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x}}$ . 04
- (b) Determine the interval of convergence of the series 07  
 $x + \frac{x^2}{2^2} + \frac{x^3}{3^2} + \dots + \frac{x^n}{n^2} + \dots$ .
- Q.2** (a) 1) Expand the polynomial  $f(x) = 2x^3 + 7x^2 + x + 6$  in powers of  $(x - 2)$ . 03
- 2) Test the convergence of the series  $\sum_{n=1}^{\infty} \frac{n! 2^n}{n^n}$ . 04
- (b) Expand  $\sin x$  in powers of  $(x - \frac{\pi}{2})$ . Hence find the value of  $\sin 91^\circ$ . 07
- Q.3** (a) 1) Express  $\int_0^{\infty} e^{-x^2} dx$  in terms of Gamma function. 03
- 2) Find the first order partial derivatives of  $z = \tan^{-1}\left(\frac{x}{y}\right)$ . 04
- (b) Trace the curve  $y^2(a - x) = x^3$ . 07
- Q.4** (a) 1) If  $x = r \cos \theta$ ,  $y = r \sin \theta$ , then find  $\frac{\partial(r, \theta)}{\partial(x, y)}$ . 03
- 2) Evaluate  $\iint_R r^2 \sin \theta dr d\theta$ , where  $R$  is the region of the circle  $r = 2a \cos \theta$  lying above the initial line. 04
- (b) Find the volume of the solid obtained by rotating the region  $R$  enclosed by the curves  $y = x$  and  $y = x^2$  about the line  $y = 2$ . 07
- Q.5** (a) 1) Evaluate  $\lim_{x \rightarrow 0} \frac{\log \sin 2x}{\log \sin x}$ . 03
- 2) Evaluate  $\int_0^2 \int_1^z \int_0^{yz} xyz dx dy dz$ . 04
- (b) Use Lagrange's method to determine the minimum distance from the origin to the plane  $3x + 2y + z = 12$ . 07
- Q.6** (a) If  $u = \sin^{-1} \frac{x+y}{\sqrt{x}+\sqrt{y}}$ , prove that 07
- a)  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$ .
- b)  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4 \cos^3 u}$ .
- (b) Trace the curve  $r = a(1 + \cos \theta)$ . 07
- Q.7** (a) Find the maximum and minimum value of  $x^4 + y^4 - 2x^2 + 4xy - 2y^2$ . 07
- (b) Evaluate  $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy dx$  by changing the order of integration. 07

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