DIMENSIONAL ANALYSIS.

AIM:-

1) Development of functional Relationships:-

Rayleigh's Method:-

Ques the time period of simple pendulum depends upon the length of the simple pendulum and accelaration due to gravity find the expression for the time period of simple pendulum.

Ans.

$$T \propto L^a g^b$$

$$[T] = C[L]^a[LT^{-2}]^b$$

$$[T] = C[L^{a+b}\, T^{-2b}]$$

$$-2b = 1$$

$$a+b = 0$$

$$b = -1/2 \text{ and } a = 1/2$$

$$T = C L^{1/2} g^{-1/2}$$

$$T = C\sqrt{\frac{L}{g}}$$

By experiments $C = 2\pi$

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

Ques

$$\underbrace{F_D}_{\text{Dependent}} \quad \underbrace{D, V, \rho\, \mu,}_{\text{Independent Variable}}$$

$$F_D \propto D^a V^b \rho^c \mu^d$$

$$[MLT^{-2}] = C[L]^a[LT^{-1}]^b[ML^{-3}]^c[M^1L^{-1}T^{-1}]^d$$

$$[MLT^{-2}] = C[L^{a+b-3c-d}\, M^{c+d}\, T^{-b-d}]$$

$$c+d = 1$$

$$a+b-3c-d = 1$$

$$-b-d = -2$$

$$b = 2-d$$

$$c = 1-d$$

$$a+2-d-3+3d-d = 1$$

$$a = 2-d$$

$$F_D = C\, D^{2-d}\, V^{2-d}\, \rho^{1-d}\, \mu^{d}$$

$$F_D = C\,(D^2 V^2 \rho)\left(\frac{\mu}{DV\rho}\right)^d$$

Note $\underbrace{\frac{F_D}{\rho V^2 D^2} = C\left(\frac{\mu}{\rho V D}\right)^d}_{\text{Dimentionless groups}} \Rightarrow \underbrace{\frac{F_D}{\rho V^2 D^2}}_{X} - C\underbrace{\left(\frac{\mu}{\rho V D}\right)^d}_{Y} = 0 = (X, Y)$ Dimensionless group

$f(X, Y) = 0.$

$f(X, Y) = 0$

$X = Cf(Y)$ or $Y = Cf(X)$

Buckingham's - π theorem :-

$\underset{\text{Dependent Variable}}{F_D} = \underbrace{V, D, \rho, \mu}_{\text{Indipendent Variable}}$

If total No. of Variables $= m$

and if total No. of fundamental parameter $= n \leq 3$

$n \leq 3$ → Geometric parameter L, D, H .....
→ Kinematic parameter V, a, g, N, ω, ν
→ Dynamic parameter ρ, μ, m .....

remaining No. of variables $= m-n$

No. of Non dimensional groups $= m-n$ ie $(\pi_1, \pi_2, \pi_3 \cdots \pi_{m-n})$.

According to Buckingham's approach

$$f(\pi_1, \pi_2, \pi_3, \pi_4 \cdots) = 0$$

$$\pi_1 = Cf(\pi_2, \pi_3, \pi_4 \cdots \pi_{m-n}).$$

or $$\pi_2 = Cf(\pi_1, \pi_3, \pi_4 \cdots \pi_{m-n}).$$

Ques $\underbrace{F_D}_{\text{Dependent Variable}}$ $\underbrace{V\ D\ \rho\ \mu}_{\text{independent variables}}$

total variable $= 5$

Fundamental Variable $= 3 \Rightarrow (V, D, \rho)$

Remaining variable $= 5-3 = 2$ (F_D, μ)

No. of Non dimentional groups $= 5-3 = 2$

π_1, π_2

π_1

$$\pi_1 = (V^a D^b \rho^c) F_D$$

$$[M^0 L^0 T^0] = [LT^{-1}]^a [L]^b [M^1 L^{-3}]^c [M^1 L^1 T^{-2}]$$

$$a+b-3c+1=0$$
$$-a-2=0$$
$$-3c+1=0$$

$$\pi_1 = \frac{F_D}{\rho V^2 D^2}$$

π_2

$$\pi_2 = (V^a D^b \rho^c)\mu$$

$$\pi_2 = \frac{\mu}{\rho V D}$$

According to Buckingham's π-Method.

$$f(\pi_1, \pi_2) = 0$$

$$f\left(\frac{F_D}{\rho V^2 D^2}, \frac{\mu}{\rho V D}\right) = 0$$

$$\boxed{\frac{F_D}{\rho V^2 D^2} = c f\left(\frac{\mu}{\rho V D}\right)}$$

Method 2

F_D, V, D, ρ, μ

$$\pi_1 = (V^a D^b \mu^c) F_D \qquad \pi_2 = (V^a D^b \mu^c)\rho$$

$$\pi_1 = \left(\frac{F_D}{\mu V D}\right) \qquad \pi_2 = \frac{\rho V D}{\mu}$$

According to Buckingham's π-method.

$$f(\pi_1, \pi_2) = 0$$

$$f\left(\frac{F_D}{\mu V D}, \frac{\rho V D}{\mu}\right) = 0$$

$$\therefore \frac{F_D}{\mu V D} = c f\left(\frac{\rho V D}{\mu}\right) \Rightarrow \boxed{F_D = (\mu V D) c f\left(\frac{\rho V D}{\mu}\right)}$$

Note

$$F_D = \frac{c(\rho V^2 D^2)\mu V D}{\rho V^2 D^2} f\left(\frac{\rho V D}{\mu}\right)$$

$$F_D = \frac{c(\rho V^2 D^2) f\left(\frac{\rho V D}{\mu}\right)}{\frac{\rho V D}{\mu}}$$

$$\boxed{F_D = c(\rho V^2 D^2) f\left(\frac{\rho V D}{\mu}\right)}$$

Ques 4)
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$$V = \sqrt{2gH}\, fn\left[\frac{D}{H}, \frac{\mu}{\rho V H}\right]$$

$\frac{M^1 L^{-1} T^{-1}}{M^1 L^{-3} \times L^1 T^{-2} L^1}$

$V \quad g \quad H \quad D \quad \mu \quad \rho$

$\pi_1 \qquad \pi_2 \quad \pi_3$

$$\pi_1 = (H^a \rho^b g^c) V = \frac{V}{\sqrt{gH}}$$

$$\pi_2 = (H^a \rho^b g^c) D = \frac{D}{H}$$

$$\pi_3 = (H^a \rho^b g^c) \mu = \frac{\mu}{\rho\sqrt{gH^3}} = \frac{\mu}{\rho H\sqrt{gH}} = \frac{\mu}{\rho V H}$$

According to Buckingham π theorem

$$f(\pi_1, \pi_2, \pi_3) = 0$$

$$f\left(\frac{V}{\sqrt{gH}}, \frac{D}{H}, \frac{\mu}{\rho V H}\right) = 0$$

$$\frac{V}{\sqrt{gH}} = c f\left(\frac{D}{H}, \frac{\mu}{\rho V H}\right) \Rightarrow V = c\sqrt{gH}\, f\left(\frac{D}{H}, \frac{\mu}{\rho V H}\right)$$

By experiments $c = \sqrt{2}$.

$$\boxed{V = \sqrt{2gH}\, f\left(\frac{D}{H}, \frac{\mu}{\rho V H}\right)}$$

Ques

$T \quad L \quad g$

π_1

total variables = 3.

Fundament variable = 2

remaining No. of variable = m-n = 1.

$$\pi_1 = T(L^a g^b)$$

$$[M^0 L^0 T^0] = [T][L^a L^b T^{-2b}]$$

$$a + b = 0$$

$$-2b + 1 = 0 \quad b = 1/2$$

$$L = -1/2$$

$$\pi_1 = T(L^{-1/2} g^{1/2})$$

$$\pi_1 = T\sqrt{\frac{g}{L}}$$

According to Buckingham Method

$$f(\pi_1) = 0$$

$$f_n\left(T\sqrt{\frac{g}{L}}\right) = 0 \Rightarrow T\sqrt{\frac{g}{L}} = C$$

$$\cancel{T = C\sqrt{\frac{L}{g}}} \qquad \boxed{T = C\sqrt{\frac{L}{g}}}$$

Different types of forces in fluid flow system

(It's order of Scale Analysis)

Let $L \rightarrow$ characteristic dimension in the flow.

Inertia force.

$$F_I = ma = \rho L^3 \frac{V}{T^2} = \rho L^2 \frac{L}{T} V$$

$$\boxed{F_I = \rho L^2 V^2}$$

1) Viscous force (F_V)

$$F_V = \left(\mu \frac{\partial u}{\partial y}\right) A = \mu \frac{V}{L} \cdot L^2$$

$$\boxed{F_V = \mu V L}$$

2) Pressure force

$$F_P = (\Delta P) A$$

$$\boxed{F_P = (\Delta P) L^2}$$

3) Gravitational force

$$F_g = mg = \rho L^3 g$$

4) Surface tension force (F_σ)

$$F_\sigma = \sigma L$$

5) Compressibility force ($F_{elastic}$)

$$F_{elastic} = KA$$

$$\boxed{F_{elastic} = KL^2}$$

Different Non-Dimensional Number in fluid flow system

1) Reynold's No. :- It is defined as the ratio of Inertia force to viscous force. this number plays most important role in those areas where viscous effects are severly dominant

Ex Boundary layer flow (pipe flow).

$$\boxed{Re_L = \frac{F_I}{F_V} = \frac{\rho L^2 V^2}{\mu V L} = \frac{\rho V L}{\mu}}$$

2) Euler's Number :- (E_u) :- It is defined as the square root of the ratio of Inertia force to pressure force.

This number plays severely important role in those areas where pressure forces are severlly dominant.

Ex pipe flow

$$E_u = \sqrt{\frac{F_I}{F_P}} = \sqrt{\frac{\rho L^2 V^2}{\Delta P L^2}} = \frac{V}{\sqrt{\frac{\Delta P}{\rho}}}$$

$$\boxed{E_u = \frac{V^2}{(\Delta P/\rho)} = \frac{\rho V^2}{\Delta P}} \qquad \boxed{E_u = \frac{\frac{1}{2}\rho V^2}{\Delta P}}$$

3) Froude Number F_r) :- This number is basically defined as the square root of the ratio of Inertia forces to gravitational forces. This number plays significant role in those areas where gravitational forces are dominating.

For Ex. Open channel flows, rivers, canals, spillways

$$\boxed{F_r = \sqrt{\frac{F_I}{F_g}} = \sqrt{\frac{\rho L^2 V^2}{\rho L^3 g}} = \frac{V}{\sqrt{gL}}}$$

4) Weber Number :- (W_b) It is defined as the square root of ratio of Inertia force to surface tension force

It becomes important in those areas where surface tension force becomes equally significant as compare to the other forces.

Ex Capillary flows

$$W_b = \sqrt{\frac{F_i}{F_\sigma}} = \sqrt{\frac{\rho V^2 L^2}{\sigma L}} = \frac{V}{\sqrt{\sigma/\rho L}}$$

5 Mech Number:- It is defined as the squrare root of ratio of inertia force to the Compressibility force. It becomes Significant in the field where Compressibility effects are important which is the high velocity gasous flows.

$$M_a = \sqrt{\frac{F_i}{F_{elastic}}} = \sqrt{\frac{\rho L^2 V^2}{K L^2}} = \frac{V}{\sqrt{K/\rho}} = \frac{V}{C}$$

V → object velocity in the medium

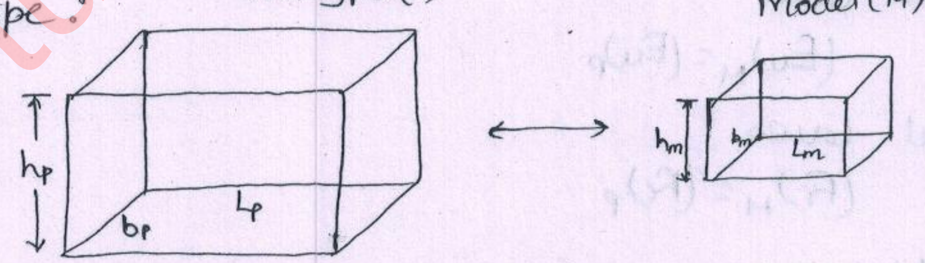
C → Velocity of sound in the medium

Modelling and Similitude:-

Modals are generally the smaller versions of prototype and simlitude is a branch of science in which the actual results of the prototype are obtained experimently just by doing the experiments on their modals, but for that there should be a perfect similarity between the modals and prototypes.

Types of Similarity:-

(1) Geometric Similarity:- A modal and a prototype are said to be geometrically similar if every dimension in the modal is reduced to the same scale as compared to prototype.



$$\frac{L_m}{L_p} = \frac{b_m}{b_p} = \frac{h_m}{h_p} \text{ - - - - - -} = L_r \Rightarrow \text{Length scale ratio (Scale Ratio)} = \text{Const}$$

$$\frac{A_m}{A_p} = \frac{L_m \times B_m}{L_p \times B_p} = L_r^2 \Rightarrow \text{Area Scale ratio}$$

$$\frac{V_m}{V_p} = \frac{L_m \times B_m \times h_m}{L_p \times B_p \times h_p} = L_r^3 \Rightarrow \text{Volume Scale Ratio.}$$

(ii) Kinematic Similarity :- It is the similarity of kinematic Parameters between Model and prototypes.

$$\frac{(V_1)_m}{(V_1)_p} = \frac{(V_2)_m}{(V_2)_p} = \frac{(V_3)_m}{(V_3)_p} - - - - - = \frac{V_m}{V_p} = V_r = \text{Velocity Scale ratio}$$

$$V_r = \frac{L_m/t_m}{L_p/t_p} = \frac{(L_m/L_p)}{(t_m/t_p)} = \frac{L_r}{t_r}$$

and $$\frac{(a_1)_m}{(a_1)_p} = \frac{(a_2)_m}{(a_2)_p} - - - - - = \frac{a_m}{a_p} = a_r = \text{Acceleration Scale ratio}$$

$$\boxed{a_r = \frac{V_r}{t_r} = \frac{L_r}{t_r^2}}$$

(iii) Dynamic Similarity :- It is the similarity of forces between Model and prototypes.

$$\frac{(F_v)_m}{(F_v)_p} = \frac{(F_\sigma)_M}{(F_\sigma)_p} = \frac{(F_P)_m}{(F_P)_p} = - - - - -$$

Note :- If all three similarity exists between the Model and prototype then we can say they are exactly similar.

Different Model Laws :-

1) Reynold's Model Law

$$(Re)_M = (Re)_p$$

2) Euler Model Laws

$$(Eu)_M = (Eu)_p$$

3) Froude Model Laws

$$(Fr)_M = (Fr)_p$$

4) Weber Model Laws

$$(Wb)_M = (Wb)_p$$

5) Mach Model Laws

$$(Ma)_M = (Ma)_p$$

Ques. A model is constructed on the basis of Reynold's model No. Calculate the velocity scale ratio and discharge scale ratio in terms of length scale ratio and kinematic viscosity ratio.

Ans.

$$(Re)_M = (Re)_P$$

$$\frac{V_M L_M}{\nu_m} = \frac{V_P L_P}{\nu_P} \Rightarrow \boxed{V_r = \frac{\nu_r}{L_r}}$$

Discharge scale ratio

$$Q_r = A_r V_r$$

$$Q_r = L_r^2 \cdot \frac{\nu_r}{L_r} \Rightarrow \boxed{Q_r = \nu_r L_r}$$

Ques A model is constructed on the basis of Reynold's Model law and Froude Model law. Find the velocity and Discharge scale ratio in ~~the~~ terms of length scale ratio.

Ans

$$(F_r)_M = (F_r)_P$$

$$\frac{V_M}{\sqrt{g L_M}} = \frac{V_P}{\sqrt{g L_P}} \Rightarrow \boxed{V_r = \sqrt{L_r}} = \frac{\nu_r}{L_r}$$

$$\nu_r = L_r^{3/2}$$

$$Q_r = \nu_r L_r^2$$

$$Q_r = L_r^{1/2} \cdot L_r^2 = L_r^{5/2} \Rightarrow \boxed{Q_r = L_r^{5/2}}$$

Ques

Prototype (ship) water	Model (Air)
$L_P = 300m$	$V_m = 30 m/s$
$\rho_P = 1030 kg/m^3$	$(F_D)_M = 60N$
$L_r = \frac{1}{100}$	$\rho_m = 1.24 kg/m^3$
$V_P = ?$	$\nu_M = 0.018 \times 10^{-4} m^2/s$
$(F_D)_P = ?$	
$\nu_P = (0.012) \times 10^{-4} m^2/s$	

Reynolds Model laws

$$\frac{V_M L_M}{\nu_M} = \frac{V_P L_P}{\nu_P}$$

$$V_r = \frac{\nu_r}{L_r} = \frac{(0.018/0.012)}{(1/100)} = \frac{V_M}{V_P} = \frac{30}{V_P}$$

$$\boxed{V_P = 0.2 m/s}$$

$$(F_D)_r = \mu_r V_r L_r$$

$$= \nu_r \rho_r V_r L_r$$

$$= \left(\frac{0.018}{0.012}\right) \times \frac{1.24}{1030} \times \frac{30}{0.2} \times \frac{1}{100} = \frac{(F_D)_M}{(F_D)_P} = \frac{60}{(F_D)_P}$$

$$(F_D)_P = 22150 N = 22.150 kN$$

Ques 11 Pg 45

Prototype Water	Model Air
$L_P = 1.5m$	$L_M = 0.3m$
$V_P = 3.5 m/s$	$V_M = 35 m/s$
$\rho_P = 998 kg/m^3$	$(\rho_{air})_M = ?$
$\mu_P = 1.0 \times 10^{-3}$ Pa-s	$(F_D)_M = 40N$
	$\mu_m = 1.90 \times 10^{-5}$ Pa-s

$\rho_{water} = 998 kg/m^3$ at std. atm pressure

$\rho_{Air} = 1.17 kg/m^3$ at std. atm pressure

$\mu_{Air} = 1.90 \times 10^{-5}$ Pa-s at local atm pressure

$\mu_{water} = 1.0 \times 10^{-3}$ Pa-s at local atm pressure

$$(R_e)_M = (R_e)_P$$

$$\frac{\rho_M V_M L_M}{\mu_M} = \frac{\rho_P V_P L_P}{\mu_P}$$

$$\frac{\rho_m \times 35 \times 0.30}{1.90 \times 10^{-5}} = \frac{998 \times 3.5 \times 1.5}{1.0 \times 10^{-3}}$$

$$\rho_m = 9.481 kg/m^3 = (\rho_{Air})_M$$

Air (Ideal gas)

$$P = \rho RT$$

$$P \propto \rho$$

$$\frac{P_1}{P_2} = \frac{\rho_1}{\rho_2} = \frac{1 atm}{P_2} = \frac{1.17}{9.481}$$

$$\boxed{P_2 = 8.1034 atm}$$

$$(P_{Air})_M = 8.1034 atm$$

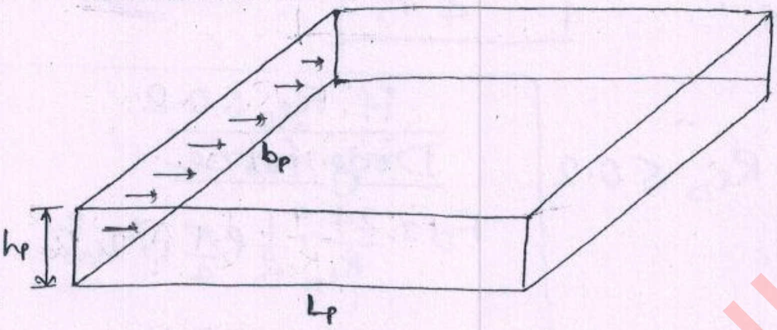
$$(F_D)_r = \mu_r V_r L_r$$

$$= \frac{1.90 \times 10^{-5}}{1.0 \times 10^{-3}} \times \frac{\cancel{350}^{\cancel{10}\,2}}{3.5} \times \frac{0.30}{1.50_{5}}$$

$$\frac{(F_D)_M}{(F_D)_P} = \frac{40}{(F_D)_P} = 1.90 \times 10^{-2} \times 2$$

$$\boxed{(F_D)_P = 1052.63 N}$$

Theory of Distorted Model :- Some times in the prototype as compare to horizontal dimensions vertical dimension are very less therefore in Model studies, Models are made by taking different scale ratios in Horizontal and vertical dimension. Such kind of models are not having Geometric similarity therefore they are known as distorted Models and these distorted Models are generally seen exaggerated on vertical scale.



$$\frac{L_M}{L_P} = \frac{b_m}{b_P} = (L_r)_H$$

Horizontal scale ratio

$$\frac{h_m}{h_p} = (L_r)_V = \text{Vertical scale ratio}$$

1) Area Scale Ratio

$$A_r = \frac{A_M}{A_P} = \left(\frac{b_M}{b_P}\right)\left(\frac{h_m}{h_P}\right) = (L_r)_H (L_r)_V$$

2) Velocity Scale Ratio

$$(F_r)_M = (F_r)_P$$

$$\frac{V_M}{\sqrt{g h_M}} = \frac{V_P}{\sqrt{g h_P}} \Rightarrow V_r = \sqrt{\frac{h_m}{h_p}} = \sqrt{(L_r)_V}$$

3) Discharge Scale ratio

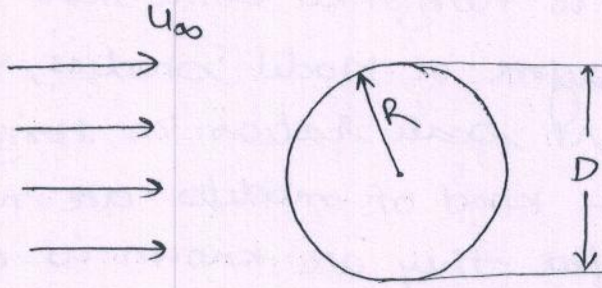
$$Q_r = A_r V_r$$

$$Q_r = (L_r)_H (L_r)_V \sqrt{(L_r)_V}$$

$$\boxed{Q_r = (L_r)_H (L_r)_V^{3/2}}$$

Drag force on sphere:-

Characteristic Dimension = D



$$Re_D = \frac{u_\infty D}{\nu}$$

$$F_D = C_D \cdot \frac{1}{2} \rho A u_\infty^2$$

$$\boxed{A = \frac{\pi}{4} D^2.}$$ projected Area

1. Stoke's Law

$$C_D = \frac{24}{Re_D} \quad \text{if } Re_D \le 0.2$$

if $Re_D \le 0.2$

Drag force

$$F_D = \frac{24}{Re_D} \cdot \frac{1}{2} \rho \frac{\pi}{4} D^2 u_\infty^2$$

$$F_D = 3 \nu \rho \pi D u_\infty$$

$$\boxed{F_D = 6 \pi \mu R u_\infty}$$

2. If $0.2 < Re_D \le 5$

$$\boxed{C_D = \frac{24}{Re_D}\left(1 + \frac{3}{16 Re_D}\right)}$$

3. If $5 < Re_D \le 1000$

$$\boxed{C_D = 0.4}$$

4. If $1000 \le Re_D < 1,00,000$

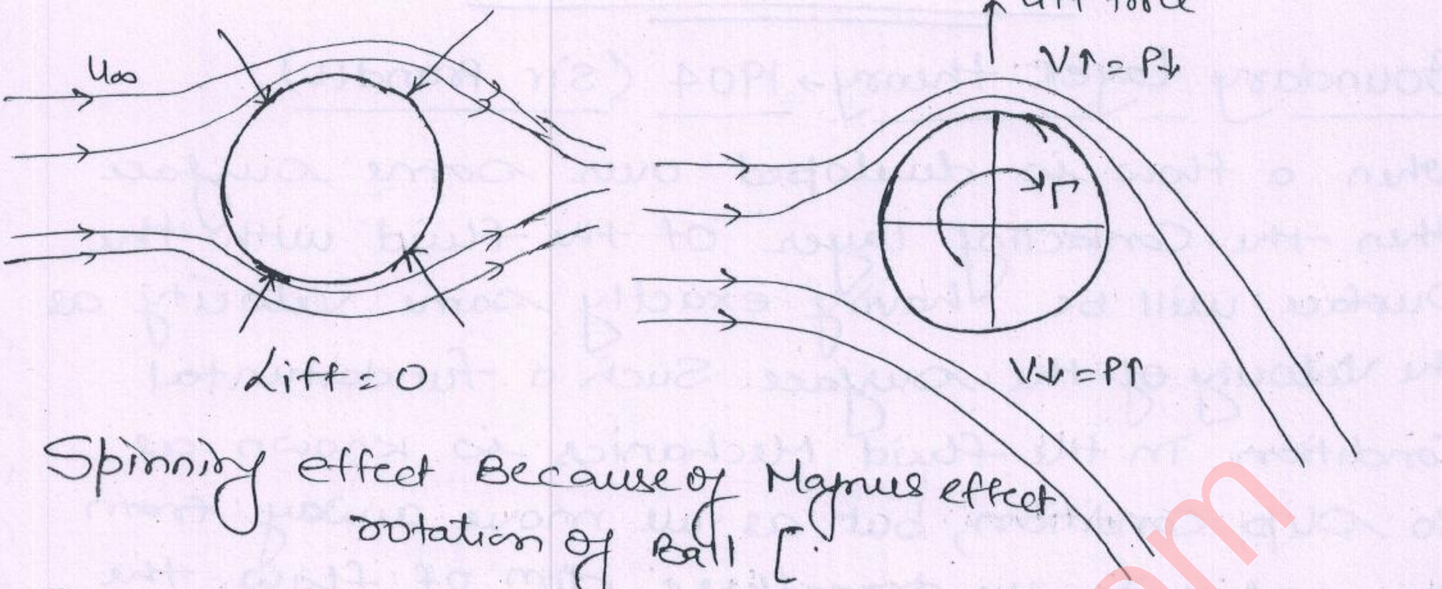
$$\boxed{C_D = 0.5}$$

5. If $Re_D > 1,00,000$

$$\boxed{C_D = 0.2}$$

Magnus Effect :- When a cylinder is rotated in a Uniform Stream, then there will be automatic generation of lift force. this effect was firstly seen by Magnus known as Magnus effect.

[Velocity of Air Not the velocity of Sphere or Ball]



Spinning effect Because of Magnus effect
rotation of Ball Γ

Swinging effect because of Surface roughness one side of Ball [Pace of Ball]

Googly effect because of Magnus + Surface roughness.
[Medium Pace]

Kutta - Juokowski.

$$\boxed{F_L = \rho u_\infty L \Gamma}$$ (only for cylinder).