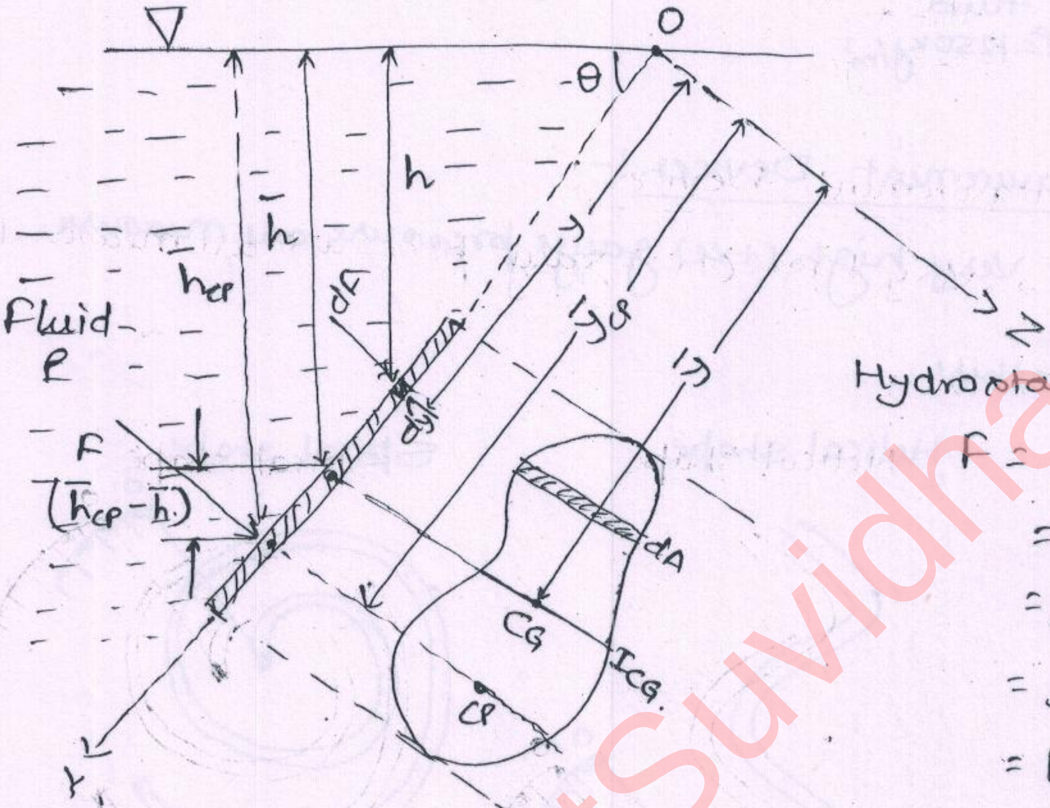
Fluid Statics

Hydrostatic forces on the plane Surfaces:-

$$\sin\theta = \frac{\bar{h}}{\bar{y}} = \frac{h}{y} = \frac{\bar{h}_{cp}}{\bar{y}_{cp}}$$

$$\bar{y} = \frac{\int y\,dA}{A}$$

$$I = \int y^2 dA$$



Hydrostatic force

$$F = \int dF$$
$$= \int P\,dA$$
$$= \int \rho g h\,dA$$
$$= \rho g \int y \sin\theta\,dA$$
$$= \rho g \sin\theta \int y\,dA$$
$$= \rho g \sin\theta(\bar{y}A)$$

$$\boxed{F = \rho g \bar{h} A}$$

this hydrostatic force is passing through a point known as Centre of pressure [C.P]

Location of CP

$$\int dF\,y = F\,\bar{Y}_{cp}$$

$$\rho g \sin\theta \int y^2 dA = \rho g \bar{h} A \frac{\bar{h}_{cp}}{\sin\theta}$$

$$\frac{\sin^2\theta}{\bar{h}A} I_{xx} = \bar{h}_{cp}$$

$$\frac{\sin^2\theta}{\bar{h}A}(I_{CG} + \bar{y}^2 A) = \bar{h}_{cp}$$

$$\bar{h}_{cp} = \frac{\sin^2\theta}{\bar{h}A}\left[I_{CG} + \frac{\bar{h}^2}{\sin^2\theta} A\right]$$

$$\boxed{\bar{h}_{cp} = \bar{h} + \frac{I_{CG}\sin^2\theta}{\bar{h}A}}$$

Note:- If surface is taken to more depth then $\bar{h}$ will ↑

$$(\bar{h}_{cp} - \bar{h}) = \frac{I_{CG}\sin^2\theta}{\bar{h}A} \Rightarrow \downarrow\downarrow$$

$$\Rightarrow (\bar{h}_{cp} - \bar{h}) \downarrow\downarrow$$

$\Rightarrow$ CP is shifting towards C.G.

For Horizontal Surface.

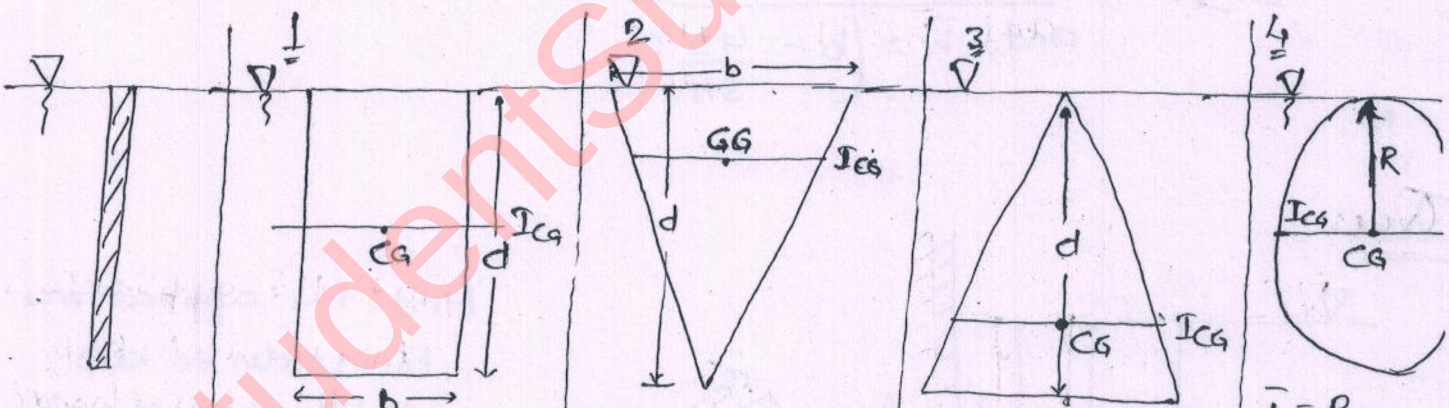
$$\theta = 0°$$

$$\bar{h}_{cp} = \bar{h}.$$

For Vertical Surfaces

$$\theta = 90°$$

$$\boxed{\bar{h}_{cp} = \bar{h} + \frac{I_{CG}}{\bar{h}A}}$$



Vertical Surface

1

$\bar{h} = \frac{d}{2}$

$A = bd$

$I_{CG} = \frac{bd^3}{12}$

$\bar{h}_{cp} = \frac{d}{2} + \frac{\frac{bd^3}{12}}{\frac{d}{2} \times bd}$

$\bar{h}_{cp} = \frac{d}{2} + \frac{d}{6}$

$\bar{h}_{cp} \frac{4d}{6}$

2

$\bar{h} = \frac{d}{3}$

$A = \frac{bd}{2}$

$I_{CG} = \frac{bd^3}{36}$

3

$\bar{h} = \frac{2d}{3}$

$A = \frac{bd}{2}$

$I_{CG} = \frac{bd^3}{36}$

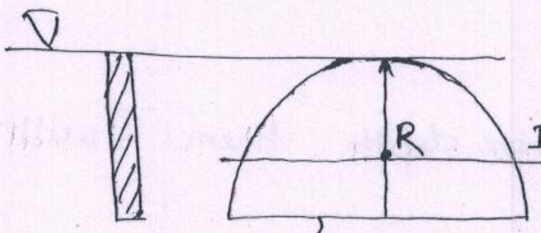
4

$\bar{h} = R$

$A = \pi R^2$

$I_{CG} = \frac{\pi R^4}{4}$

5



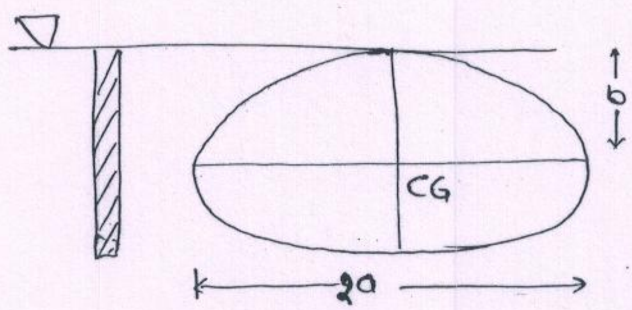
$$\bar{h} = \left(R - \frac{4R}{3\pi}\right)$$

$$A = \frac{\pi R^2}{2}$$

$$\boxed{I_{CG} + \left(\frac{4R}{3\pi}\right)^2 \frac{\pi R^2}{2} = \frac{\pi R^4}{8}}$$

$$I_{CG} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) R^4 = \underline{0.1097 R^4}.$$

6



$$\bar{h} = b.$$

$$A = \pi ab$$

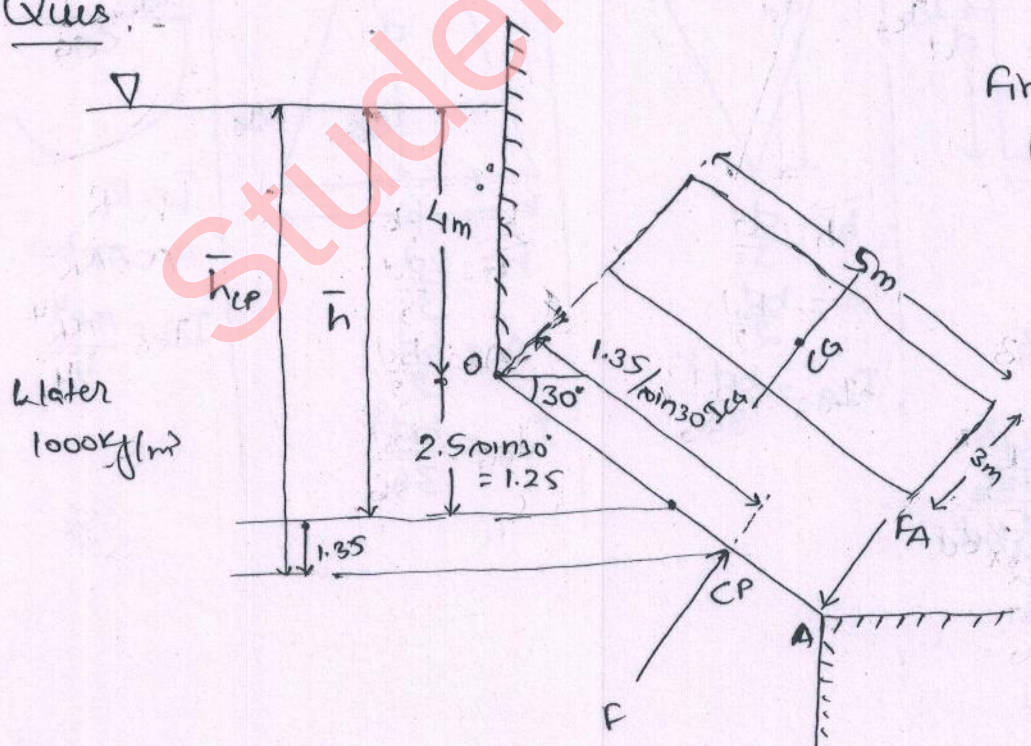
$$I_{CG} = \frac{\pi a b^3}{4}.$$

for <u>semi ecliptical disc</u>

Imp $\Longrightarrow$ $\boxed{I_{CG} = 0.1097\, a b^3}$

and; $\bar{h} = \left(b - \frac{4b}{3\pi}\right)$

Ques.



Find the applied force FA in order to keep this door closed?

$\bar{h} = 4 + 1.25$

$\bar{h} = 5.25$

$A = 5 \times 3 = 15 m^2$

$I_{CG} = \frac{3(5)^3}{12} m^4.$

Hydrostatic force

$$F = (1000)(9.81)(5.25)(15) \text{ N.}$$

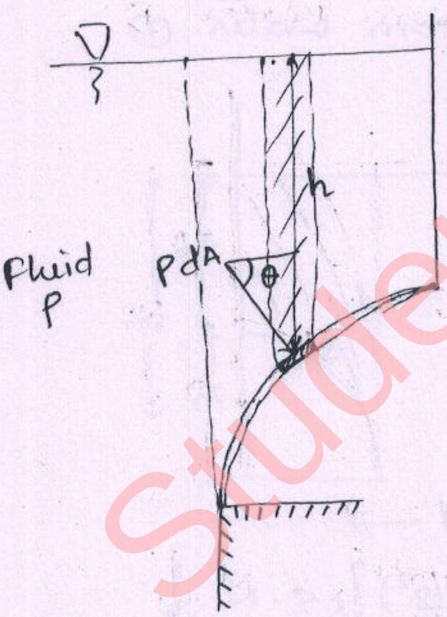
$$\bar{h}_{cp} = \bar{h} + \frac{I_{CG} \sin^2\theta}{\bar{h} A} = 5.35 \text{ meter}$$

— taking Moments about O.

$$F \times 2.7 = F_A \times 5$$

$$F_A = \frac{2.7}{5.0} F = \underline{417.170 \text{ kN}}.$$

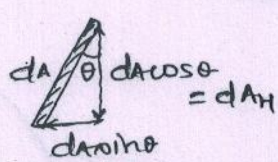
Hydrostatic forces on Curved Surfaces :-



Horizontal force

$$F_H = \int P dA \cos\theta$$

$$= \int \rho g h \, dA \cos\theta$$



$$= \int \rho g h \, dA_H.$$

↳ differential Projected Area in Horizontal dim

$$F_H = \rho g \bar{h} A_H$$

↓ Total Horizontal Projected Area.

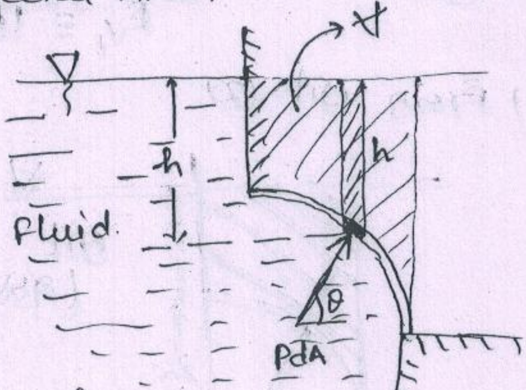
Vertical force.

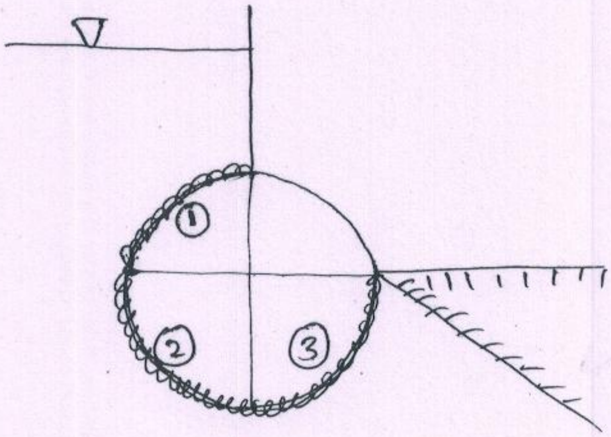
$$F_V = \int P dA \sin\theta$$

$$\int \rho g h \, dA \sin\theta$$

$$\int \rho g \, d\forall$$

$= \rho g \forall \rightarrow$ Volume above curved surface upto free surface





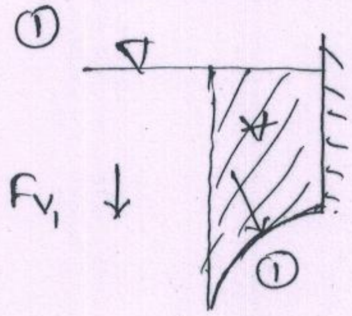
Horizontal force

① + ② $F_{H_1} \longrightarrow$

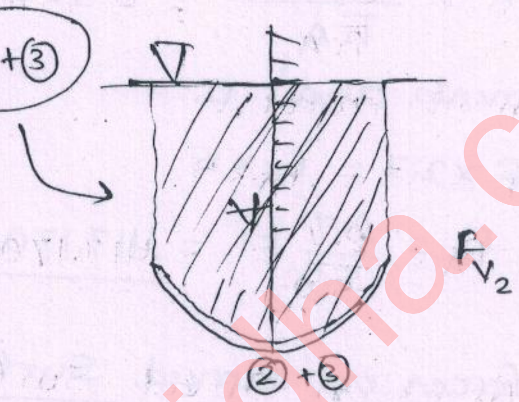
③ $F_{H_2} \longleftarrow$

Vertical force

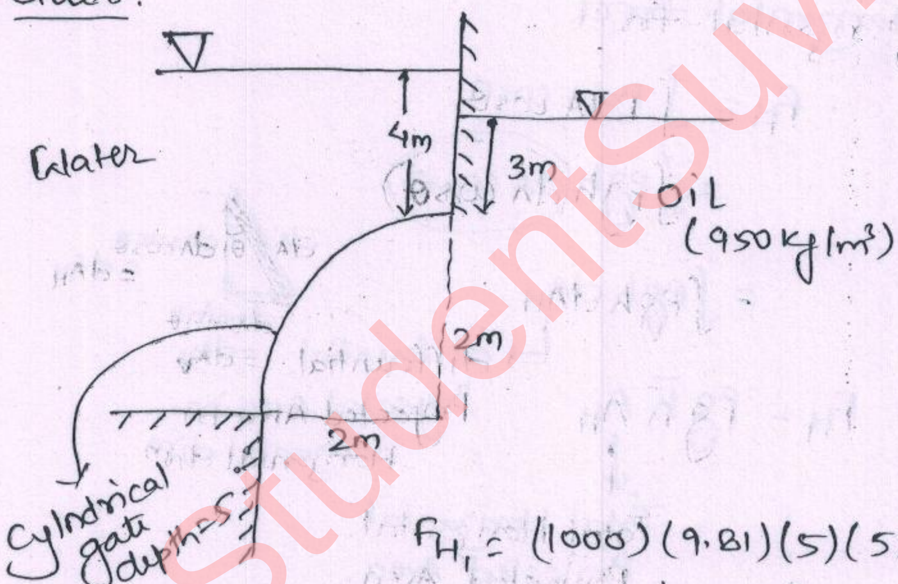
①



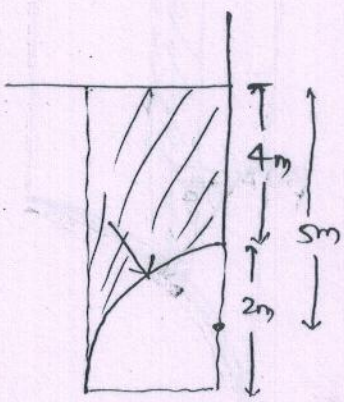
②+③



Ques.



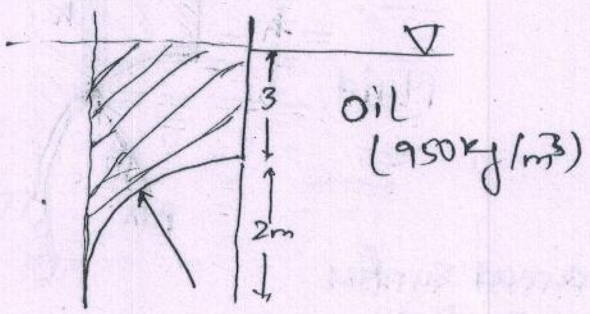
(i) From Water ①



$$F_{H_1} = (1000)(9.81)(5)(5\times 2)\ N \longrightarrow$$

$$F_{V_1} = 1000 \times (9.81)\left[(6\times 2) - \frac{\pi(2^2)}{4}\right]\times 5\ N \downarrow$$

(ii) From Oil (2)



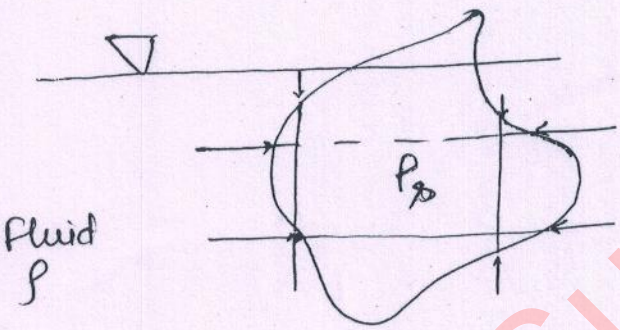
$$F_{H_2} = (950)(9.81)(4)(5\times 2)\ N \longleftarrow$$

$$F_{V_2} = 950 \times 9.81 \times \left[(5\times 2) - \frac{\pi 2^2}{4}\right]\times 5\ N \uparrow$$

Hydrostatic forces on the Bodies (~~Arkem~~ Archimidies Principal).

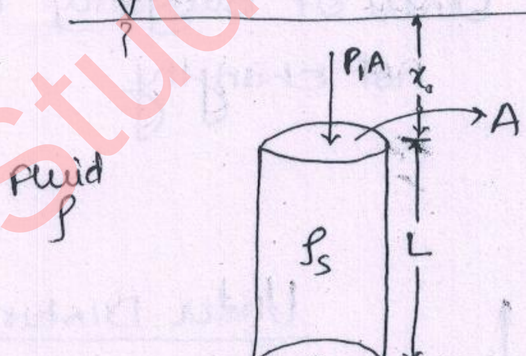
(Buoyency)

"When the bodies are submerged or immerxed fully or Partially in a static fluid then the resultant Hydrostatic force acts on the body in the Vertical upward dirn. This force is known as Buoyant Force or upthrust and the Value of this force is same as weight of the displaced fluid by the body. This force acts from a point on the body known as Centre of Buoyancy represented as 'B'.



Net Horizontal force on this body by the fluid = 0.

Resultant Hydrostatic force on the body is in Vertical upward dirn ⇒ upthrust (Buoyant force) F_B or F_{up}



Hydrostatic force on the body in upward dirn

$$F_B = (P_2A - P_1A)$$

$$F_B = A\left[\rho g(x+L) - \rho g x\right]$$

$$= \rho g (AL)$$

$$= \rho g V'_{body}$$

$$\boxed{F_B = \rho \forall g} \qquad \boxed{\forall = V'_{body}}$$

V'_{body} ⇒ Volume of body submerged $\forall$ = Volume of fluid displacement

$V'_{body} \leq V_{body}$

$$F_B = \rho (V'_{body}) g$$

$$= \rho \frac{m'}{\rho_s} g = \frac{m'g}{(\rho_s/\rho)}$$

$$\boxed{F_B = \frac{m'g}{RD}}$$

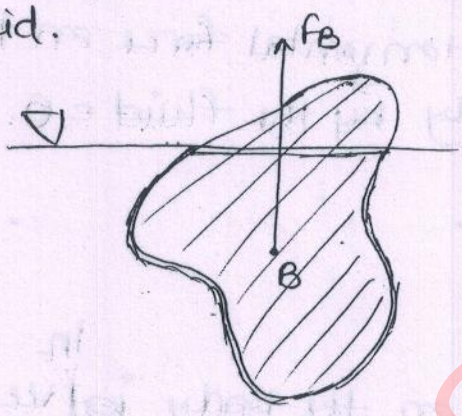
$m' \Rightarrow$ mass of body submerged

$m' \leq m$

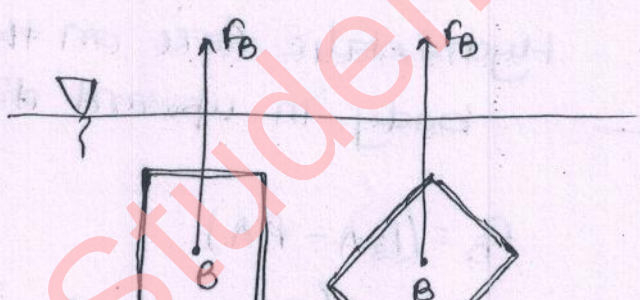
$RD \Rightarrow$ Relative density of Body w.r.t fluid

Centre of Buoyancy: $\rightarrow$ It is the point on the body from where Buoyant force acts.

This point is the centre of Gravity of the displaced fluid.



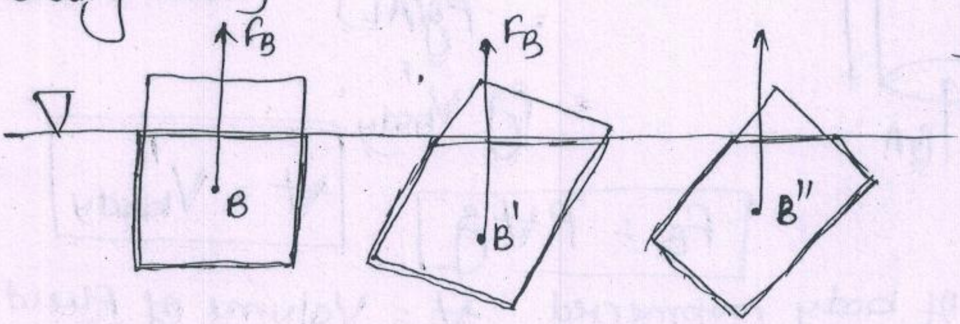
Submerged Body [totally Inside]



Under Disturbance

Centre of Buoyancy is not changing

Floating Body

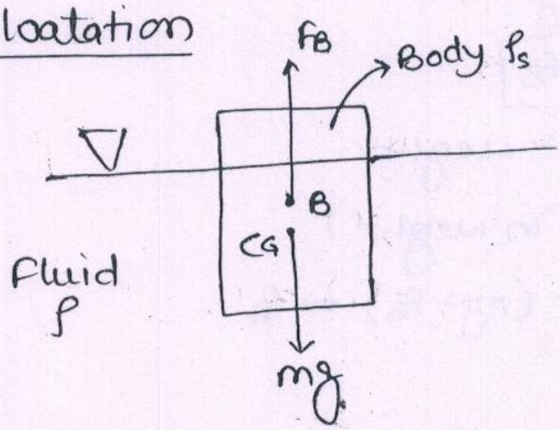


Under Disturbance

Centre of Buoyancy is changing.

Principal of Floatation :-

Floatation



$$\boxed{\rho_s \le \rho}$$

Body is floating

$$mg = F_B$$

$$mg = \frac{m'g}{RD}$$

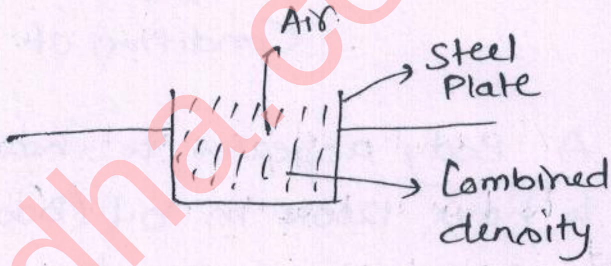
$$m' = m(RD)$$

$$m' \le m \Rightarrow RD \le 1$$

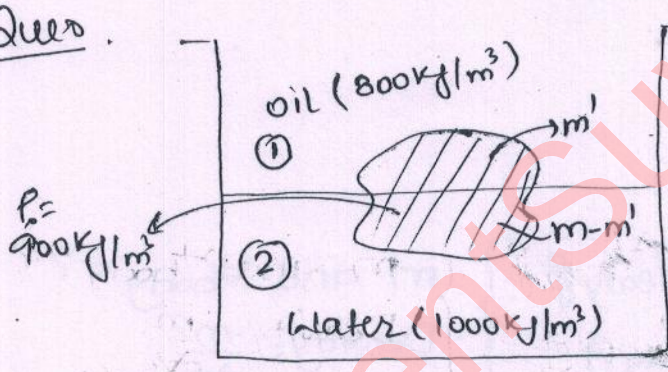
$$\frac{\rho_s}{\rho} \le 1 \Rightarrow \boxed{\rho_s \le \rho}$$

Condition of floatation.

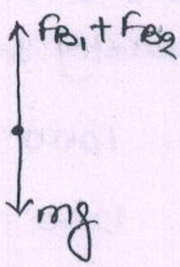
that's why steel plate float over the water.



Ques. What fraction of body is in oil and in water



Floatation



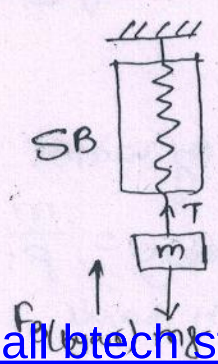
$$mg = F_{B_1} + F_{B_2}$$

$$mg = \frac{m'g}{\frac{900}{800}} + \frac{(m-m')g}{\frac{900}{1000}} \Rightarrow 2m' = m$$

$$\boxed{\frac{m'}{m} = \frac{1}{2}}$$

CONCEPT OF APPARANT WEIGHT :-

Real weight taken in air

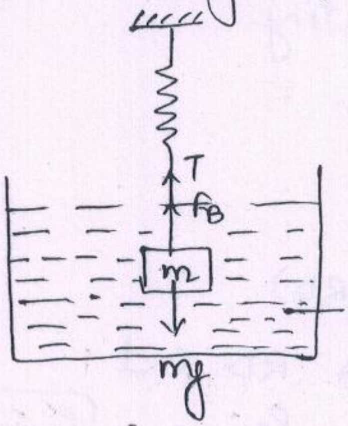


$$T + F_{B(\text{By Air})} = mg$$

$$T = mg - \cancel{F_{B(\text{by Air})}}^{\,0}$$

$$T = mg \text{ (Real Weight)}$$

Apparant Weight ($\rho_s > \rho$)



$$T + F_B = mg$$

$$\boxed{T = mg - F_B}$$

Apparant weight

Reduction in weight:

$mg - (mg - F_B) \Rightarrow F_B$.

Note if $\rho_s \le \rho$

$$mg = F_B$$

$$T = 0$$

Condition of Weightlessness.

Ex. A Body appears to have 1000N in water and appears to have 1200N in oil ($800 kg/m^3$).

Find (i) Real Weight of the body

(ii) Mass of Body

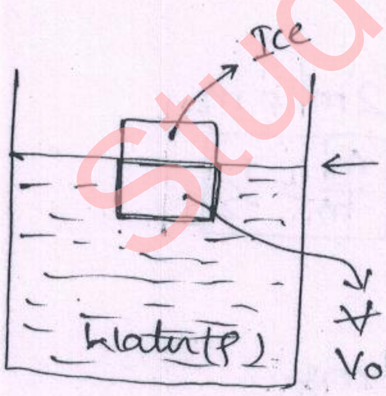
(iii) Volume of Body

(iv) Density of Body

So

$$\left.\begin{aligned} 1000 &= mg - 1000 \times \forall_{Body}\, g \\ 1200 &= mg - 800\, \forall_{Body}\, g \end{aligned}\right\} \quad m \text{ and } \forall_{body} = ? \quad \text{Density} = \frac{m}{\forall_{body}}$$

Ex



Floatation

$$mg = F_B$$

$$mg = \forall \rho g$$

$$\forall = \frac{m}{\rho}$$

When Ice is melted

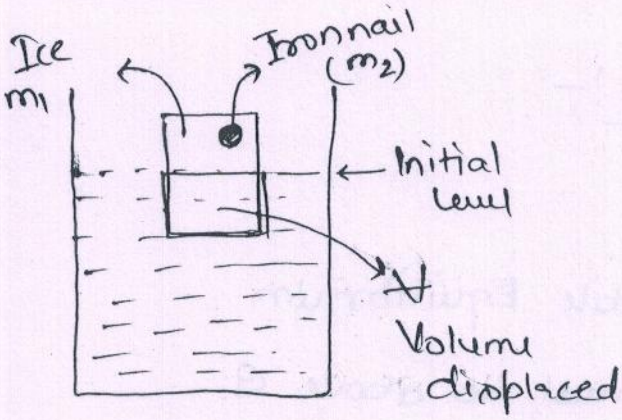
Ice $\longrightarrow$ Water

m $\longrightarrow$ (m) kg

Recovery of water

$$\forall_{recovery} = \frac{m}{\rho}$$

Level will remain same



Floatation

$$(m_1+m_2)g = F_B = \forall \rho g$$

$$\boxed{\forall = \frac{m_1}{\rho} + \frac{m_2}{\rho}}$$

Recovery ice → water ⇒ $\forall_{recovery} = \frac{m_1}{\rho}$

Nail → $\forall_{recovery} = \frac{m_2}{\rho_{Nail}}$ $\rho_{nail} >> \rho_{water}$

$$\left(\forall_{recovery} = \frac{m_1}{\rho} + \frac{m_2}{\rho_{nail}}\right) \downarrow\downarrow$$

Level will go down

Ex

Level of Boat will go up little bit

The ship enters from sea water to river water

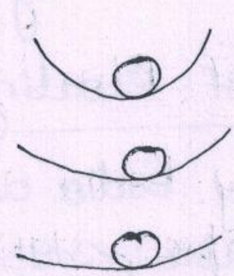
Shill will go down little bit.

$$\boxed{\rho_{sea} > \rho_{water}}$$

EQUILIBRIUM AND TYPES OF EQUILIBRIUM :-

If. $\vec{F}=0,\ \vec{M}=0$] EQ^m.

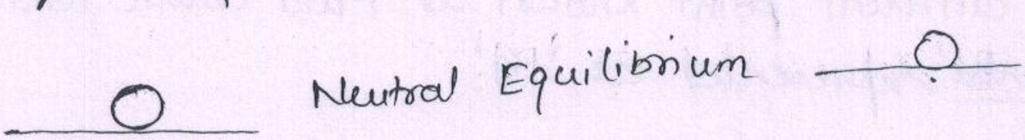
stable Equilibrium



Highly stable

stability is less

Unstable Equilibrium

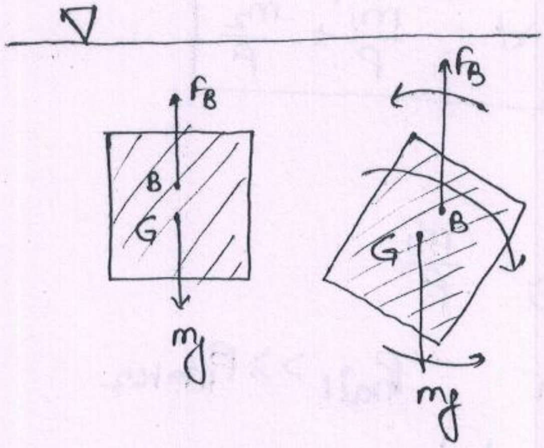


Neutral Equilibrium

Zero stability

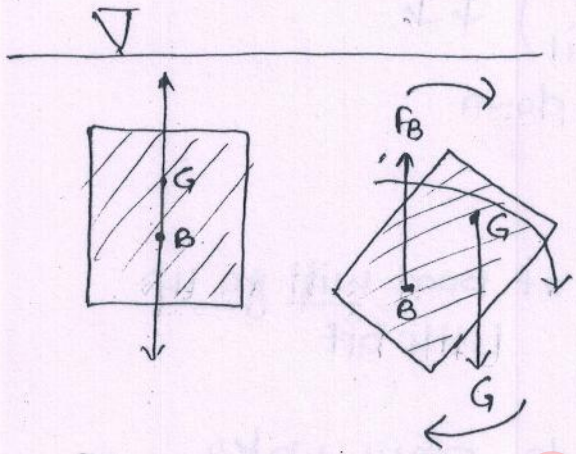
Neutral Equilibrium

Stability of Submerged Bodies :-



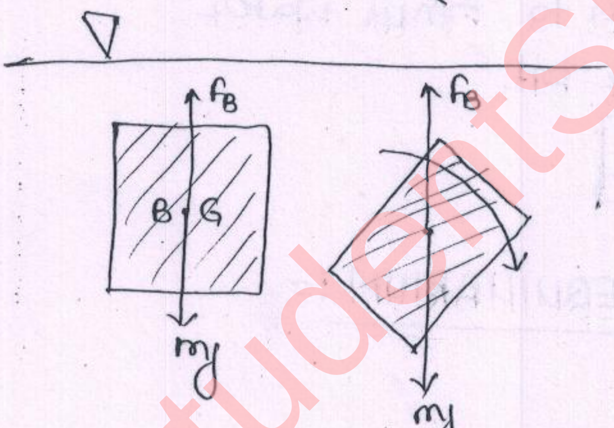
Stable Equilibrium

B must lie above G.



Unstable Equilibrium

B must lie below G.



Neutral Equilibrium

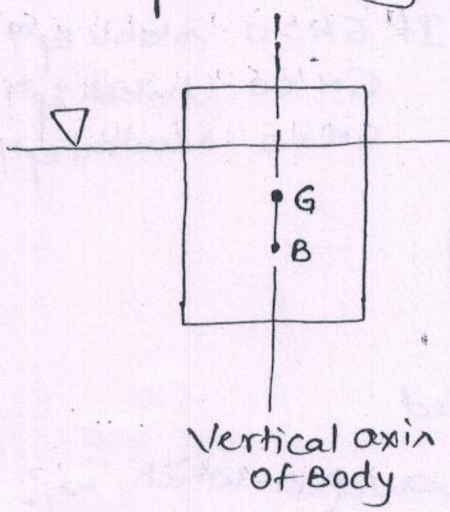
B, G both coincides

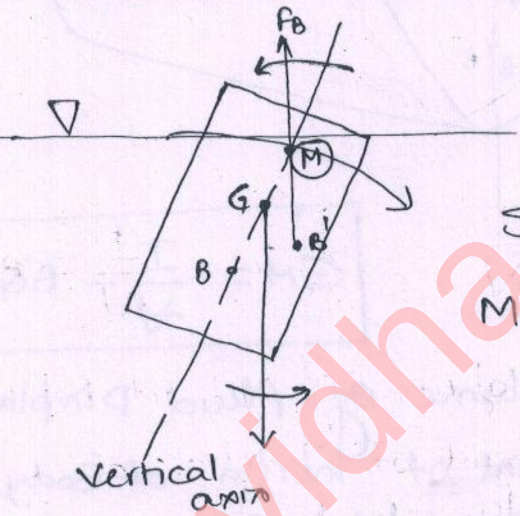
Stability of Floating Bodies :-

When floating bodies are disturbed their centre of buoyancy changes, therefore stabilities of floating bodies are analyzed through a different point known as 'Meta Centric Point' or 'Meta Centre' represented as 'M'.

It is also defined as a point about which bodies are ocillarating when they are disturbed from their position and released.

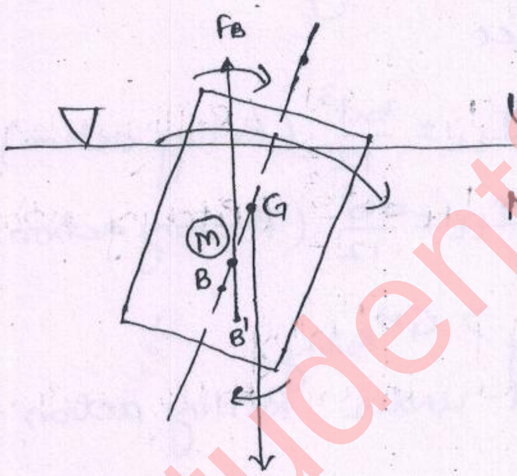
Meta Centric Point :- It is defined as the point of intersection of the vertical axis of the body with the Buoyant force line under small angular disturbance given to the body. This point may be inside or outside the body.





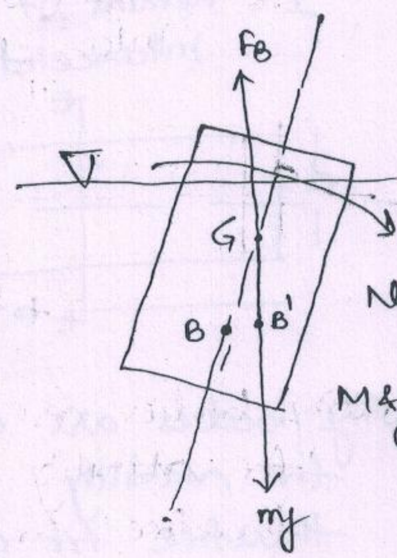
Stable Eq^m

M must lie above G.



Unstable Eq^m

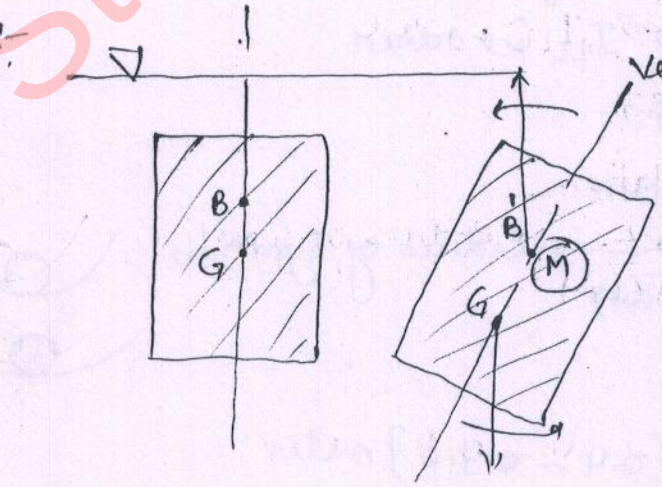
M lies below G.



Neutral Eq^m

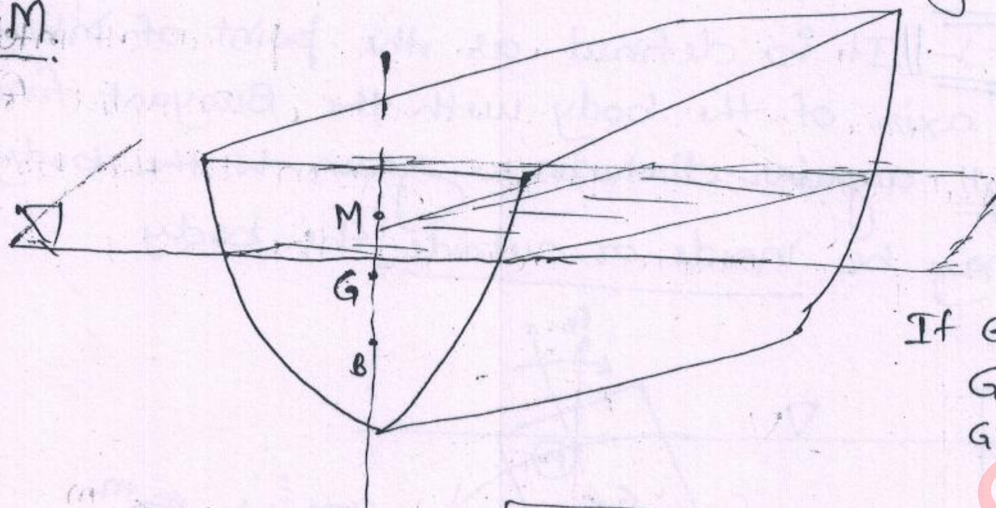
M & G both Coincides

Note:-



Ocillarate about Centre of Buoyancy

the distance between the Centre of gravity and Meta Centre is known as Meta Centric Height. represented as <u>GM</u>



GM = Metacentric Height

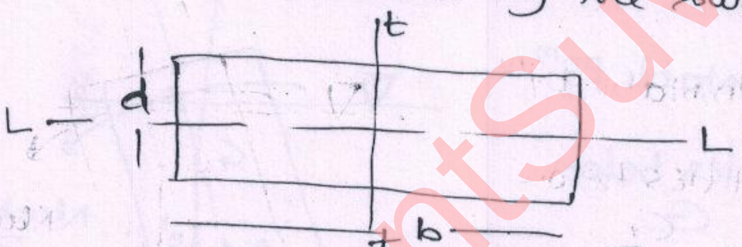
$GM = (BM - BG)$

If $GM > 0$ stable Eq^m
$GM < 0$ Unstable Eq^m
$GM = 0$ Neutral Eq^M

$BM = \frac{I}{\forall}$ $\boxed{GM = \frac{I}{\forall} - BG}$

$\forall$ = Volume of fluid Displaced

I = Moment of Inertia of Body surface which is intersected by free surface



$I_{LL} = \frac{bd^3}{12}$ (Rolling action)

$I_{tt} = \frac{db^3}{12}$ (Pitching action)

$GM_{Pitching} > GM_{Rolling}$

Floating bodies are always designed under Rolling action for stability

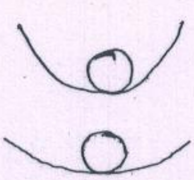
therefore for ~~day~~ design

$I_{min} \Rightarrow I_{LL}$ is taken

$GM = \frac{I_{LL}}{\forall} - BG$

Time period of oscillation

$T = 2\pi\sqrt{\frac{k^2}{g(GM)}}$ $k \rightarrow$ Radius of Gyration

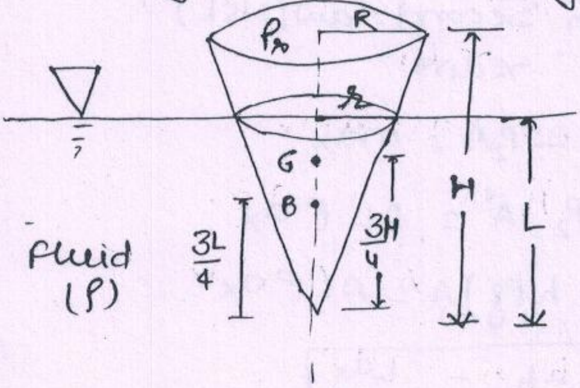


$T \propto \frac{1}{\sqrt{GM}}$

Passenger ships $\rightarrow GM \in [0.4 - 1.0]$ meter

War ships $\longrightarrow GM \in [1 - 1.5]$ meter

Ques:- A Cone having a maximum radius R and height H density ρ_o is floating in a fluid of density ρ with its axis vertical and apex down. Find the Condition for the stability of this Floating Cone.



Floatation

$$mg = F_B$$

$$\frac{1}{3}\pi R^2 H \rho_o g = \forall \rho g$$

$$\frac{1}{3}\pi R^2 H \rho_o g = \frac{1}{3}\pi r^2 L \rho g$$

$$r^2 L = R^2 H\left(\frac{\rho_o}{\rho}\right) \qquad \boxed{RD = \frac{\rho_o}{\rho}}$$

$$r^2 L = R^2 H (RD)$$

and $\frac{R}{H} = \frac{r}{L} \Rightarrow r = \frac{R}{H} L$

$$\frac{R^2}{H^2} L^3 = R^2 H (RD) \Rightarrow \boxed{L = H(RD)^{1/3}}$$

and $\boxed{r = R(RD)^{1/3}}$

Now $BG = \frac{3H}{4} - \frac{3L}{4} = \frac{3}{4}\left[H - H(RD)^{1/3}\right] = \frac{3H}{4}\left[1 - (RD)^{1/3}\right]$

$$BM = \frac{I}{\forall} = \frac{\frac{\pi r^4}{4}}{\frac{1}{3}\pi r^2 L} = \frac{3r^2}{4L}$$

$$= \frac{3R^2 (RD)^{2/3}}{4H(RD)^{1/3}} = \frac{3}{4}\frac{R^2}{H}(RD)^{1/3}$$

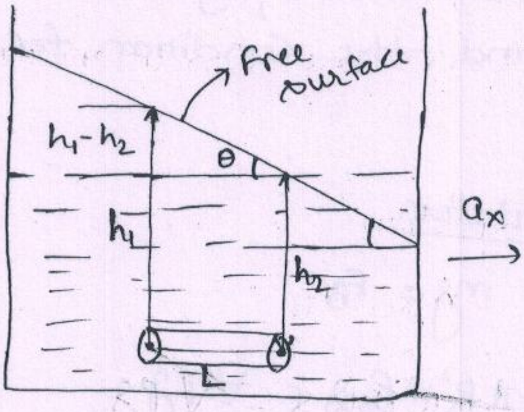
For stability $GM = BM - BG > 0$

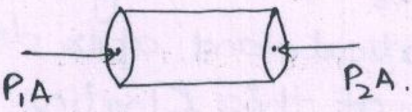
$$BM > BG$$

$$\frac{3}{4}\frac{R^2}{H}(RD)^{1/3} > \frac{3}{4}H\left[1 - (RD)^{1/3}\right]$$

$$\frac{R^2}{H^2} > \frac{1 - (RD)^{1/3}}{(RD)^{1/3}} \Rightarrow \boxed{\frac{R}{H} > \sqrt{\frac{1 - RD^{1/3}}{RD^{1/3}}}}$$

Concept of Accelarated Vessels Containing fluid





Neuton Second law (NSL)
x dim

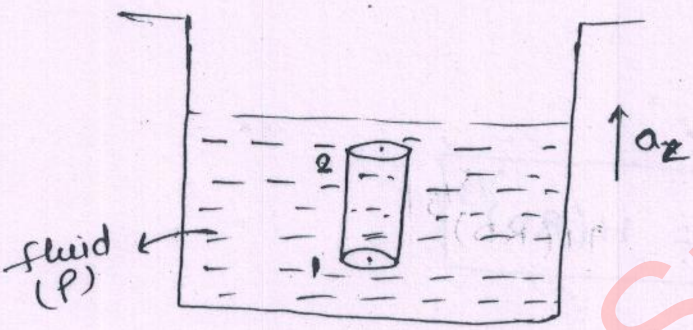
$$P_1A - P_2A = ma_x$$

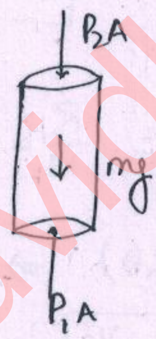
$$(P_1 - P_2)A = AL\rho a_x$$

$$(h_1\rho g - h_2\rho g)A = AL\rho a_x$$

$$\boxed{h_1 - h_2 = \frac{La_x}{g}}$$

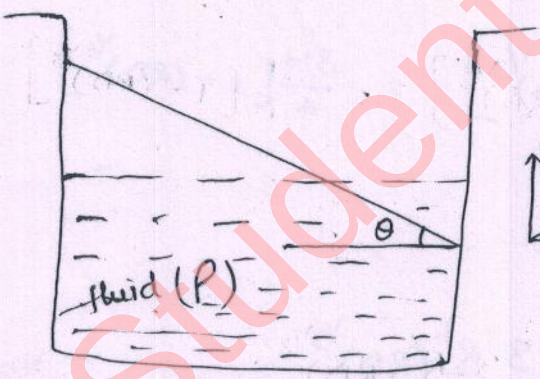
$$\boxed{\frac{h_1 - h_2}{L} = \frac{a_x}{g} = \tan\theta}$$

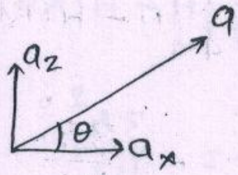




$$P_1A - P_2A - mg = ma_z$$

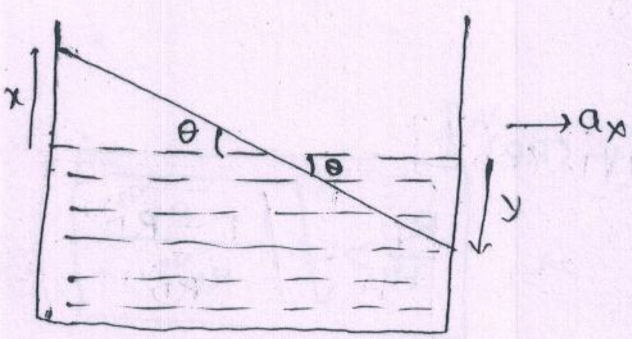
$$(P_1 - P_2)A = m(g + a_z)$$





$$\boxed{\tan\theta = \frac{a_x}{g + a_z}}$$

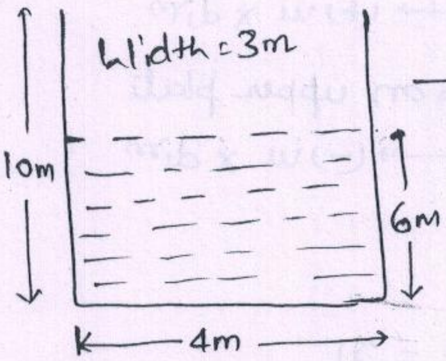
Conservation of Volume



Conservation of volume

$$\boxed{x = y}$$

Ques.

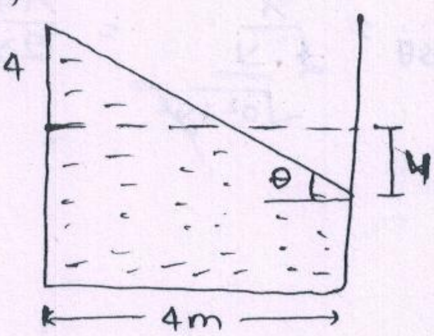


(Ques 1(i)) What max^m acc^n a_x can be there such that water does not spil out from Vessal

(ii) If $a_x = 20 m/s^2$ what volume of water has been spilled out from the Vessal.

(iii) If $a_x = 30 m/s^2$ $\forall_{spilled} = ??$

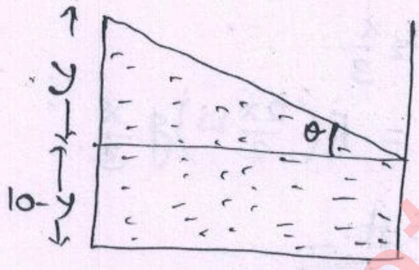
Ans (i)



$$\tan\theta = \frac{a_x}{g} = \frac{8}{4}$$

$$(a_x)_{max} = 2g = 19.62 m/s^2$$

(ii) If $a_x = 20 m/s^2$



$$\tan\theta = \frac{a_x}{g} = \frac{20}{9.81} = 2.0387 = \frac{y}{4}.$$

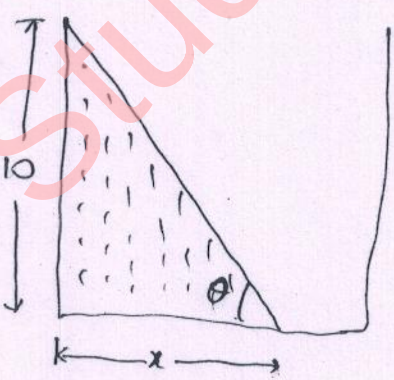
$$y = 8.1549 \text{ meter.}$$

$$V_f = \left[\frac{y4}{2} + (10-y)4\right] \times 3$$

$$V_i = 4 \times 6 \times 3$$

so $V_{spilled} = V_i - V_f$

(iii)



$$\tan\theta = \frac{a_x}{g} = \frac{30}{9.81} = 3.038 = \frac{10}{x}$$

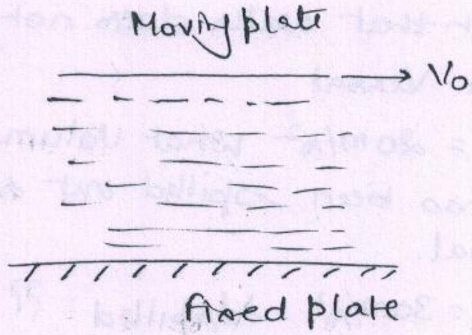
$$x = \frac{10}{3.038} = 3.27 \text{ meter}$$

$$V_f = \left(\frac{1}{2} \times 10 \times 3.27\right) \times 3.$$

$$V_i = 6 \times 4 \times 3$$

so $V_{spilled} = V_i - V_f$

Ques



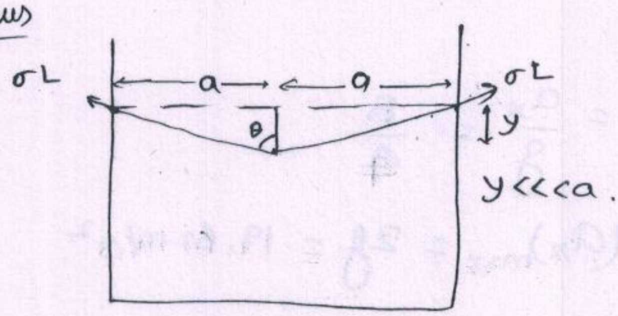
Stress on lower plate

$\longrightarrow$ (+) ve x dirn

Shear stress on upper plate

$\longrightarrow$ (−) ve x dirn

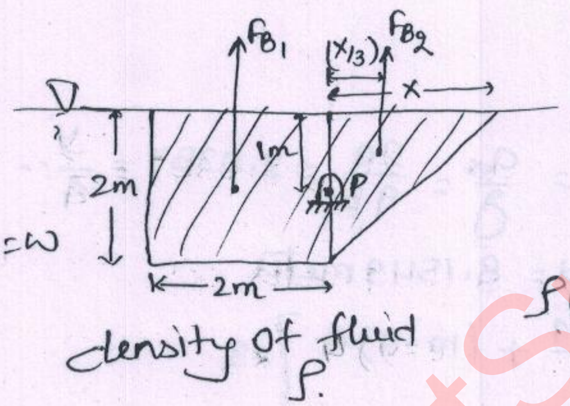
Ques



$$2\sigma L\cos\theta = \lambda L$$

$$\sigma = \frac{\lambda}{2\cos\theta} = \frac{\lambda}{2\frac{y}{\sqrt{a^2+y^2}}} = \frac{\lambda a}{2y}.$$

Ques



About P.

$$F_{B_1}\times 1 = F_{B_2}\frac{x}{3}$$

$$\rho(2\times2\times w)g\times 1 = \rho\left(\frac{2x}{2}w\right)g\frac{x}{3}.$$

$$\frac{x^2}{3} = 4.$$

$$\boxed{x = 2\sqrt{3}.}$$