KINEMATICS OF FLUID FLOW

Study of fluid flow without the consideration of basic cause of flow ie force.

1) Lagrangian Approach :- This approach is particle concentration approach that is entire concentration goes to a particular fluid particle and its motion is analysed.
After completing the flow study of one particle, the concentration goes to another particle and its motion is analysed. Since the No of particles even in a small system are very very large therefore the time consumed in this study is very high. Hence we donot prefer this approach for the kinematical study of fluid flow in classical fluid Mechanics.

2) Eulerian Approach :- This approach is space concentration approach that is entire concentration goes to particular space or zone and all the fluid particles crossing through that space are analysed as a Bulk simultaneously and on an average their motions are analysed in 1 stroke. therefore the results obtained by Eulerian approach are not correct particle by particle but on as average as a bulk flow these results are 100% correct results. the time consumption using this approach is very less Hence we prefer this approach for the kinematical study of fluid flow in classical fluid Mechanics.

Different types of flows in fluid flow system :-

1) Steady and Unsteady flows :- when the properties in the fluid flow are not changing with respect to time then such a flow is known as steady flow

R → fluid property

$$\left.\frac{\partial R}{\partial t}\right|_{space} = 0 \longrightarrow \text{steady flow}$$

2). Uniform flow and Non Uniform flow :- If the properties in a flow are not changing with respect to space then such a flow is known as Uniform flow.

$$\left.\frac{\partial R}{\partial \text{space}}\right|_{\text{time}} = 0.$$

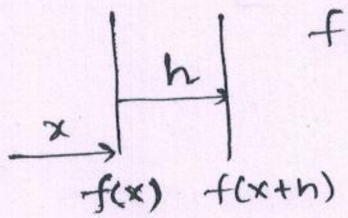
3). Incompressible flow and Compressible flow :- If the density of a fluid in a flow is not changing with respect to pressure, then such a flow is known as incompressible flow.

4) Irrotational flow and rotational flows :- When the fluid particles are moving in their stream lines if the particles are also rotating w.r.to their own centre of masses then such a flow is known as rotational flow. and if these ~~are~~ particles are not rotating with respect to their own centre of masses then Such a flow is known as irrotational flow.

5) Laminar and Turbulent Flow :- If all the fluid particles lying in a plane are having their velocity component in the same dirn, then these particles will form the layer and there will not be any kind of interjumping of fluid particles b/w the adjacent layers such a well organised flow of fluid particles in the laminated ~~flow~~ form of a layer is known as laminar flow. this flow is also known as Stream line flow.

If all the particles lying in a plane are having their velocity components ~~is~~ in different-different dirn then these particles will intermix between the adjacent layers such a most chaotic flow of the fluid particle is known as turbulent flow (chaotic).

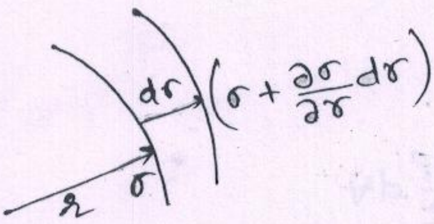
Taylor's Contribution

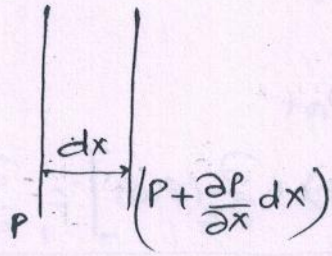


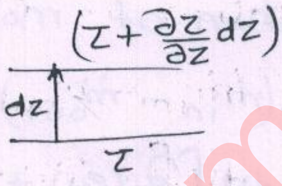
$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \dots\dots$

if h is very small $\longrightarrow$ Neglect.

taylor series approx. upto I^{st} order







If a function f is function of many variables

$f = fn(x, y, z)$

total change = sum of partial changes

$$\boxed{df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz}$$

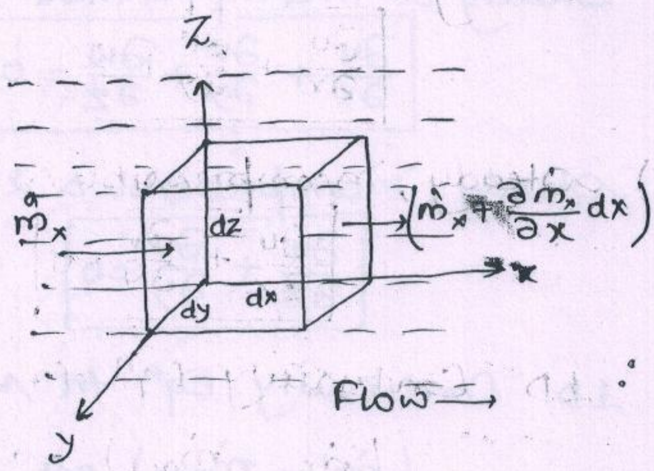
Conservation of Mass in Fluid Flow Systems:-

(Continuity Equation).

In general $\vec{V} = ui + vj + wk$

u, v, w] velocity components in x, y, z dim $\longrightarrow fn(x, y, z, t)$

$\vec{V} = fn(x, y, z, t)$



Conservation of Mass.

$\dot{m}_{in} - \dot{m}_{out} = \dot{m}_{st}$

$(\dot{m}_{in} - \dot{m}_{out})_x = \dot{m}_x - (\dot{m}_x + \frac{\partial \dot{m}_x}{\partial x}dx)$

$= -\frac{\partial \dot{m}_x}{\partial x}dx = -\frac{\partial}{\partial x}[\rho(dydz)u]dx$

$(\dot{m}_{in} - \dot{m}_{out})_x = -\frac{\partial}{\partial x}(\rho.u)d\forall$

$(\dot{m}_{in} - \dot{m}_{out})_y = -\frac{\partial}{\partial y}(\rho.v)d\forall$

$(\dot{m}_{in} - \dot{m}_{out})_z = -\frac{\partial}{\partial z}(\rho.w)d\forall$

$$(\dot{m}_{in} - \dot{m}_{out}) = -d\forall\left[\frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w\right]$$

$$\dot{m}_{st} = \frac{\partial}{\partial t} m_{st} = \frac{\partial}{\partial t}(\rho d\forall)$$

$$\dot{m}_{st} = d\forall \frac{\partial \rho}{\partial t}$$

Conservation of mass.

$$(\dot{m}_{in} - \dot{m}_{out}) = \dot{m}_{st}$$

$$-d\forall\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] = \frac{\partial \rho}{\partial t} d\forall$$

$$\boxed{\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0}$$

Most General Continuity Eq^n in 3D flow.

Assumption in flow

1) Steady flow $\frac{\partial \rho}{\partial t} = 0$

$$\frac{\partial (\rho u)}{\partial x} + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0.$$

2) Steady & incompressible flow.

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

3) Steady, incompressible & 2D flow.

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

1D Continuity Eq^n for steady flow

$$(\dot{m}_{in} - \dot{m}_{out}) = 0$$

$$\dot{m}_x - \left(\dot{m}_x + \frac{\partial \dot{m}_x}{\partial x} dx\right) = 0$$

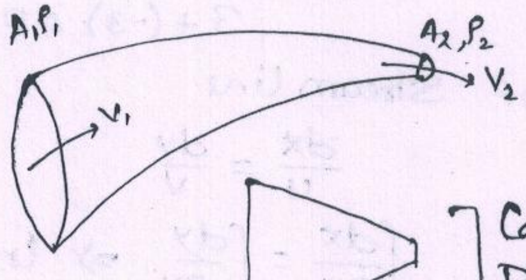
$$\frac{\partial \dot{m}_x}{\partial x} = 0$$

$$\frac{\partial}{\partial x}(\rho (dydz).u) = 0$$

$$\frac{\partial}{\partial x}(\rho.A.u) = 0 \Rightarrow \frac{\partial}{\partial x}(\rho AV) = 0$$

$$\rho AV = \text{Const}$$

$$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$$

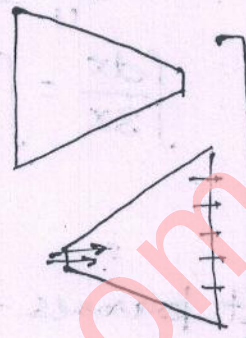


$$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$$

If flow is incompressible

$\rho = \text{Const}$

$$\boxed{A_1 V_1 = A_2 V_2}$$

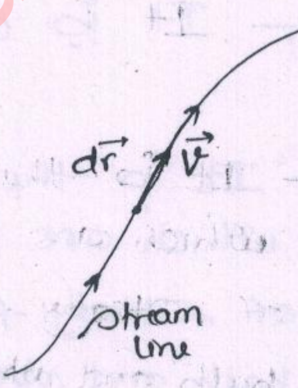


Converging Diverging Nozzles as used in fighter planes.

Stream Line :- It is an imaginary line drawn in a flow field in such a way such that tangent drawn at any point in this line directly represents the dirn of the velocity vector of this particle at that point.

Eqn of stream line →

taking differential ~~eqn~~ position vector along stream line



$$d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$d\vec{r} \times \vec{V} = 0$$

$$\begin{vmatrix} i & j & k \\ dx & dy & dz \\ u & v & w \end{vmatrix} = 0 \qquad \hat{i}(w\,dy - v\,dz) + \hat{j}(w\,dx - u\,dz) + \hat{k}(v\,dx - u\,dy) = 0$$

$$\frac{dy}{v} = \frac{dz}{w} \qquad \frac{dx}{u} = \frac{dz}{w} \qquad \frac{dx}{u} = \frac{dy}{v}$$

Eqn of stream line in differential form

$$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$$

Ques The 2D steady, incompressible flow field is given by $\vec{V} = 3x\,\hat{i} - 3y\,\hat{j}$. Find the eqn of stream line passing through (1,2).

Ans $u = 3x$ $\quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$v = -3y$

$3 + (-3) = 0$ Flow is possible

Eq^n of Stream line

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\int \frac{dx}{3x} = \int \frac{dy}{-3y} \Rightarrow \ln x = -\ln y + \ln c.$$

$$\ln(xy) = \ln c.$$

$$\boxed{xy = c}$$

It passes through (1,2)

so $\boxed{c = 2}$

Eq^n of streamline

is $xy = 2$ Rectangular Hyperbola.

$$\boxed{y = \frac{2}{x}}$$

Path Line :- It is an actual path ~~traced~~ traced by fluid particle.

Streak line :- It is the locus of all fluid particles at a moment which are crossing from the same point

In the case of steady flow, mathematically all three lines (stream, path and streak) look same.

Accelaration of fluid particles :-

$$V = u\hat{i} + v\hat{j} + w\hat{k}$$

$$\left.\begin{matrix} u \\ v \\ w \end{matrix}\right\} \rightarrow fn(x, y, z, t)$$

$$\vec{V} = fn(x, y, z, t)$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial x}\frac{dx}{dt} + \frac{\partial \vec{V}}{\partial y}\frac{dy}{dt} + \frac{\partial \vec{V}}{\partial z}\frac{dz}{dt} + \frac{\partial \vec{V}}{\partial t}\frac{dt}{dt}$$

$$\vec{a} = \underbrace{u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}}_{\text{Convective Accelaration}} + \underbrace{\frac{\partial \vec{V}}{\partial t}}_{\text{Temporal / Local Accelaration}}$$

$$a_x = \underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}}_{\text{Convective}} + \underbrace{\frac{\partial u}{\partial t}}$$

$$a_y = \underbrace{u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}}_{\text{Convective}} + \underbrace{\frac{\partial v}{\partial t}}$$

$$a_z = \underbrace{u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}}_{\text{Convective}} + \underbrace{\frac{\partial w}{\partial t}}$$

Local/ Temporal Accelaration

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} \text{ Units.}$$

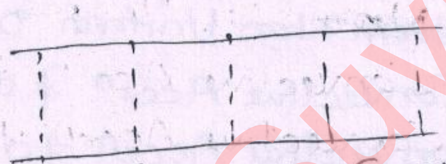
Uniform flow

Convective ~~flow~~ Acce^n = 0

Steady flow

Local Accelaration = 0

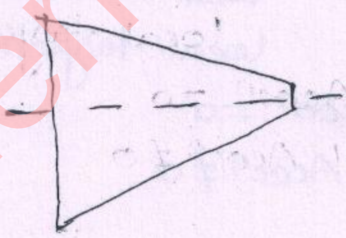
Liquid flow in Uniform Dia in straight pipe



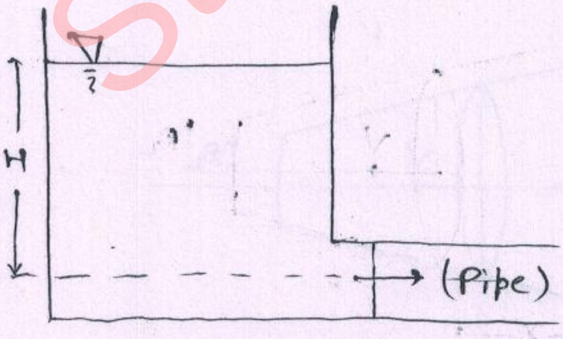
Uniform flow ⇒ Convective Accelaration = 0

Liquid flow in Non Uniform Dia in straight pipe

Non-Uniform



Convective Accelaration ≠ 0.



Uniform Dia straight pipe

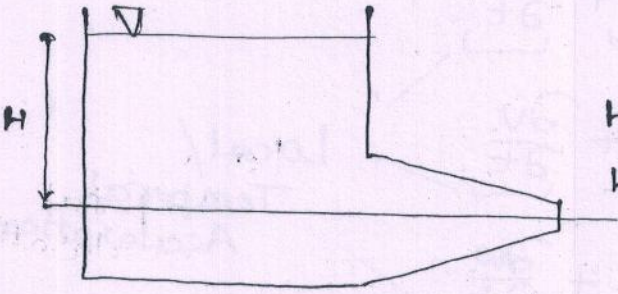
Convective Accelaration = 0

H = Const (steady)

Local Accelaration = 0

H = Variable (Unsteady)

Local Accelaration ≠ 0

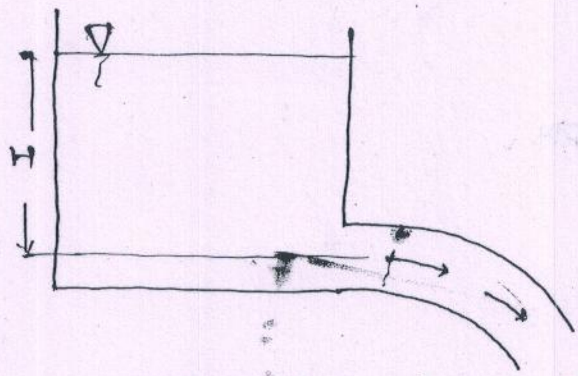


Non Uniform Dia straight pipe

Convective Accelaration $\neq 0$

H const → Local Accelabation $= 0$

H Variable ⇒ Local Accelaration $\neq 0$



Uniform Dia. Curved pipe

Convective Accen $\neq 0$ (Normal)

Convective Accen $= 0$ (tangential)

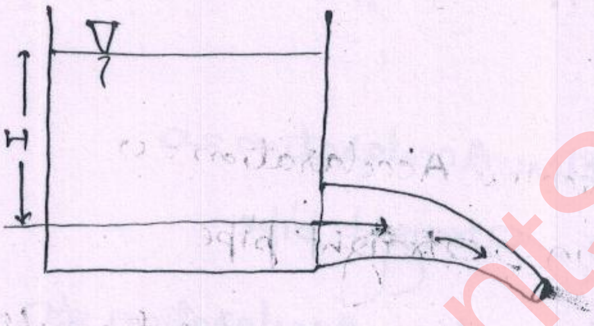
H = const

local Normal Accen $= 0$

Local tangential Accen $= 0$

H = Variable   local Normal Accen $= 0$

local tangential Accen $\neq 0$



~~Non~~ Non Uniform Dia Curved pipe

Convective Accen $\neq 0$ (Normal)

Convective Accen $\neq 0$ (tangential)

H = const

local Normal Accen $= 0$

local tangential Accen $= 0$

H = Variable

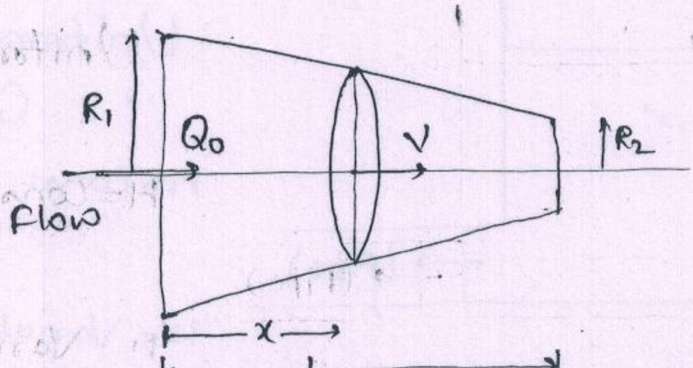
local Normal Accen $= 0$

local tangential Accen $\neq 0$

Ques

Q_0 = Const Dischage

Find the accen in flow at the exit of Converging tube.



$a_{exit} = ?$

Ans Q_0 = const so steady flow.

$$a = v\frac{\partial v}{\partial x}$$

$$Q_0 = \pi r^2 v$$

$$v = \frac{Q_0}{\pi r^2}$$

$$\frac{R_1 - R_2}{L} = \frac{R_1 - r}{x}$$

$$r = R_1 - \left(\frac{R_1 - R_2}{L}\right)x$$

$$r = R_1 - Ax \qquad \boxed{A = \frac{R_1 - R_2}{L}}$$

$$v = \frac{Q_0}{\pi (R_1 - Ax)^2}$$

$$a = \frac{Q_0}{\pi (R_1 - Ax)^2} \cdot \frac{Q_0}{\pi}\frac{\partial}{\partial x}\left\{\frac{1}{(R_1 - Ax)^2}\right\}$$

$$a = \frac{2AQ_0^2}{\pi^2 (R_1 - Ax)^5}$$

at exit put $x = L$

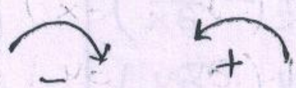
$$a_{exit} = \frac{2AQ_0^2}{\pi^2 (R_1 - AL)^5} \qquad A = \frac{R_1 - R_2}{L}$$

$$\boxed{a_{exit} = \frac{2AQ_0^2}{\pi^2 R_2^5}}$$

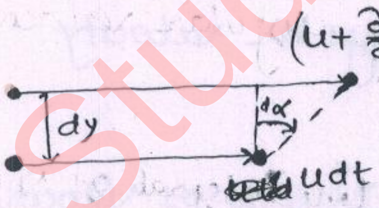
Rotational Components in a flow

2D Flow

$$\vec{V} = u\hat{i} + v\hat{j}$$



u share in Rotation

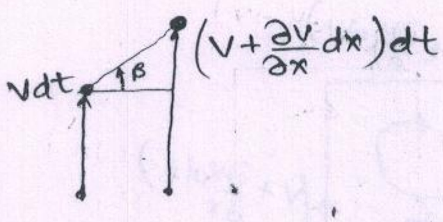


$$\tan d\alpha = -\frac{\partial u}{\partial y}\frac{dy\,dt}{dy}$$

$$\frac{d\alpha}{dt} = -\frac{\partial u}{\partial y}$$

V share in Rotation

$$\tan d\beta = \frac{\partial v}{\partial x}\frac{dx\,dt}{dx}$$



$$\frac{\partial \beta}{\partial t} = \frac{\partial v}{\partial x}$$

Angular velocity of fluid particle w.r.t its own centre of mass

$$\omega_z = \frac{\frac{d\beta}{dt} + \frac{d\alpha}{dt}}{2}$$

$$\boxed{\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)}$$

If $\omega_z = 0 \Rightarrow$ Irrotation flow

$\omega_z \neq 0 \Rightarrow$ rotational flow

In General 3D flows,

$$\vec{\omega} = \frac{1}{2}(\nabla \times \vec{V})$$

$$\vec{\omega} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$\vec{\omega} = \frac{1}{2}\left[\hat{i}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) - \hat{j}\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right) + \hat{k}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\right]$$

$$\left.\begin{aligned} \omega_x &= \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \\ \omega_y &= \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \\ \omega_z &= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \end{aligned}\right]$$

If $\omega_x = \omega_y = \omega_z = 0$

$\Rightarrow \vec{\omega} = 0$

Flow is irrotational.

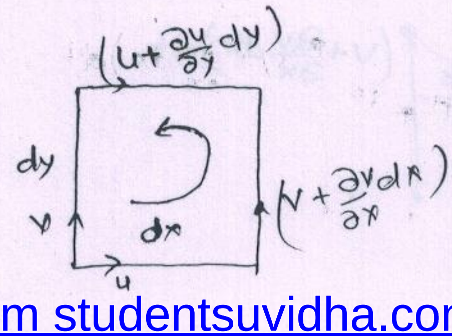
Vorticity :- It is defined as twice of angular velocity

Vorticity $= 2\omega$

Circulation :- (Γ) :- It is defined as the line integral of velocity vector taking along a closed loop

$$\boxed{\Gamma = \oint \vec{V} \cdot d\vec{r}}$$

2D flow



$$\Gamma = \oint \vec{V}\cdot d\vec{r} = u\,dx + \left(v + \frac{\partial v}{\partial x}dx\right)dy - \left(u + \frac{\partial u}{\partial y}dy\right)dx - v\,dy$$

$$= \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)dx\,dy$$

$$\boxed{\Gamma = \text{Vorticity} \times \text{Area}}$$

Ex Circle $r = 2$

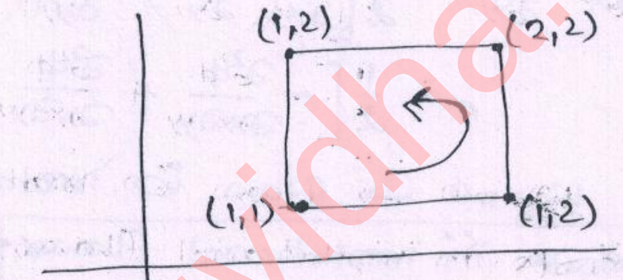
$u = 2x + 3y \qquad v = -2y$

Vorticity $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 - 3 = (-3)$

$\Gamma = (-3)\pi r^2 = -12\pi.$

Ex $u = x^2$

$v = 2xy$



$$\boxed{\begin{aligned} v|_{x=2} &= 4y \\ v|_{x=1} &= 2y \end{aligned}}$$

$$\Gamma = \oint \vec{V}\cdot d\vec{r} = \int_1^2 x^2 dx + \int_1^2 4y\,dy + \int_2^1 x^2 dx + \int_2^1 2y\,dy.$$

$= 3$ Units.

Velocity potential Function (ϕ):-

"In general this is the function defined as a function of space and time in such a way such that the negative derivative of this function w.r.t space directl. give velocity in that dirn.

$$\left.\begin{aligned} u &= -\frac{\partial \phi}{\partial x} \\ v &= -\frac{\partial \phi}{\partial y} \\ w &= -\frac{\partial \phi}{\partial z} \end{aligned}\right]$$ By Definition

Boundation on ϕ.

2D steady Incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$+\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right) = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$\boxed{\nabla^2 \phi = 0.}$$ Laplace Eq^n.

Laplace operator

ϕ must satisfy laplace Eq^n.

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}\left[\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial x}\right)\right]$$

$$= \frac{1}{2}\left[-\frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial x \partial y}\right] = 0.$$

$\omega_z = 0 \rightarrow$ flow is irrotational.

ϕ only exists in irrotational flows. $\Rightarrow$ physical significance.

Equipotential lines :- (ϕ = const line). :- It's a line joining the points having same potential function values.

In Irrotational, 2D steady, Incompressible flow

$$\phi = f(x, y)$$

$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy$$

$$d\phi = (-u\,dx - v\,dy)$$

For Equipotential line

$$\phi = \text{Const}$$

$$d\phi = 0$$

$$-u\,dx - v\,dy = 0$$

$$v\,dy = -u\,dx$$

$$\boxed{\frac{dy}{dx} = -\frac{u}{v}}$$ → Slope of Equipotential line

→ After integration Eq^n of Equipotential line

Stream function (ψ)

In general this function is defined in 2D as the function of space and time in such a way such that continuity Eq^n is satisfied and flow is possible.

$$u = -\frac{\partial \psi}{\partial y}$$

$$v = +\frac{\partial \psi}{\partial x}$$

Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

$$= \frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial x}\right)$$

$$= 0$$

Continuity Eq^n is automatically satisfied.
flow is possible.

ψ exists in both rotational and irrotational flows.

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= \frac{1}{2}\left(\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial y}\right)\right)$$

$$= \frac{1}{2}\left[\nabla^2 \psi\right]$$

If ψ satisfy laplace eq^n $\nabla^2\psi = 0 \Rightarrow \omega_z = 0$
Irrotational flow.

If ψ ~~satisfy~~ does not satisfies laplace eq^n
$\nabla^2\psi \neq 0$ $\omega_z \neq 0$ Ratational flow

Equi-stream function line (ψ const line)

It is the line joining the points in a flow which are having same stream function value.

In 2D steady, Incompressible flow

$$\psi = f(x,y)$$

$$d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy$$

$$\partial \psi = v\,dx - u\,dy$$

For the Equi stream function line

$\Rightarrow \psi = \text{Const.}$

$d\psi = 0$

$v\,dx - u\,dy = 0$

$$\frac{dy}{dx} = \frac{v}{u} \Rightarrow \boxed{\frac{dy}{v} = \frac{dx}{u}} \text{ stream line}$$

Equi-stream line are stream lines.

$$\left(\frac{dy}{dx}\right)_{\substack{\phi = \text{const}\\ \text{line}}} \times \left(\frac{dy}{dx}\right)_{\substack{\psi = \text{const}\\ \text{line}}} = -1.$$

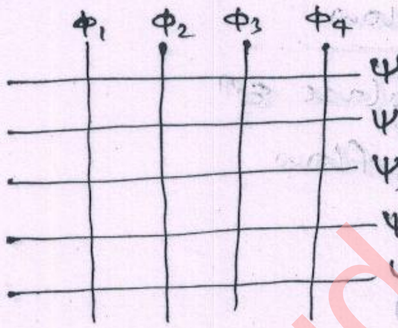
In Irrotational flow

Equipotential line & stream lines both are orthogonal to each other.

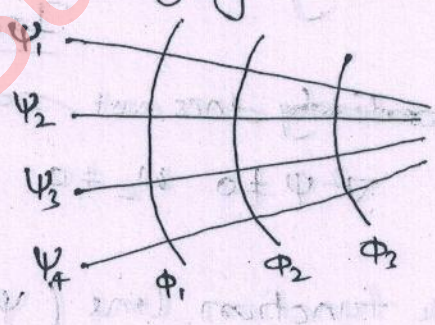
The graphical representation of equi-potential line & stream lines in an irrotational flows is a diagram known as "Flow Net"

In Irrotational Flows.

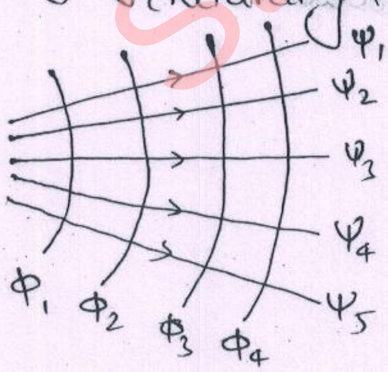
Uniform flow



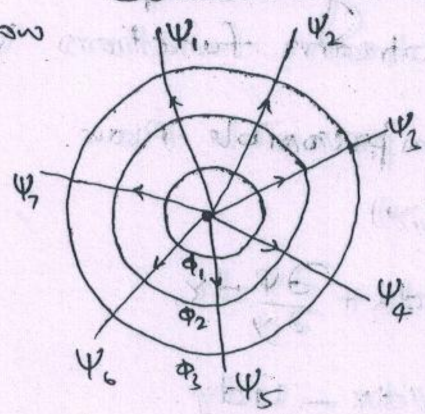
Converging Flow (Accelarated flow)



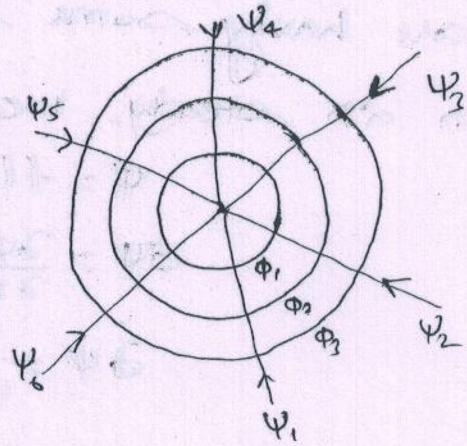
Diverging flow (Retarding flow)



Source flow

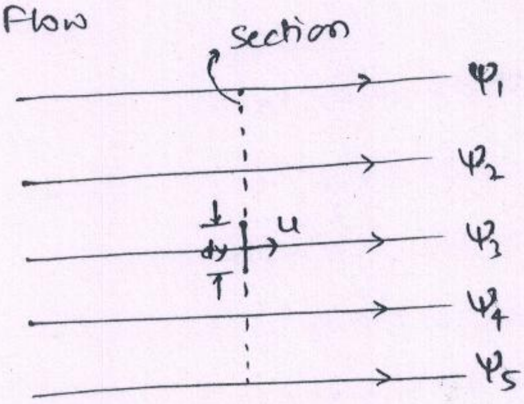


Sink flow



Practical Utilization of Stream function:-

Flow



$$\psi = f(x, y)$$

$$d\psi = \frac{\partial \psi}{\partial x}\cancel{dx}^{\,0} + \frac{\partial \psi}{\partial y}dy$$

$$d\psi = \frac{\partial \psi}{\partial y}dy = -u\,dy$$

$$-d\psi = u\,dy = u \times (dy \times 1)$$

$$-\int d\psi = \int dQ_{\text{Per unit width of flow}}$$

$$\boxed{\Delta\psi = Q_{\text{Per unit width of flow}}}$$

Cauchy - Riemann Eq^n

In Irrotational flows

$$\boxed{\begin{aligned} u &= -\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial y} \\ v &= -\frac{\partial \phi}{\partial y} = +\frac{\partial \psi}{\partial x} \end{aligned}}$$

Ques the flow field of 2D flow is given as

$$\vec{V} = 3xy\,\hat{i} + \left(\frac{3}{2}x^2 - \frac{3}{2}y^2\right)\hat{j}$$

find the relevent potential and stream function for this flow.

Ans $u = 3xy \quad , \quad v = \frac{3}{2}x^2 - \frac{3}{2}y^2$

Continuity $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \Rightarrow 3y + (-3y) = 0$

Flow is possible.

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(3x - 3x) = 0$$

Flow is irrotational

ϕ will exist.

$$u = -\frac{\partial \phi}{\partial x} \Rightarrow \int \partial\phi = -\int 3xy\,\partial x$$

$$\phi = -\frac{3}{2}x^2y + f(y) + c \quad \text{— (1)}$$

↳ There can be the pure function of y

$$v = -\frac{\partial \phi}{\partial y} \Rightarrow \int \partial\phi = \int\left(\frac{3}{2}y^2 - \frac{3}{2}x^2\right)\partial y$$

$$\phi = \frac{y^3}{2} - \frac{3}{2}x^2y + f(x) + c \quad \text{— (2)}$$

↳ There can be pure function of x

By (1) & (2)

$$\boxed{\phi = -\frac{3}{2}x^2y + \frac{y^3}{2} + c}$$ Ans.

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