INTRODUCTION

Effect of temperature on the Viscosity of fluid :-

Basic reasons of Viscosity → Cohesive forces b/w the molecule [cohesion]

$$(\text{cohesion})_{gas} <<<< (\text{cohesion})_{liquid}$$

$$\mu_{gas} <<<< \mu_{liquid}$$

Liquids.

If $temp_{liq} \uparrow \Rightarrow$ Cohesion ↓↓
$\Rightarrow \mu_{liq} \downarrow\downarrow$

$\nu_{liq} = \frac{\mu_{liq}}{\rho_{liq}}$ If $T_{liq} \uparrow \Rightarrow \rho_{liq} \downarrow$ (very slightly)

If $T_{liq} \uparrow \Rightarrow \mu_{liq} \downarrow$ as well as $\nu_{liq} \downarrow$

but ↓ in μ_{liq} is more as compared to ↓ in ν_{liq}

Gases [cohesion ⟶ 0 (Nil)]

If $T_{gas} \uparrow \Rightarrow \bar{C}_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow \bar{C}_{rms} \propto \sqrt{T}$ ↑↑

Randomness Will ↑↑

This will introduced an additional resistance in the path of fluid flow $\Rightarrow \mu_{gas} \uparrow\uparrow$

$\nu_{gas} = \frac{\mu_{gas}}{\rho_{gas}}$ If $T \uparrow = \rho_{gas} \downarrow\downarrow$

$P = \rho RT \Rightarrow \rho = \frac{P}{RT}$ $\rho \propto \frac{1}{T}$

When $T_{gas} \uparrow \Rightarrow \mu_{gas} \uparrow$ & $\nu_{gas} \uparrow\uparrow$

the ↑ in μ_{gas} is less than ν_{gas} ↑

Effect of pressure on Viscosity

If P ↑

→ μ_{liq} → No change (Incompressible)

— ν_{liq} → No change

- $\mu_{gas} \Rightarrow \overline{C_{rms}} = \sqrt{\frac{3RT}{M}}$ = No change

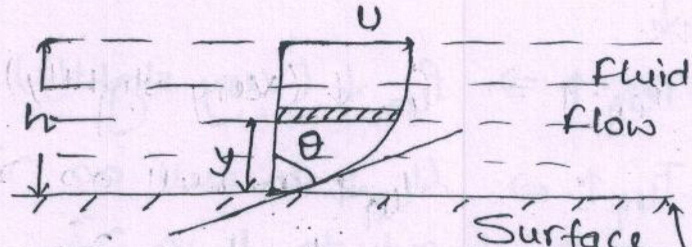
Randomness → No change

$\therefore \mu_{gas}$ → No change

- $\nu_{gas} = \frac{\mu_{gas}}{\rho_{gas}}$ $(P = \rho RT)$ $P \propto \rho$

$\nu_{gas} \downarrow\downarrow$

Linearization of Newton's law of Viscosity:-



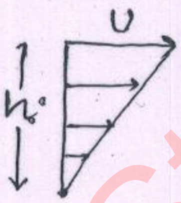
Newton's law of Viscosity

$$\tau = \mu \frac{dU}{dy}$$

Surface Shear Stress.

$$\tau_{(y=0)} = \tau_0 = \mu\left(\frac{\partial u}{\partial y}\right)\Big|_{y=0}$$

If h → very very small of the order of mm.
the velocity profile can be treated as straight line

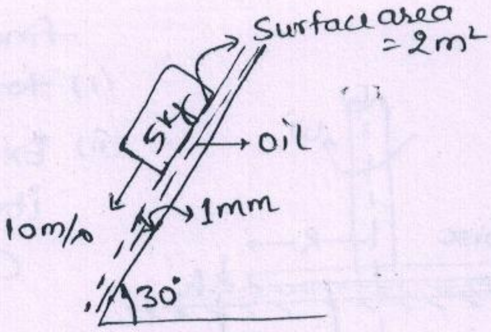


$$\frac{dU}{dy}\Big|_{y=0} = \frac{U-0}{h}$$ ← Surface velocity

$$\tau_0 = \mu\left(\frac{U-0}{h}\right) = \frac{\mu U}{h}$$

$$F_{drag} = \tau_0 A = \frac{\mu U A}{h}$$ A = Surface Area of plate

Ques 1)

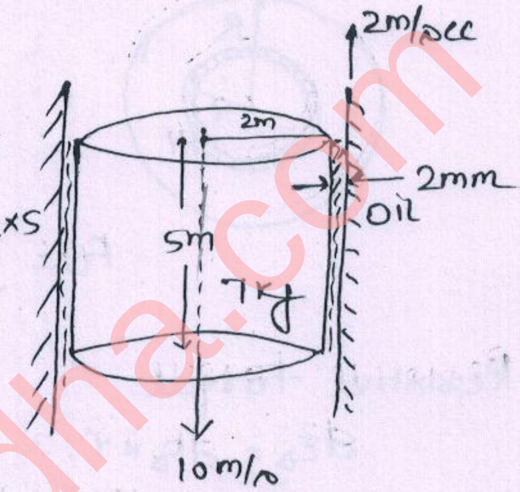


$mg\sin\theta = F_{Drag}$

$$5\times 9.81 \sin 30^\circ = \frac{\mu(10-0)}{1\times 10^{-3}}\times 2$$

$\mu = ?$

Ques 2)

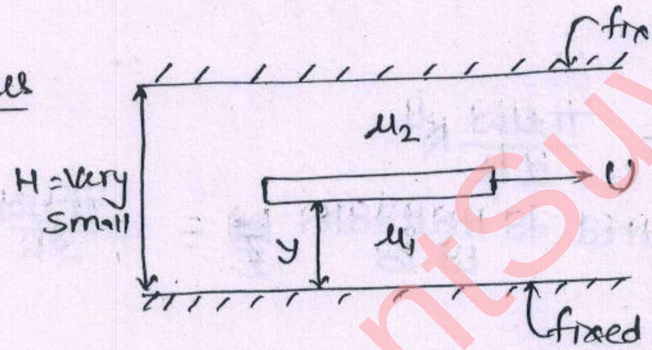


$mg = F_{Drag}$

$$7\times 9.81 = \frac{\mu\left(10-(-2)\right)}{2\times 10^{-3}}\times 2\pi\times 2\times 5$$

$$\mu = \frac{7\times 9.81\times 2\times 10^{-3}}{12\times 2\pi\times 10}$$

Ques



find y such that
(i) Drag on moving plate by both of fluids is same
(ii) total drag on moving plate is minimum.

Ans (i) $F_{D_1} = F_{D_2}$

$$\frac{\mu_1(U-0)}{y}A = \frac{\mu_2(U-0)}{H-y}A$$

$$\boxed{y = \left(\frac{\mu_1}{\mu_1+\mu_2}\right)H}$$

(ii) $F_D = F_{D_1} + F_{D_2}$

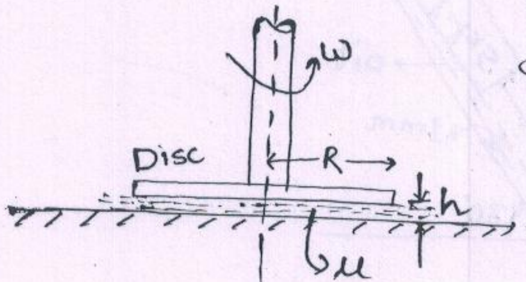
$$F_D = UA\left[\frac{\mu_1}{y} + \frac{\mu_2}{H-y}\right]$$

for F_D to be Minimum $\frac{dF_D}{dy} = 0$

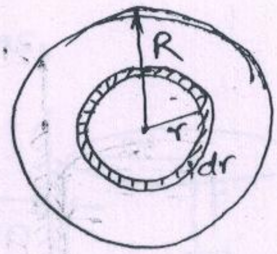
$$-\frac{\mu_1}{y^2} + \frac{\mu_2}{(H-y)^2} = 0 \qquad \frac{\sqrt{\mu_1}}{y} = \frac{\sqrt{\mu_2}}{H-y}$$

$$y = \left(\frac{\sqrt{\mu_1}}{\sqrt{\mu_1}+\sqrt{\mu_2}}\right)H$$

Ques.



find
(i) total Drag on the Disc
(ii) External torque required to maintain its angular velocity ω to be Constant.



(i) Differential Drag force on Element

$$dF_D = \frac{\mu(r\omega - 0)}{h} \cdot 2\pi r\, dr$$

$$dF_D = \frac{2\mu\pi\omega}{h} r^2 dr$$

$$F_D = \int_0^R \frac{2\pi\mu\omega}{h} r^2 dr = \frac{2}{3}\frac{\pi\mu\omega}{h} R^3.$$

(ii) Resistive torque

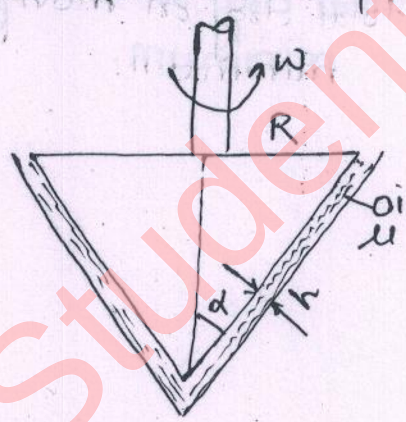
$$d\tau_D = dF_D \times r.$$

$$d\tau_D = \frac{2\pi\mu\omega}{h} r^3 dr.$$

$$\tau_D = \int_0^R \frac{2\pi\mu\omega}{h} r^3 dr = \frac{\pi\mu\omega}{2h} R^4.$$

the External torque required to maintain ω is $\tau = -\frac{\pi\mu\omega}{2h} R^4$.

Ques

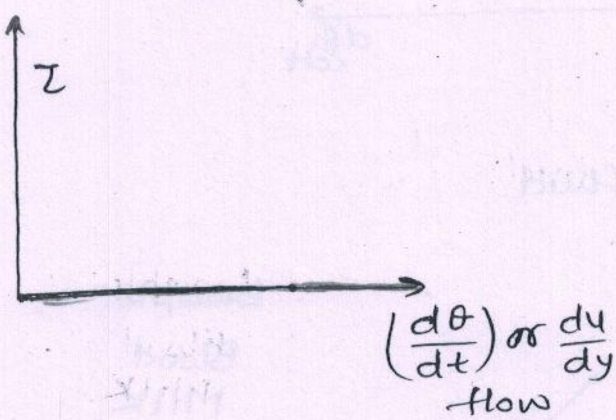


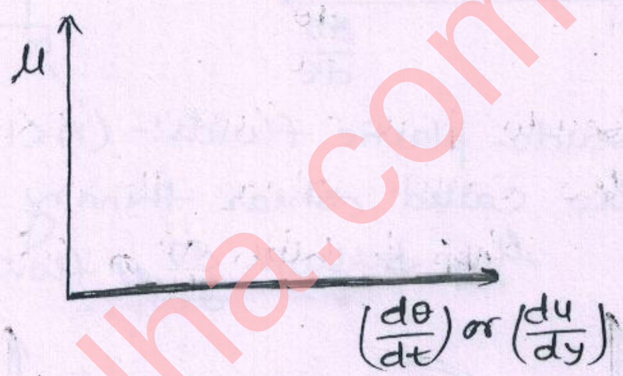
Find
(i) total Drag
(ii) External torque required to maintain its ~~average~~ angular velocity ω to be Constant.

Rheology :- It is a branch of Science which deals with the study of different types of fluid behaviours

1. Ideal fluid.

that fluid which is
- Inviscid ($\mu = 0$)
- Incompressible
- Having Zero Surface tension.

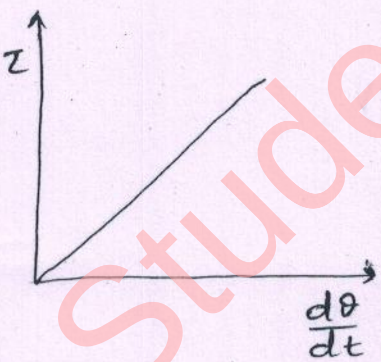


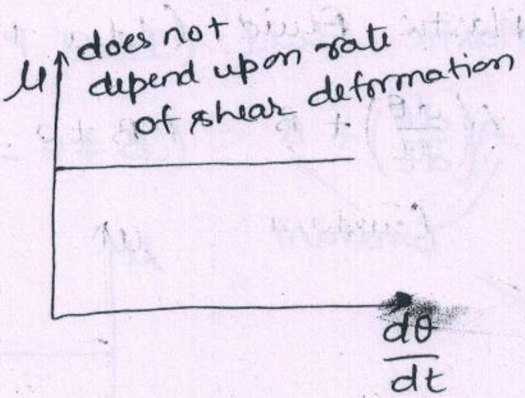


2. Newtonian Fluid.

those fluids which obey Newton law of Viscosity like water, oil, Air, Kerosene

$$\tau = \mu\left(\frac{d\theta}{dt}\right) \text{ or } \tau = \mu\left(\frac{du}{dy}\right)$$





3. Non Newtonian Fluid.

those fluid which do not obey Newton's law of Viscosity

$$\tau = A\left(\frac{d\theta}{dt}\right)^n \quad (n \neq 1)$$

$$\tau = \boxed{A\left(\frac{d\theta}{dt}\right)^{n-1}}\frac{d\theta}{dt}$$

$\Downarrow$

μ_{app}

$n > 1$
→ Dilatent Fluids

$n < 1$
→ Pseudoplastic Fluids

a) Dilatent fluid ($n>1$)

also called Shear thickning fluid.

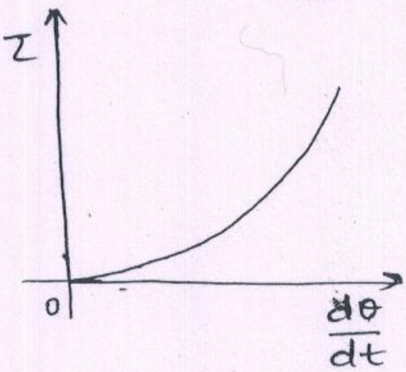
because $\mu_{App}\uparrow$ with $\frac{d\theta}{dt}$

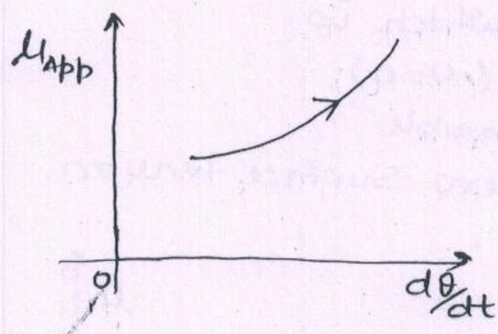
Example -

Rice Starch

Honey

Saturated solution of Sugar in water





b) Pseudo-plastic fluids:- ($n<1$)

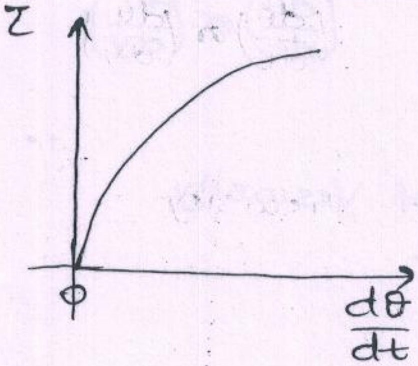
also called shear thinning fluid.

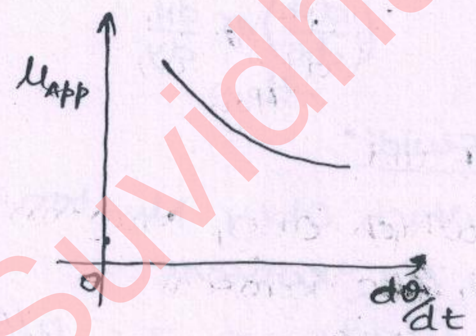
$\mu_{App}\downarrow$ with $\frac{d\theta}{dt}$ or flow.

Example:-

Blood

Milk.

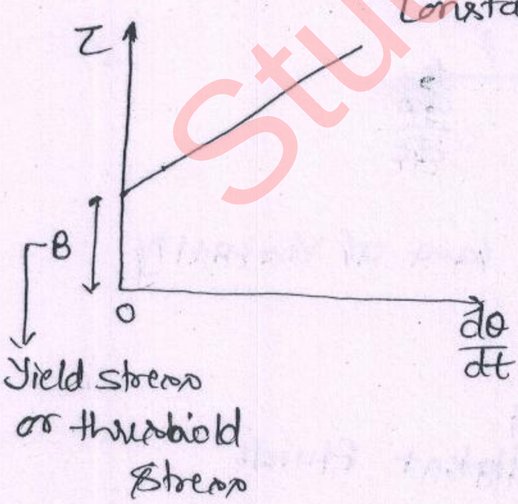


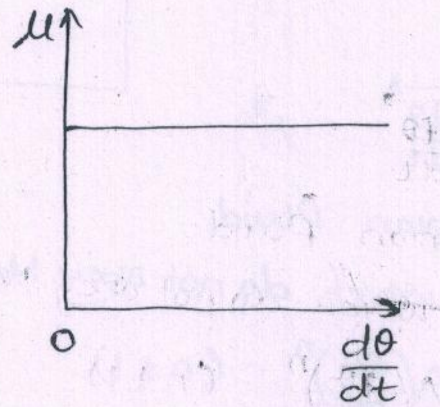


4. Bingham Plastic Fluid. (Ideal Plastic)

$$\tau = A\left(\frac{d\theta}{dt}\right) + B \qquad (B \neq 0 \text{ and } B>0)$$

(A, B — Constant)





Example :- tooth paste

Creams

Gel

5. Thixotropic Fluid.

$$\tau = A\left(\frac{d\theta}{dt}\right)^n + B.$$

$n \neq 1,\ n > 1,\ n < 1$

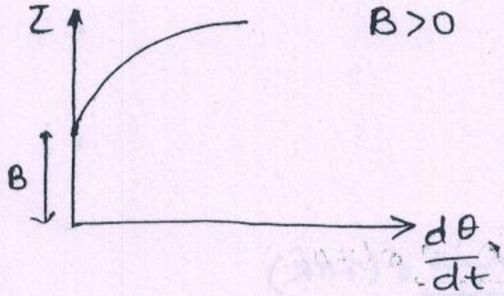
$B > 0$

μ also depends upon time.

In thixotropic fluids

Example few paints

Printer's ink



Note:- Visco-plastic fluid :- these fluids exibit the property of elasticity upto certain limits.

Eg Molten form of rubber

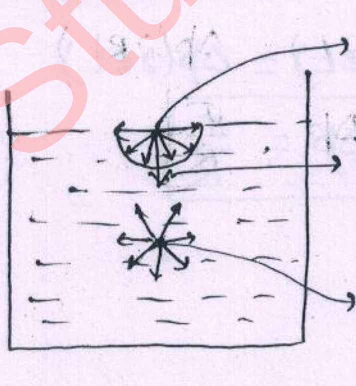
Surface Tension :-

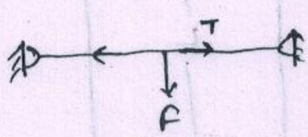
"Fluid is having a very important property by virtue of it tries to minimise its Surface Area upto its maximum extent such a property is known of fluid is known as Surface tension."

fixed Volume

then Surface Area is minimum for Circular shape

Basic reason of Surface tension is (cohesive force)





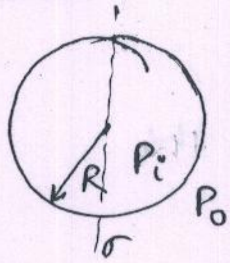
Mathematically it is defined as force per unit length

$$\sigma = \frac{F}{L} \quad N/m$$

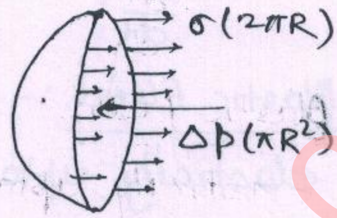
Drop of water

Minimise its Surface Area.

Drop :-



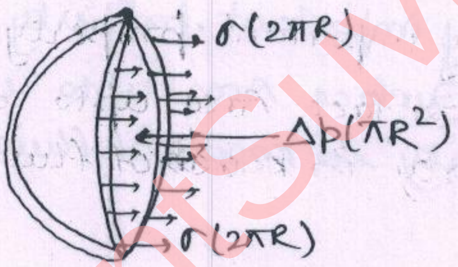
$P_i - P_o = \Delta p$ (Excess pressure)



$\Delta p \pi R^2 = \sigma(2\pi R)$

$$\boxed{\Delta p = \frac{2\sigma}{R}}$$

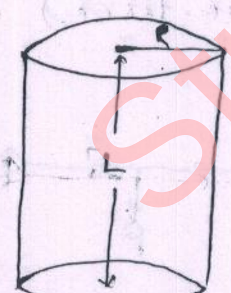
Bubble

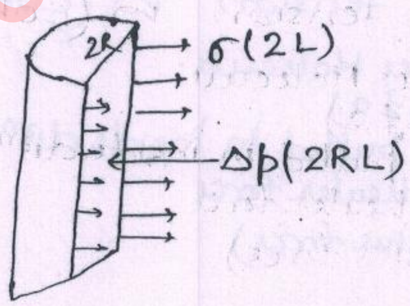


$\sigma(4\pi R) = \Delta p \pi R^2$

$$\boxed{\Delta p = \frac{4\sigma}{R}}$$

Jet





$\sigma(2L) = \Delta p(2RL)$

$$\boxed{\Delta p = \frac{\sigma}{R}}$$

Air Bubble $\equiv$ Air Drop

$$\boxed{\Delta p = \frac{2\sigma}{R}}$$

Concept of Surface Energy:-

Surface tension force on L length $= \sigma L$

to create the fluid layer (surface area) External force required $= \sigma L$

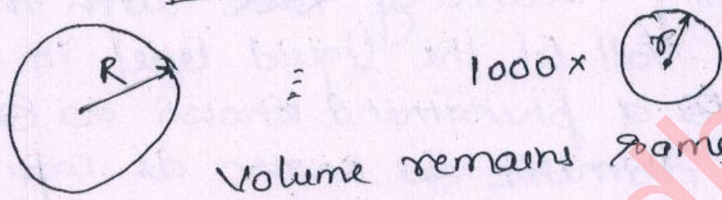
Work done by external agent in displacement of L length by x is $W = \sigma (L \times x)$

$W = \sigma A$

This is stored in the form of surface energy (potential Energy of Surface)

$$\boxed{E = \sigma A}$$

Ex



Volume remains same

$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3 \Rightarrow R = 10r$

$$\boxed{r = R/10}$$

$E_1 = \sigma(4\pi R^2)$ — Work is required → $E_2 = \sigma(40\pi R^2)$; No Work is required (←)

$E_2 = 1000 \times \sigma(4\pi r^2)$

$= 1000 \times \sigma \left(4\pi \frac{R^2}{100}\right)$

$E_2 = \sigma(40\pi R^2)$

Wetting & Non-Wetting Liquids:-

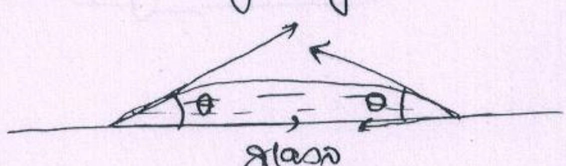
It is a Mutual Property of liquid - Surface

It depends upon cohesion & Adhesion both

If Adhesion >> cohesion

liquid is wetting on the surface

Water - Glass :- Angle of contact $\theta < \pi/2$



Pure Water on glass

$\theta = 0°$

If Cohesion >> Adhesion.

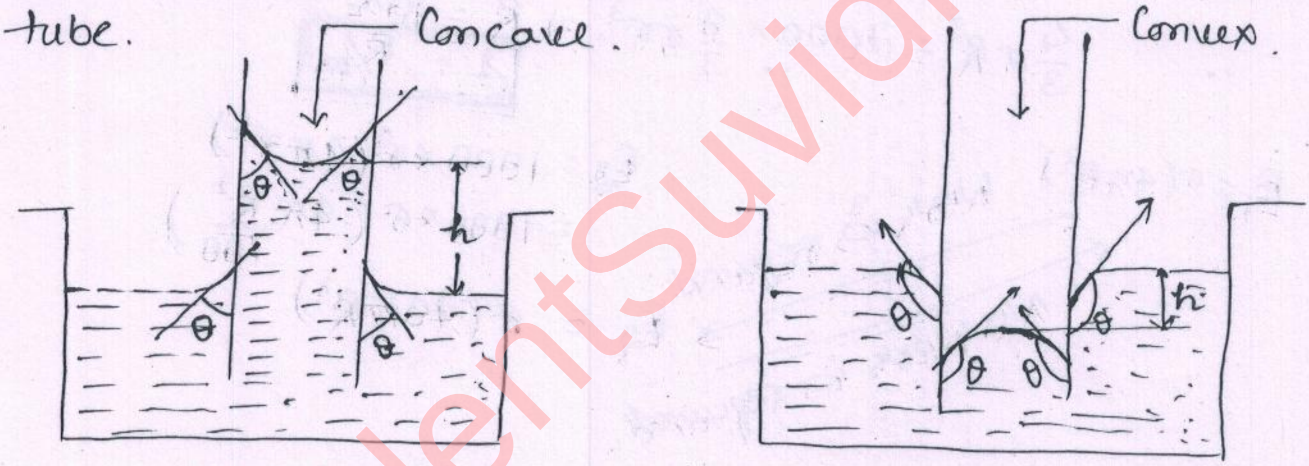
liquid is Non-wetting on the surface

Hg-glass

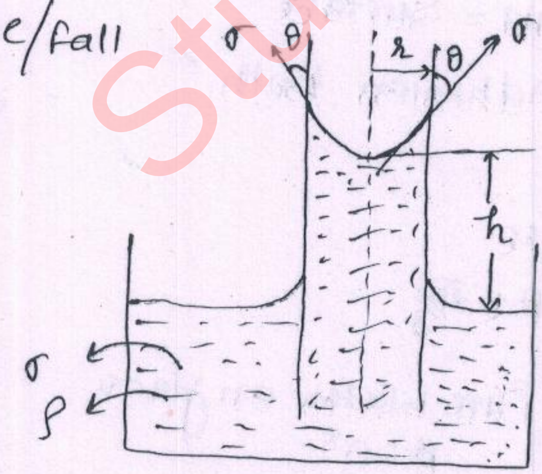
Angle of contact $\theta > \pi/2$

$\theta_{Hg\text{-}glass} \Rightarrow 130^\circ - 140^\circ$

Capillarity :- "when a tube of very fine diameter is immersed in a liquid. then there will be the rise or fall of liquid level inside the tube depending upon the wetting or Non-wetting nature of liquid with the tube Surface. This rise or fall of the liquid level in a tube of very fine diameter is a phenomena known as Capillarity and this tube of fine diameter is known as capillary tube.



Wetting   Non wetting



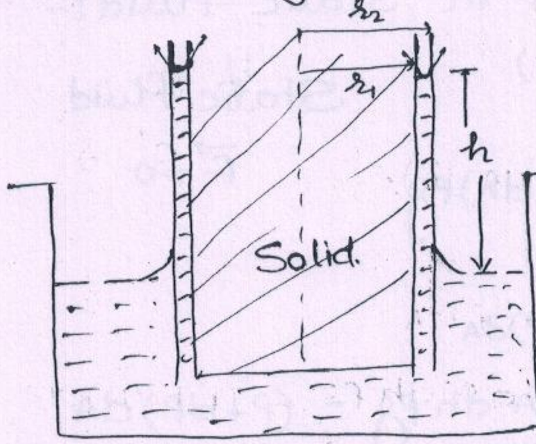
r = radius of capillary tube

θ = Angle of contact

$$(\sigma \cos\theta)\, 2\pi r = (\pi r^2 h)\rho g$$

$$\boxed{h = \frac{2\sigma \cos\theta}{r\rho g}}$$

Annular Capillary



$$\sigma \cos\theta \, 2\pi r_1 + \sigma \cos\theta \, 2\pi r_2 = \pi (r_2^2 - r_1^2) h \rho g$$

$$\boxed{h = \frac{2\sigma \cos\theta}{(r_2 - r_1)\rho g}}$$

- If Inside Material is different than the outside Material the Contact angle will be different

Pressure & Pressure Measurement :-

Pressure :- Existence of fluid mass is the existence of pressure

Mathematically it is defined as External Normal force per Unit Area.

"Scalar Quantity" $P = \frac{F}{A}$ → External Normal force, Thrust

Unit of pressure is $\frac{N}{m^2}$ or Pa (Pascal)

Units of Pressure :-

1.) $1 \text{ Pa} = 1 \text{ N/m}^2$

2) $1 \frac{\text{Kgf}}{\text{cm}^2} = \frac{9.81}{10^{-4}} \frac{\text{N}}{\text{m}^2} = 9.81 \times 10^4 \text{ Pa}$

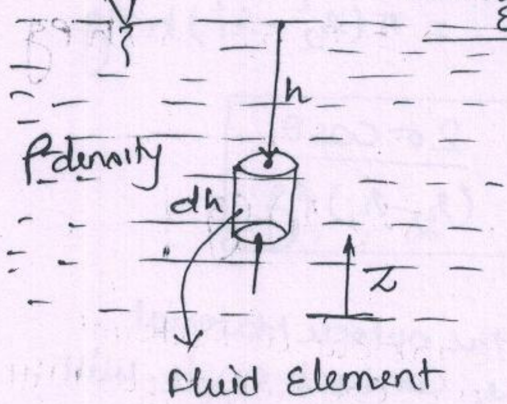
3) $1 \text{ atm} = 101325 \text{ Pa}$.

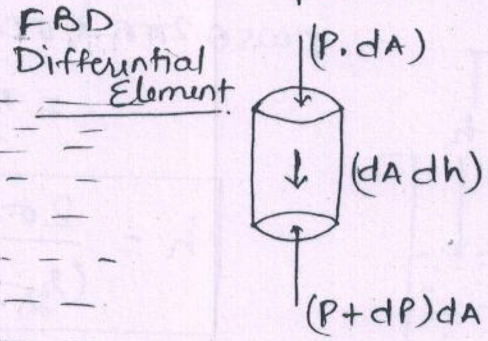
4) $1 \text{ bar} = 10^5 \text{ Pa}$

5) $1 \text{ P.S.I} = 1 \frac{\text{Lbf}}{\text{inch}^2} = \frac{(0.453) \times 9.81}{(2.54)^2 \times 10^{-4}} \text{ Pa}$.

In tyre pressure is around 2 ~ 2.5 atm

Hydrostatic Pressure at a point in static fluid:-





Static fluid

$\vec{F} = 0$

$$P\,dA + dA \cdot dh\,\rho g = (P+dP)\,dA$$

$$\frac{dP}{dh} = \rho g \quad \text{and} \quad \frac{dP}{dz} = -\rho g$$

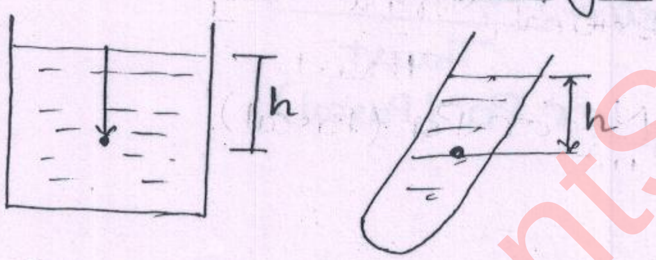
$\frac{dP}{dh}$ is pressure Gradient in downward dirn

P ↑ in downward dirn

P ↓ in upward dirn

$$\frac{dP}{dh} = \rho g \Rightarrow \int_0^P dP = \rho g \int_0^h dh$$

$$\boxed{P = \rho g h}$$



Pressure ~~for~~ Represented as a height of fluid column

1 atm = 101325 Pa

Water column $101325 = h_{water} \times 1000 \times 9.81$

$h_{water} = 10.3\,m$

Hg column $101325 = h_{Hg} \times 13600 \times 9.81$

$h_{Hg} = 0.760\,m = 760\,mm$

so $\boxed{1\text{ atm} = 101325\text{ Pa} = 10.3\,m\,(\text{Water}) = 760\,mm\,(Hg)}$

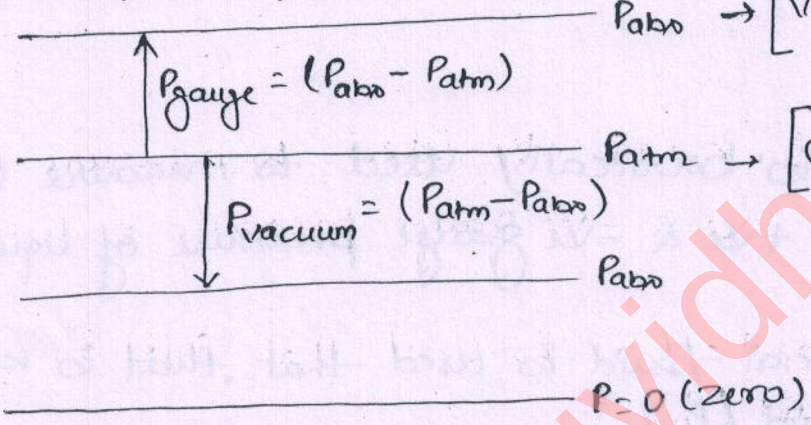
Different types of pressures:-

1) Atmospheric pressure:- the pressure exerted by the enviomment mass is known as Atmospheric pressure.

2) Absolute Pressure:- It is the total pressure of the system

3) Gauge Pressure:- It is the pressure of the system above atmospheric pressure Values.

4) Vacaume Pressure:- It is the pressure of the system below atmospheric pressure Values.

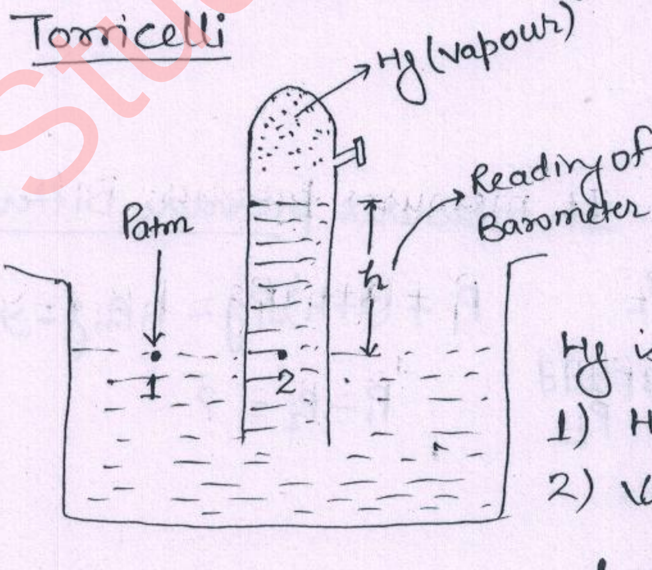


Pabs → [Vacuum pressure are −ve gauge pressure]

Patm → [Gauge pressure are −ve Vacuum pressure]

CONVENTIONAL PRESSURE MEASUREMENT DEVICES:-

1) BAROMETER:- It is a device which is basically used to measure local atmospheric pressures.
This device was made by a big mathematician Named Torricelli



$$P_1 = P_2$$

$$P_{atm} = \rho_{Hg} g h + P_{vapour(Hg)}$$ ($P_{vapour(Hg)} \to 0$)

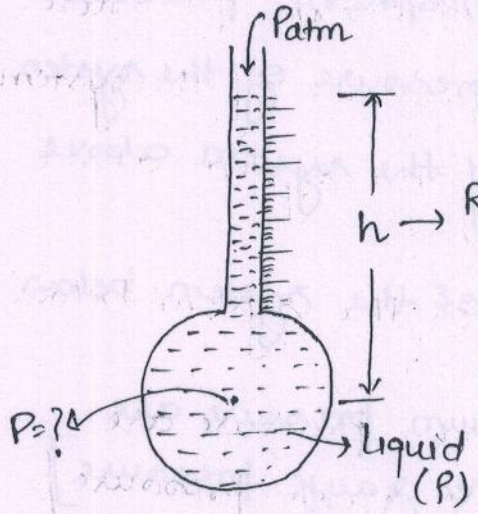
$$\boxed{P_{atm} = \rho_{Hg} g h}$$

Hg is having very important proper

1) High Density Fluid
2) Very low Vapour pressure

1 mm (Hg) ≃ 1 torr → Unit of pressure

2) **PIEZOMETER** :- It is basically used to measure moderate +ve gauge pressure of liquid only.



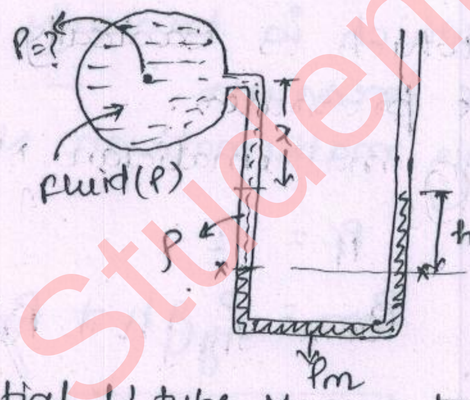
$P = \rho g h$ (gauge)

3) **Manometer** :- It is basically used to measure low, medium and high, +ve & -ve gauge pressure of liquid & Gases.

In Manometer additional fluid is used that fluid is known as Manometric fluid (ρ_m)

Simple U-tube Manometer

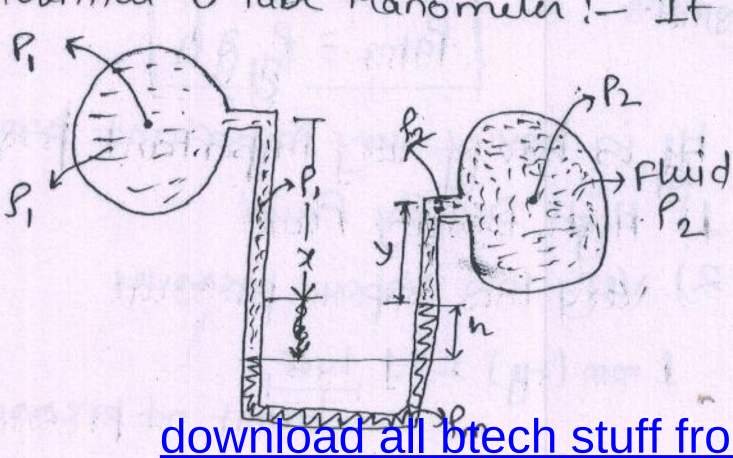
It measure Pressure



$P + (x+h)\rho g - h\rho_m g = 0$

$P = ?$ (gauge)

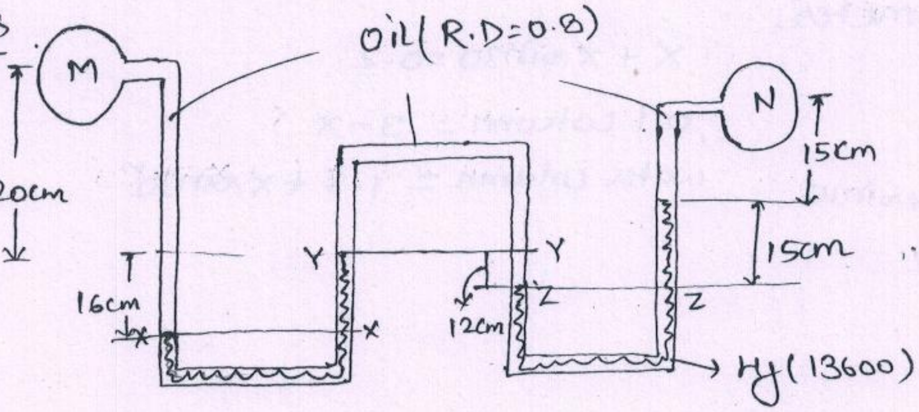
Differential U tube Manometer :- It measures pressure Difference



$P_1 + (x+h)\rho_1 g - h\rho_m g - y\rho_2 g = P_2$

$P_1 - P_2 = ?$

Ques



$RD = 0.8 = \frac{\rho_{oil}}{\rho_{water}}$

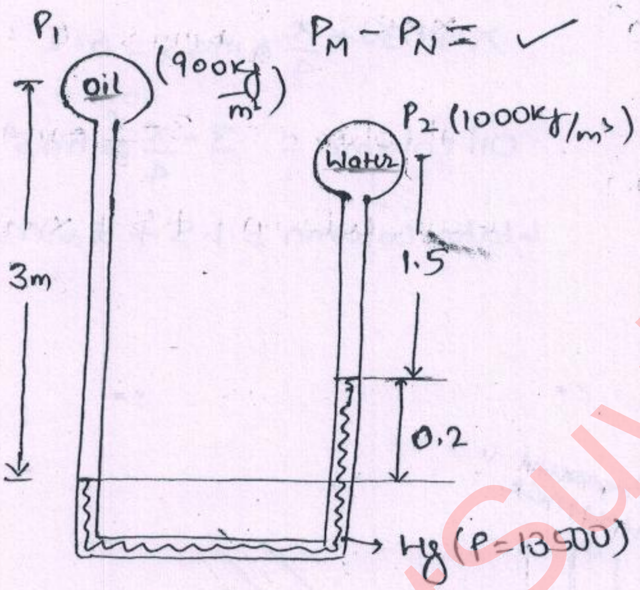
$\rho_{oil} = 1000 \times 0.8$

$= 800$

$$P_M + 0.36\times800\times9.81 - 0.16\times13600\times9.81 + 0.12\times800\times9.81$$
$$-0.15\times13600\times9.81 - 0.15\times800\times9.81 = P_N$$

$P_M - P_N =$ ✓

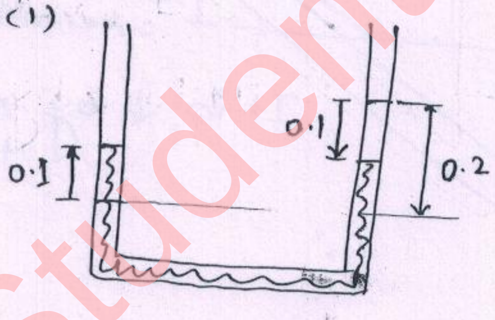
Ques



$$P_1 + 3\times900\times9.81 - 0.2\times13500\times9.81$$
$$- 1.5\times1000\times9.81 = P_2$$

$P_1 - P_2 =$ ✓

Now (i)



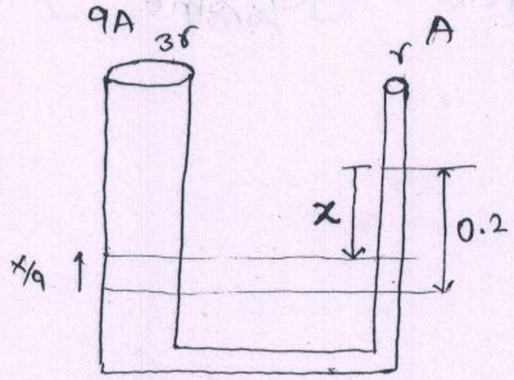
Oil column = 2.9m

Water column = 1.6m

$$P_1 + 2.9\times900\times9.81 - 1.6\times1000\times9.81$$
$$= P_2 + \Delta P$$

$\Delta P =$ ✓

(ii) Different areas of both limbs.



Volume will remain same

so $x + \frac{x}{9} = 0.2$

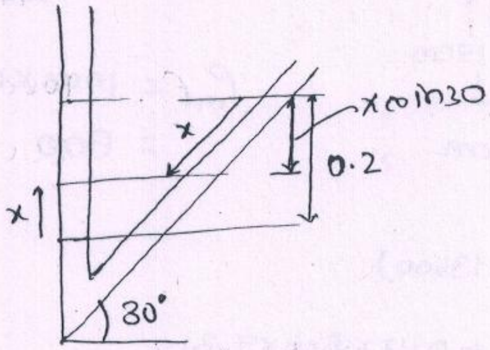
Oil column $= 3 - \frac{x}{9}$

Water column $= 1.5 + x$

$x(\pi R^2) = y(\pi(3r)^2)$

$y = \frac{x}{9}$

III) Inclined Manometer

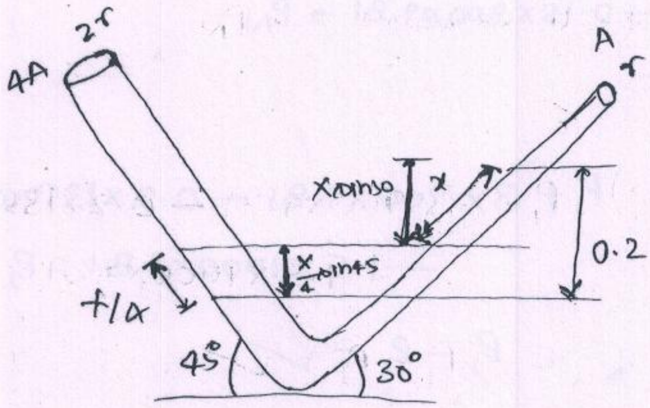


$x + x \sin 30 = 0.2$

$\therefore$ oil column $= 3 - x$

water column $= 1.5 + x \sin 30°$

(IV)

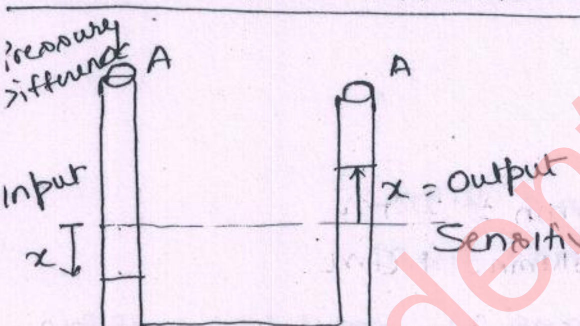


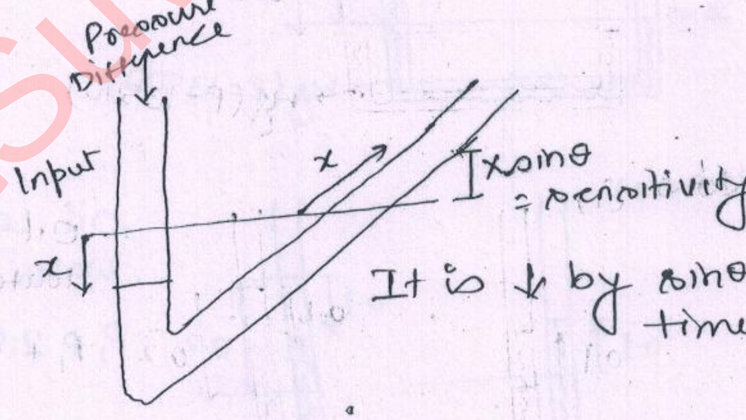
$x \sin 30 + \frac{x}{4} \sin 45 = 0.2$

Oil column $= 3 - \frac{x}{4} \sin 45°$

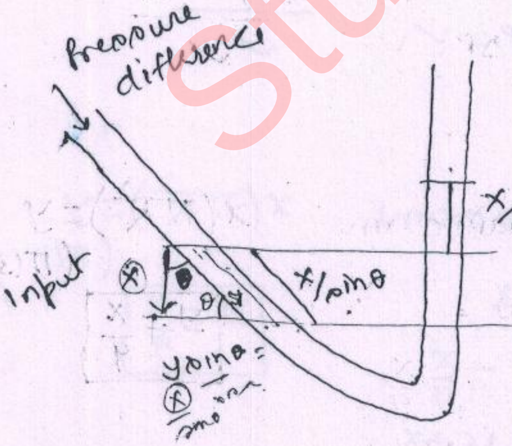
Water column $= 1.5 + x \sin 30°$

SENSITIVITY OF MANOMETER





It is ↓ by sinθ times.



$x/\sin\theta$ = sensitivity [it is ↑ by $\frac{1}{\sin\theta}$ times]

= output

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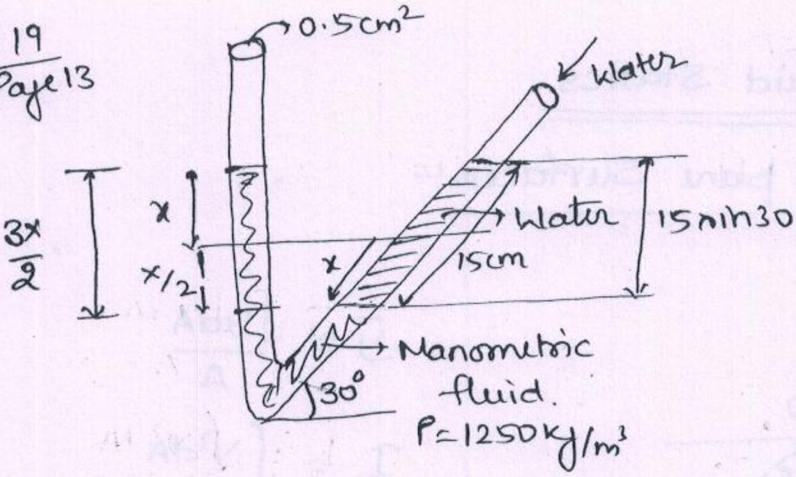


$$P_{atm} + \frac{3x}{2} \times 1250 \times 9.81 - \frac{7.5}{100} \times 1000 \times 9.81 = P_{atm}$$

$x = 0.04$ meter

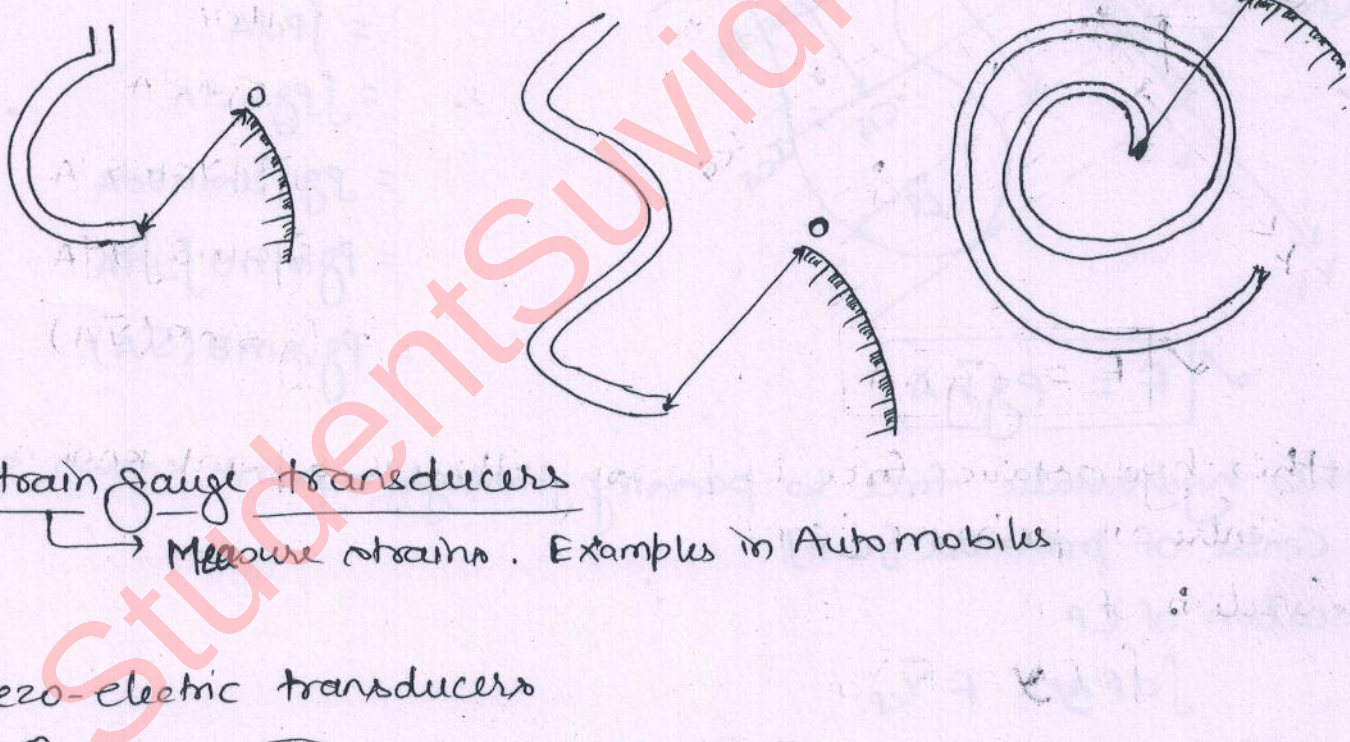
$\boxed{x = 4\,cm}$

Modern Pressure Measurement Devices :-

Bourdon gauge ⟶ Very high (+ve) gauge pressure are measure

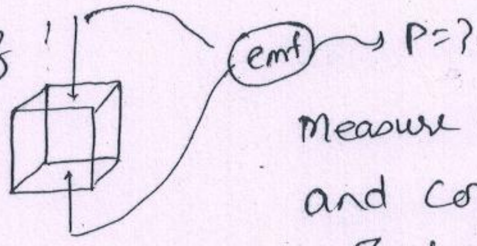
Inside this
Flexible tube is there

C shape   Helical shape   Spiral shape



Strain gauge transducers
↳ Measure strains. Examples in Automobiles

Piezo-electric transducers
Quartz !



Measure Electromagnetic
and converts in Pressure
Ex. Electronics Devices