FREE ELECTRON THEORY

Conductivity of metal decreases with increase in temprature.
According to ohm's Law, Current density vector $\vec{J}$ is directly proportional to Electric feild.

$$\vec{J} \propto \vec{E}$$

or $$\vec{J} = \sigma \vec{E}$$

where σ is electrical conductivity.

Drude proposed free electron theory to explain properties of metal.
Assumptions made by him are:

1). Metal contains large number of conduction electrons which behave like a molecule of perfect gas and moves throughout volume of metal.

2). Free electron moves randomly in all possible directions with different velocity. The distribution of velocity is in accordance with the Maxwell.

Notes

Average kinetic energy of free electron is $\frac{3}{2}KT$. (K boltzmann constant)

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3). Free electron makes collision from time to time with positive ion of lattice.

4). In the absence of electric feild current density vector is zero
$$\vec{J} = \sigma \vec{E} = 0$$

5). When electric feild is applied on the metal then electrons get drifted with some average velocity is known as V_d.

6). The distance travelled by free electron between any two successive collisions is called mean path.

If there are 'n' number of free electrons and electric feild is applied on metal then
$$\vec{F} = -e\vec{E}$$
and
$$\vec{F} = ma = \frac{mV_d}{t}$$
m - mass of electron.

Notes
$$V_d = \frac{-e\vec{E}t}{m}$$
Substance having 'n' number of electrons

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of charge e drifted upon by drift velocity V_d then

$$\vec{J} = -neV_d$$

$$\vec{J} = -ne\left(\frac{-e\vec{E}t}{m}\right)$$

$$= \frac{ne^2\vec{E}t}{m}$$

Now, τ is the average time between any two successive collisions

then $\tau = t + t = 2t$

$$\vec{J} = \frac{ne^2\vec{E}\tau}{2m}$$

But acc. to ohm's law $\vec{J} = \sigma\vec{E}$

$$\therefore \sigma = \frac{ne^2\tau}{2m} \quad \text{--- (1)}$$

But when λ is the mean free path, $\vec{v}$ is mean speed and τ is the average time between two successive collisions.

$$\text{time} = \frac{\text{dis}}{\text{vel.}} = \frac{\lambda}{\vec{v}}$$

$$\therefore \sigma = \frac{ne^2\lambda}{2m\vec{v}}$$

This is expression for electrical conductivity and independent on Temp but depends only on 'n' that varies

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from metal to metal.

Thermal Conductivity

Free electron behaves like a molecule of perfect gas. Free electron possess kinetic energy of thermal ignition more at hot end than at cold end. But due to their random motion of free electron energy is transfered at both ends on account of transfer of electrons.

Suppose that there are 'n' no of free electron that varies from metal to metal having possible random motion in all direction. Lets the motion of the electron is resolved in one direction. $\frac{n\bar{v}}{6}$ electron crosses a plane P upwards or downwards.

Consider a plane P_1 at temp T_1 and λ distance from plane P and another plane P_2 at temp T_2 and λ distance from another side of plane P.

Notes

If $T_1 > T_2$ then energy is transfered from P_1 to P_2, total no of electron transfered is $\frac{n\bar{v}}{6}$ and each electron

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has energy $\frac{3}{2}kT_1$. Total energy transferred from P_1 to P_2 =

$$\frac{n\bar{v}}{6} \cdot \frac{3}{2}kT_1$$

Simillarly, from P_2 to P_1 = $\frac{n\bar{v}}{6}\frac{3}{2}kT_2$

Net energy transfered:

$$Q = (P_1 \text{ to } P_2) - (P_2 \text{ to } P_1)$$

$$Q = \frac{n\bar{v}}{6} \cdot \frac{3}{2} k \left[T_1 - T_2\right]$$

we know that $Q = \frac{k'}{2\lambda}(T_1 - T_2)$

thus, $\frac{n\bar{v}}{4}\frac{k}{2} = \frac{k'}{2\lambda}$

$$k' = \frac{n\bar{v}k\lambda}{2}$$

k' represents thermal conductivity.

<u>Widemann-Frenz Law:</u>

Widemann-Frenz law States that ratio of thermal to electrical conductivity is Same for all metals at ordinary temprecture.

But lorentz modified this law and

Show that $\frac{k'}{\sigma}$ is directly propor-
-tional to temprature.

$$\frac{k'}{\sigma} = \frac{n\bar{v}k\lambda}{2} \Big/ \frac{ne^2\lambda}{2m\bar{v}}$$

$$= \frac{kmv^2}{e^2}$$

$$\frac{1}{2}mv^2 = \frac{3}{2}kT \quad \therefore$$

$$= \left(\frac{k}{e}\right)^2 T \cdot 3$$

$$\therefore \quad \frac{k'}{\sigma} \propto T$$

This is valid only in case of ordinary temp. but not on low temp.

Limitations

1). Widemann Law:
Free electron theory beautifully explains widemann frenz law at ordinary temprature but fails to explain it at low temprature.

2). Paramagnetism:
Paramagnetism of metal is indepe-ndent of temprature which can't be explain by Free e⁻ theory.

3). Electrical Conductivity
The electrical conductivity of a metal depends on n and not on absolute temp.
Thus, free e⁻ theory fails to explain that $\sigma \propto \frac{1}{\text{Temperature}}$.

Quantum theory of Conduction:

Quantum theory of Conduction modified free electron theory. Assumptions made were:

1). A metal contains large number of conduction-eletrons which were not completely free but partly.

2). The interior of the metal is covered with the region of uniform potential.

3). The force between the conduction electron and ion core is neglected. The whole energy of the electron is K.E as potential energy is neglected.

Notes
finally, it is considered that electron is present in one dimensional crystal of length l.

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Secondly, the particular integral everywhere in the Crystal is zero (constant)

Therefore Schrodinger equation has form

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0$$

let $\frac{2m}{\hbar^2}E = \omega^2$

and $\frac{d}{dx} = D$

$\therefore$

$$(D^2 + \omega^2)\psi = 0$$

$$D = \pm i\omega$$

General Solution: $C_1 \cos\omega x + C_2 \sin\omega x$

$$\psi(x) = C_1 \cos\sqrt{\frac{2mE}{\hbar^2}}\,x + C_2 \sin\sqrt{\frac{2mE}{\hbar^2}}\,x$$

Now value of C_1 and C_2 can be calculated by two boundary condition that exist at $x=0$ and $x=L$

at $x=0$, $\psi = 0$

Notes

$$0 = C_1(1) + C_2(0)$$

$$C_1 = 0$$

$\therefore$ $\psi(x) = C_2 \sin\sqrt{\frac{2mE}{\hbar^2}}\,x$

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at $x = L$, $\psi = 0$

$$0 = C_2 \sin \sqrt{\frac{2mE}{\hbar^2}}\, L \qquad (C_2 \neq 0)$$

$$\therefore \; n\pi = \sqrt{\frac{2mE}{\hbar^2}}\, L$$

$$\frac{n^2\pi^2 \cdot \hbar^2}{2mL^2} = E \quad -(1)$$

Value of C_2 can be calculated by

$$\int_0^L (\psi x)^2 \, dx = 1$$

$$\int_0^L C_2^2 \cdot \sin^2 \sqrt{\frac{2m}{\hbar^2} \cdot \frac{n^2\pi^2\hbar^2}{2mL^2}}\, x$$

$$C_2^2 \int_0^L \sin^2 \frac{n\pi}{L} x \, dx$$

$$C_2^2 \int_0^L \frac{1 - \cos 2\frac{n\pi}{L} x}{2} \, dx$$

$$C_2^2 \left(\frac{L}{2}\right) \qquad (\text{Since, } \sin 2n\pi = 0)$$

$$C_2 = \sqrt{\frac{2}{L}}$$

Notes

Required Sol: $\sqrt{\frac{2}{l}} \sin \frac{n\pi}{L} x = \psi(x)$

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FERMI DIRAC

Fermi dirac Statistics is applicable to all those identical and indistinguishable particles that obeys pauli exclusion principle.
All the particles having half spin integral angular momentum obeys pauli exclusion principle. That means all protons, electrons and nuetrons Show F-D statistics.

Consider a box having number of Compartment, in that Compartment there are no of cells out of this Consider i_{th} Compartment.

Now, we have to distribute N_i no of indistinguishable particles among g_i no of cell then pauli exclusion principle must be remind that not more than one particle can occupy a given cell that is $g_i > N_i$

The g_i no of cell can be arranged

Notes

in different no of ways. But N_i permutation between particles is meaningless and Same, permutation

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ERMI DIRAC

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n of particles can be different number of

$$g_i C_{N_i} = \frac{g_i!}{(g_i - N_i)!\, N_i!}$$

approximation $x \log x - x$

(both sides)

$$\log g_i! - \log (g_i - N_i)! - \log N_i!$$

$$g_i - g_i - \log (g_i - N_i)^{g_i - N_i} + g_i - N_i - N_i \log N_i + N_i$$

$$-\log (g_i - N_i) \cdot g_i - N_i - N_i \log N_i$$

relate w.r.t N_i

$$(g_i - N_i) \frac{1}{(g_i - N_i)} (dN_i) + \log (g_i - N_i)\, dN_i$$

$$N_i \cdot \frac{1}{N_i}\, dN_i - \log N_i\, dN_i$$

$$\left(\log (g_i - N_i) - \log N_i\right) dN_i$$

In most probable State $d(\log w) = 0$

$$\sum_{i=1}^{N} \left(\log (g_i - N_i) - \log N_i\right) dN_i = 0 \quad -(A)$$

The distribution is Subjected to the condition that total no of particles and energy of system is constant.

$$N = \sum_{i=1}^{N} N_i \quad \text{and} \quad E = \sum_{i=1}^{N} E_i N_i$$

Differentiate this

$$0 = \sum_{i=1}^{N} dN_i \quad -(1) \qquad \text{and} \qquad 0 = \sum_{i=1}^{N} E_i dN_i \quad -(2)$$

multiply (1) by α and (2) by β and add to (A)

$$\sum_{i=1}^{N} \left[\log (g_i - N_i) - \log N_i - \alpha - \beta E_i\right] = 0$$

$$\therefore \log \frac{g_i - N_i}{N_i} = \alpha + \beta E_i$$

Notes

$$\frac{g_i}{N_i} - 1 = e^{\alpha + \beta E_i}$$

$$\frac{g_i}{N_i} = 1 + e^{\alpha + \beta E_i}$$

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$$\frac{N_i}{g_i} = \frac{1}{1 + e^{\alpha + \beta E_i}} = f(E)$$

Called fermi dirac distribution.

FERMI ENERGY

The level which divides filled and empty level is called fermi level at absolute temprature (T=0). The corrosponding energy at this level is called fermi energy

According to fermi dirac distribution

$$\frac{N_i}{g_i} = \frac{1}{1 + e^{(\alpha + \beta E_i)}} = f(E)$$

No. of particles having momentum lying between p and p+dp is:

$$g(p)\,dp = g(s)\,\frac{4\pi v}{h^3}\,p^2\,dp$$

$$g(s) = 2s+1 \quad \text{for electron spin } S = \frac{1}{2}$$

$$= 2$$

Notes

$$g(p)\,dp = \frac{8\pi v}{h^3}\,p^2\,dp.$$

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8.00 If m is the mass of electron, when electric feild is applied on the
9.00 metal, electron gain k.E:

$$k.E = \frac{1}{2} m v^2 = \frac{p^2}{2m}$$

10.00

$$E = \frac{p^2}{2m}$$

11.00

$$p^2 = 2mE$$

12.00 on differentiation

$$2p\,dp = 2m\,dE$$

1.00

$$p\,dp = m\,dE \quad \text{and} \quad dp = \frac{m}{\sqrt{2mE}}\,dE$$

2.00

3.00 Now,

$$g(p)\,dp = \frac{8\pi V}{h^3}\,2mE\,\frac{m}{\sqrt{2mE}}\,dE$$

4.00

5.00 No. of electrons having having energy lying between E and E+dE:

6.00

$$g(E)\,dE = \frac{8\pi V}{h^3}\sqrt{2mE}\,m\,dE$$

7.00

$$dn = f(E) \cdot dE$$

8.00

$$n = \int dn = \int \frac{8\pi V}{h^3}\sqrt{2mE}\,m\,dE \cdot \frac{1}{e^{\alpha+\beta E_i}+1}$$

Notes

Now, $\alpha = -E_F / kT$ $\quad \beta = \frac{1}{kT}$

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if $e^{E-E_F/kT} = -\infty$ then

$$\frac{1}{1+e^{E-E_F/kT}} = 1$$

if $e^{E-E_F/kT} = \infty$ then $\frac{1}{1+e^{E-E_F/kT}} = 0$

$$\therefore n = \int_0^{E_F(0)} \frac{8\pi V}{h^3} \sqrt{2m} \cdot m \sqrt{E}\, dE$$

$$n = \frac{8\pi V}{h^3} \sqrt{2m} \cdot m \cdot \frac{2}{3} (E_F)^{3/2}$$

$$\therefore E_F(0) = \frac{h^2}{8m} \left(\frac{3}{\pi} \cdot \frac{n}{V}\right)^{2/3}$$

Density of State: The term density of state refers to number of energy levels per unit energy range at a given (range) energy

$$\text{Density of State} = \frac{8\pi V}{h^3} \cdot m\sqrt{2mE_{F(0)}}$$

when $E < E_F$

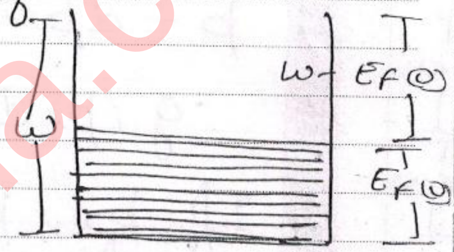
It is zero, when $E > E_F$.

$\frac{n_i}{g_i} = \frac{1}{1+e^{\alpha+\beta E_i}} = f(E)$

Thermionic emission:

It is phenomenon in which the Surface of metal is heated so that electrons start emitting from the Surface.

Free electron theory assumes that there is a constant potential with in the metal. If W is the constant potential the minimum energy which is supplied to the electron so that it can escape out through metal is:



$W - E_F(0)$ and often known as work function.

RICHARDSON'S EQUATION: $\boxed{15\text{-}20}$

Acc. to Fermi dirac Statistics:

$$\frac{N_i}{g_i} = \frac{1}{1+e^{\alpha+\beta E_i}} = f(E)$$

Notes

The minimum number of particles having momentum lying between p, $p+dp$ is:-

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$$g(p)\,dp = \frac{4\pi V}{h^3}\, g(s)\, p^2\, dp$$

$g(s) = 2s+1$ (For electron spin $s = \frac{1}{2}$)

$g(s) = 2$

$$g(p)\,dp = \frac{8\pi V}{h^3}\, p^2\, dp$$

The total no. of particles per unit volume lying between p and $p+dp$ is,

$$g(p)\,dp = \frac{8\pi}{h^3}\, p^2\, dp$$

If p_x, p_y and p_z are the component of momentum along x, y and z-axis.

then, $p^2 = p_x^2 + p_y^2 + p_z^2$

Volume of momentum space lying between p_x, $p_x + dp_x$, p_y, $p_y + dp_y$, p_z, $p_z + dp_z$.

$$4\pi p^2\, dp = dp_x\, dp_y\, dp_z \quad \text{--- (1)}$$

Now, $g(p)\,dp = \frac{2}{h^3}\, dp_x\, dp_y\, dp_z$

If $p = mv$ then v_x, v_y, v_z be the component of velocity along x, y and z-axis.

$-\frac{E_F}{KT}$ $\beta = \frac{1}{KT}$

$dp_x = m\,dv_x$, $dp_y = m\,dv_y$ and

$dp_z = m\,dv_z$

$$g(p)\,dp = \frac{2}{h^3} m^3\, dv_x\, dv_y\, dv_z$$

$$f(E) = \frac{1}{1 + e^{\alpha + \beta E_i}}$$

where $\alpha = -E_F/kT$ and $\beta = \frac{1}{kT}$

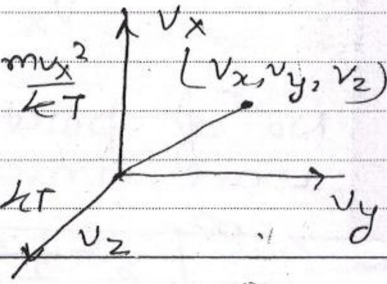
$$= e^{-(E_i - E_F/kT)}$$

$$dn = \frac{2m^3}{h^3} dv_x\, dv_y\, dv_z \cdot e^{(-\frac{1}{2}mv^2)/kT} \cdot e^{E_F/kT}$$

If the surface of metal is along x-z plane and normal to x-axis.

$$dn_x = \frac{2m^3}{h^3} dv_x\, dv_y\, dv_z \cdot e^{-\frac{1}{2}\frac{m(v_x^2+v_y^2+v_z^2)}{kT}} \cdot e^{+E_F/kT}$$

$$\int dn_x = \int_{-\infty}^{\infty} \frac{2m^3}{h^3} dv_x\, dv_y\, dv_z\, e^{-\frac{1}{2}\frac{mv_x^2}{kT}} \cdot e^{-\frac{1}{2}\frac{mv_y^2}{kT}} \cdot e^{-\frac{1}{2}\frac{mv_z^2}{kT}} \cdot e^{+E_F/kT}$$



Notes

Solving $\int_{-\infty}^{\infty} e^{-\frac{1}{2}\frac{mv_x^2}{kT}}\, dv_x$

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$$\frac{1}{2}\frac{mv_x^2}{kT} = z$$

$$dv_x = \sqrt{\frac{kT}{m}} \times \frac{1}{2\sqrt{z}}$$

$$\sqrt{\frac{2kTz}{m}} = v_x \quad \text{and} \quad dv_x = \sqrt{\frac{2kT}{m}} \cdot \frac{1}{2\sqrt{z}}$$

$$2\int_0^\infty e^{-z} \cdot z^{-\frac{1}{2}} \cdot \sqrt{\frac{kT}{2m}}$$

$$= \sqrt{\frac{2kT}{m}} \cdot \sqrt{\pi} \quad \text{[using Beta and Gamma function]}$$

Simillarly, $\int_{-\infty}^{\infty} e^{-\frac{1}{2}\frac{mv_y^2}{kT}} dv_y = \sqrt{\frac{2kT}{m}} \cdot \sqrt{\pi}$

$$n_x = \frac{2kT\pi}{m} \cdot \frac{2m^3}{h^3} \cdot e^{+E_F/kT} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\frac{mv_x^2}{kT}} dv_x$$

If the metal is heated, electron will escape out through metal only when $\frac{1}{2}mv_x^2 \geq W$

$$v_x \geq \sqrt{\frac{2W}{m}}$$

$\frac{1}{2}mv_x^2 \geq W$, $v_x \geq \sqrt{\frac{2W}{m}}$

No. of particles per unit area per unit time is:

$$\int_{\sqrt{\frac{2W}{m}}}^{\infty} e^{-\frac{1}{2}\frac{mv_x^2}{kT}} \cdot v_x \, dv_x$$

$$\int_{\sqrt{\frac{2W}{m}}}^{\infty} e^{-\frac{1}{2}\frac{mv_x^2}{kT}} v_x \, dv_x$$

$\frac{1}{2}\frac{m v_x^2}{KT} = P$

$v_x^2 = \frac{2KTP}{m}$

$2 v_x dv_x = \frac{2KT}{m} dP$

$$\frac{1}{2}\frac{m v_x^2}{kT} = P$$

$$2 v_x \, dv_x = \frac{2kT}{m} dP$$

$$\int_{\omega/kT}^{\infty} \frac{kT}{m} e^{-P} dP \Rightarrow -\frac{kT}{m}\left[e^{-\infty} - e^{-\omega/kT}\right]$$

$$= \frac{kT}{m} e^{-\omega/kT}$$

$$\therefore n = \frac{kT}{m} \cdot \frac{2kT\pi}{m} \cdot \frac{2m^3}{h^3} e^{-(\omega - E_F)/kT}$$

$$= \frac{4\pi k^2 T^2 m}{h^3} e^{-(\omega - E_F)/kT}$$

$(J = nevd)$

current density

$$\boxed{J = A T^2 e^{-\phi/k_B T}}$$

where A = Emission Coefficient $= [1.20 \times 10^6]$ amp/m^2 - deg^2 K

Notes

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$-P\,dP$

$\frac{kT}{m}\left(e^{-\infty} - e^{-\omega/kT}\right)$

$\frac{kT}{m}\left(e^{-\omega/kT}\right)$

$n = \frac{2kT}{m} \times \pi \times \frac{2m^2}{h^3} \times \frac{kT}{m} e^{-\omega/kT} \times e^{E_F/kT}$

$e^{-(\omega - E_F)/kT}$

$n = \frac{4\pi k^2 T^2 m}{h^3}$

$\frac{1}{2} M v_x^2 = \frac{P\,kT^2}{m}$

$J = nevd$

$\int v_x\, dx = \frac{P\,dP}{m}\left(\frac{kT}{m}\right)$

$\boxed{J = A T^2 e^{-\phi/kT}}$

$A = 1.20 \times 10^6\ Amp/m^2$

$\int \frac{kT}{m} e^{-\omega/kT}$

$\frac{kT}{m}\left(e^{-\infty} - e^{-\omega/kT}\right)$

$\frac{kT}{m}\left(e^{-\omega/kT}\right)$

$n = \frac{kT}{m} \times \frac{2kT}{m} \times \pi \times \frac{2m^3}{h^3} \times e^{E_F/kT}\, e^{-\omega/kT}$

$n = \frac{4\pi k^2 T^2 m}{h^3} e^{-(\omega - E_F)/kT}$

$J = nevd$

$\boxed{J = A T^2 e^{-\phi/kT}}$

Make your will the master of your body in small things and when you want something big it will not be difficult for you to get it.

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