London Equation & Penetration Depth:

In order to explain Meissner effect in superconduc. H. London proposed superconductivity. The maxwell equation of electromagnetic wave fail to explain the ideal diamagnetism and zero electrical resistance. So in 1935 London Brother modified the four maxwell equation and desired new equation, that explain the magnetic and electrical properties of superconductor. Now the four maxwell equation are.

(i) $\text{Div } H = 0$ ——— (1)

(ii) $\text{Curl } E = -\frac{dB}{dt}$ ——— (2)

(iii) $\text{Curl } H = J$ ——— (3).

We know that when a current flow through the superconductor electron are accelerated with constant vel. They moves this vel. year to year. Thus force experience by the electron $\boxed{F = ma = m\frac{dv}{dt}}$

If each electron having charge is $(-e)$ then force experienced by electron

$F = -eE$

Bohemian

Now equation eq. of force we get

$m\frac{dv}{dt} = -eE$ or $\boxed{\frac{dv}{dt} = -\frac{eE}{m}}$ — (4)

Suppose a substance contain n electron and each having a charge $(-e)$ they are accelerated with constant vel. (v) then the current density is given as. $J = -nev$ Diff. this both side

$\frac{dJ}{dt} = -ne\frac{dv}{dt}$ — (5) putting value of (4) in (5) we get

$\boxed{\frac{dJ}{dt} = \frac{ne^2}{m}E}$ → (6) This is known as Ist london equation. According to london theory there are two type of electron in superconductors (i) Normal electron (ii) Super electron.

Normal electron does not respond to electric field only super electron are assumed to respond to electric field. Taking maxwell eq. (2). That is

$\text{Curl } E = -\frac{dB}{dt} = -\mu_0\frac{dH}{dt}$ → (7) $[B = \mu_0 H]$ →

Taking curl of eq. (6) on both side we get

$\text{curl } \frac{dJ}{dt} = \frac{ne^2}{m}\text{curl } E$ — (8)

Putting value of eq. (7) in eq. (8) we get

$\text{curl}\frac{dJ}{dt} = -\frac{ne^2}{m}\mu_0\frac{dH}{dt}$

March
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21
22 23 24 25 26 27 28 29 30 31
S M T W T F S S M T W T F S S M T W T F S

Thus $\text{curl}\,\frac{dJ}{dt} = -\frac{\mu_0 n e^2}{m}\cdot\frac{dH}{dt}$.

Integrating we get $\int \text{curl}\,\frac{dJ}{dt} = -\frac{\mu_0 n e^2}{m}\int\frac{dH}{dt}$

$\Rightarrow \text{curl}\,J = -\frac{\mu_0 n e^2}{m}(H - H_0)$.

Where H_0 is constant of integration and represent field inside the super conductor at $t=0$. But field inside of super conductor is zero ($H_0 = 0$) According to meissner effect. Putting $H_0 = 0$ we get

$$\boxed{\text{curl}\,J = -\frac{\mu_0 n e^2 H}{m}} \rightarrow (9)$$

This is known as london second eq.

Penetration Depth:- Taking maxwell eq. that is eq.

$\text{curl}\,H = J \rightarrow (10)$ [From eq. 3]

Taking curl of eq. (10) on both side we get

$\text{curl}\,\text{curl}\,H = \text{curl}\,J$.

By Def. $\text{curl}\,\text{curl}\,H = \text{grad}\,\text{div}\,H - \text{grad}.\text{grad}\,H$.

$\text{grad}\,\text{div}\,H - \text{grad}\,\text{grad}\,H = \text{curl}\,J$.

But $\text{div}\,H = 0$.

$-\text{grad}\,\text{grad}\,H = \text{curl}\,J$.

$\Rightarrow -\nabla.\nabla H = \text{curl}\,H \Rightarrow -\nabla^2 H = \text{curl}\,J$

Putting value of curl J in eq. (9) we get

$$-\nabla^2 H = -\frac{\mu_0 n e^2}{m}H. \quad \text{or} \quad \nabla^2 H = \frac{\mu_0 n e^2}{m}H. \rightarrow (11)$$

$$\boxed{\text{Let } \lambda^2 = \frac{m}{\mu_0 n e^2}}$$

Then the eq. (11) become

$$\nabla^2 H = \frac{H}{\lambda^2}.$$

Where λ is called Penetration depth.