

# Fresnel's Bi Prism

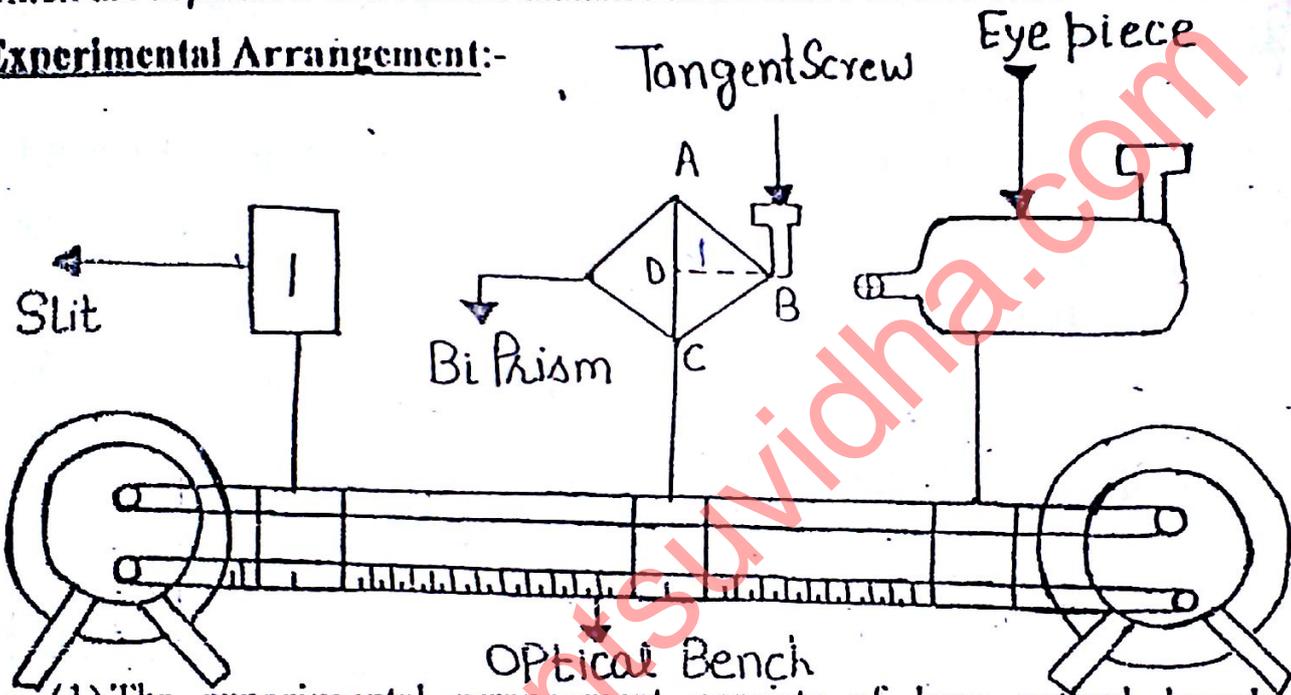
## Construction:-

Fresnel Bi Prism is a combination of two acute angle prism placed base to base in such a way that whose obtuse angle are made  $179$  degree and other two angles are  $\frac{1}{2}$  Degree each.

## Use of Fresnel Bi Prism:-

Fresnel Bi Prism is used to produce two coherent image of given slit which are separated at a certain distance and behave as two coherent sources.

## Experimental Arrangement:-



- (1) The experimental arrangement consists of long optical bench and contains three stands first stand for the slit, second stand for Bi Prism and third stand for eye piece.
- (2) The slit, Bi Prism, and eye piece are adjusted at the same height.
- (3) All the three stands can move along as well as right angle to the length of the optical bench.
- (4) The slit is made narrow, vertical and illuminated by source of monochromatic light.
- (5) The light emerging from the slit and allowed to fall on the Bi Prism. Then edge B of Bi Prism divide incident wave front into two parts. One is ABD and other BCD. But edge B of Bi Prism is combination of two Prisms. The light emerging from the edge B fall away from the normal. (When light rays goes from denser medium to rare medium bends away from the normal)

Fresnel's Bi Prism

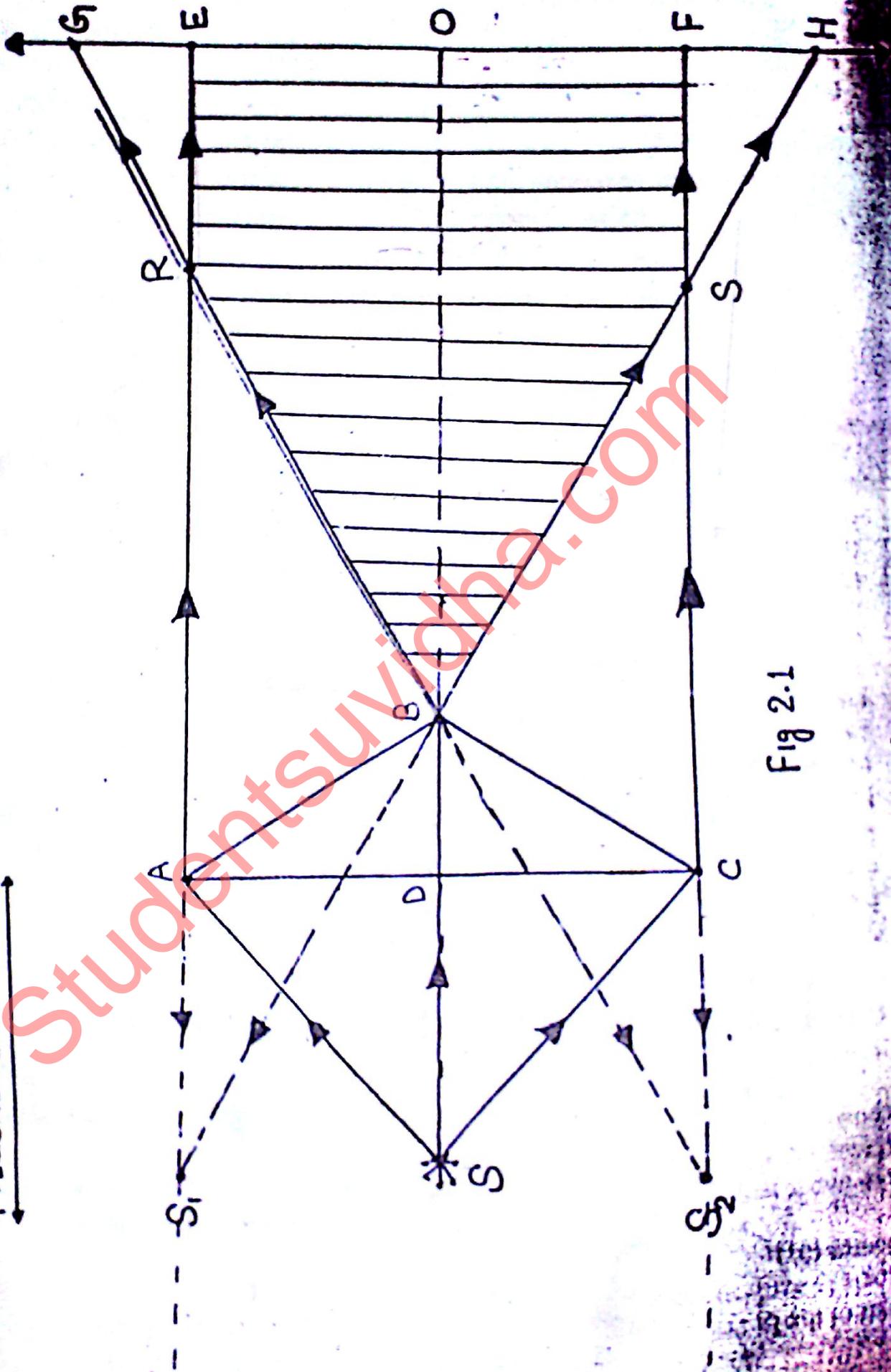


Fig 2.1

When the light from the source (S) is allowed to fall on the upper part of the Bi Prism then the upper part of the bi prism deviate it through a small angle towards the lower side of the diagram and appears to come from any other source (S<sub>1</sub>)

Similarly when the light from the source (S) is allowed to fall on the lower part of the Bi Prism then the lower part of the Bi prism deviate it through a small angle towards the upper side of the diagram and appears to come from any other source (S<sub>2</sub>) instead of S.

So Bi Prism can be used to produce two vertical images of the slit which are separated by a distance 2d and behave as two coherent sources.

Interference Pattern:-

The interference patterns are obtained in the overlapping region EF and can be seen in the field of view of eye piece. Because in EF region AE and BC overlap at R, and BH and CF overlap at S. So interference takes place in the region EF. That is shown in fig 2.1

Determination of the wave length of light

Let us suppose that S be the source of monochromatic light and S<sub>1</sub> and S<sub>2</sub> are two virtual image produced by the Bi prism and separated at certain distance and behave like as a two coherent source. Let us suppose that XY be the screen that is eye piece placed parallel to S<sub>1</sub> and S<sub>2</sub>. Let us suppose that P be the point on the screen at a distance (x). here we want to find out intensity of light is either maximum or minimum that depend upon path difference between both of the two ray thus path difference between both of the two rays will be: - S<sub>2</sub>P - S<sub>1</sub>P

First of all calculate value of S<sub>2</sub>P and S<sub>1</sub>P.

In order to calculate value of S<sub>2</sub>P. Draw a perpendicular from S<sub>2</sub> on XY.

By taking right angle triangle S<sub>2</sub>RP.

In this right angle triangle PR = x+d, S<sub>2</sub>R = D.

Applying Pythagoras Theorem S<sub>2</sub>P<sup>2</sup> = S<sub>2</sub>R<sup>2</sup> + RP<sup>2</sup>

(S<sub>2</sub>P)<sup>2</sup> = D<sup>2</sup> + (x+d)<sup>2</sup>

Or S<sub>2</sub>P = [D<sup>2</sup> + (x+d)<sup>2</sup>]<sup>1/2</sup> = D [1 + (x+d)<sup>2</sup>/D<sup>2</sup>]<sup>1/2</sup>

Apply Binomial Theorem

S<sub>2</sub>P = D [1 + (x+d)<sup>2</sup>/D<sup>2</sup>]<sup>1/2</sup> Negative higher Power  
{Because X << D, d << D}

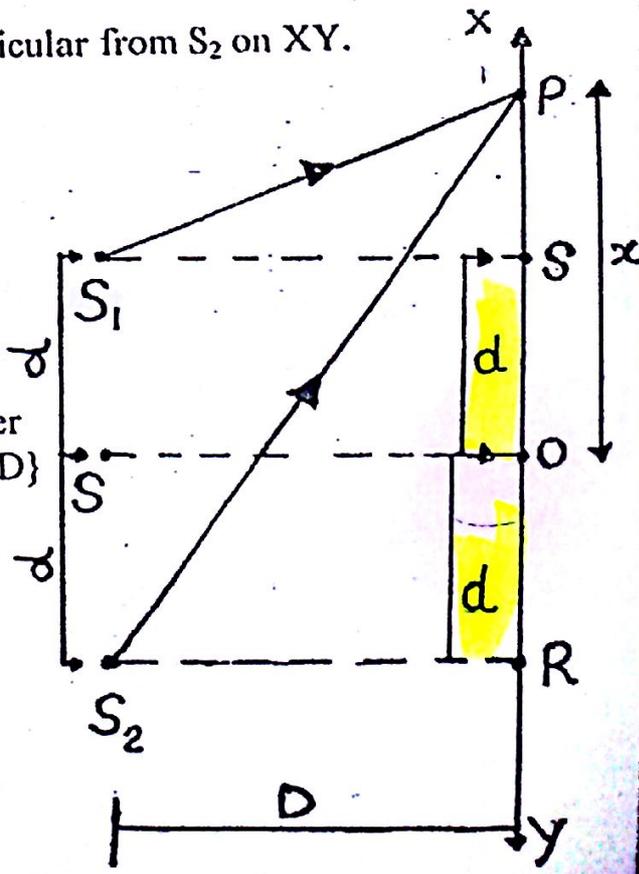
S<sub>2</sub>P = D + (x+d)<sup>2</sup>/2D

Similarly we can calculate value of S<sub>1</sub>P

Draw a perpendicular From S<sub>1</sub> on XY.

By taking right angle triangle S<sub>1</sub>SP

In this right angle triangle S<sub>1</sub>S = D, SP = x-d.



Applying Pythagoras Theorem

$$(S_1P)^2 = (SP)^2 + (S_1S)^2$$

$$(S_1P)^2 = (x-d)^2 + D^2$$

$$\text{or } (S_1P) = [(D^2 + (x-d)^2)]^{1/2}$$

$$(S_1P) = \frac{D [1 + (x-d)^2]^{1/2}}{D}$$

Apply Binomial Theorem

$$S_1P = \frac{D [1 + (x-d)^2]^{1/2}}{2D} \quad \text{Neglecting higher Power}$$

$2D)^2 \quad \{ \text{Because } x \ll D, d \ll D \}$

$$S_1P = \frac{D + (x-d)^2}{2D}$$

Putting value of  $S_1P$  and  $S_2P$  in eq. (1) we get.

$$S_2P - S_1P = \left[ \frac{D + (x+d)^2}{2D} \right] - \left[ \frac{D + (x-d)^2}{2D} \right]$$

$$= \frac{(x+d)^2}{2D} - \frac{(x-d)^2}{2D}$$

$$= \frac{4xd}{2D} = \frac{2xd}{D}$$

Now intensity at a point P is maximum only when path difference is equal to  $n\lambda$ .

Thus in such a case

$$\frac{2xd}{D} = n\lambda \quad \text{or} \quad x \Rightarrow \frac{n\lambda D}{2d}$$

For first bright fringe  $n=1$

$$x_1 = \frac{D\lambda}{2d}$$

For second bright fringe putting  $n=2$

$$x_2 = \frac{2D\lambda}{2d}$$

For third bright fringe putting  $n=3$

$$x_3 = \frac{3D\lambda}{2d} \quad \text{and so on}$$

Fringe Width:-

The differences between any two consecutive bright fringes are called fringe width and are denoted by " $\beta$ " which is given by:

$$\beta = x_2 - x_1 = \frac{2\lambda D}{2d} - \frac{\lambda D}{2d} = \frac{\lambda D}{2d}$$

$$\text{Thus } \beta = \frac{\lambda D}{2d} \quad \text{or} \quad \lambda = \frac{2d \cdot \beta}{D}$$

For determining the wave length of the light following measurements are

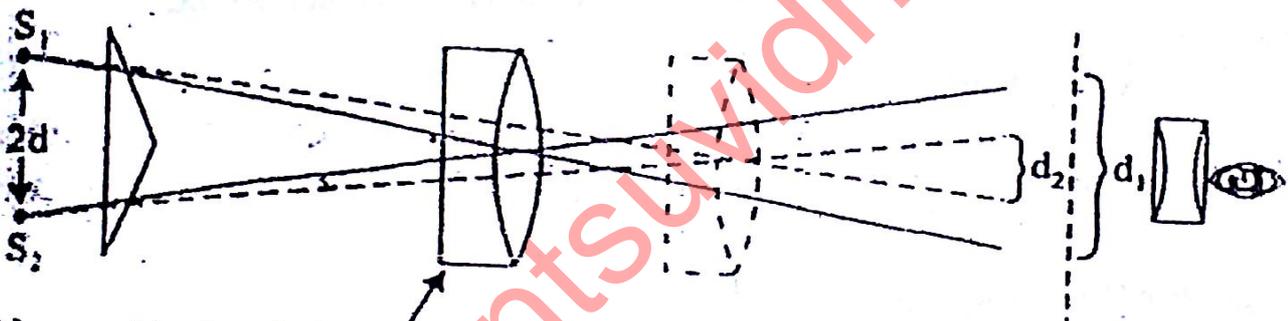
### Measurement of Fringe Width :-

The cross wire of eye piece are adjusted on the first bright fringe and the reading of the screw is noted after that eyepiece is moved on the second bright fringe and the cross wire of the eye piece is adjusted and reading of screw is noted. This procedure is repeated on each and every bright fringes and reading of screw is noted thus the difference between any two constructive bright fringes which gives the fringe width saying ( $\beta$ ).

(2) Measurement of D: -Distance between slit and eye piece can be read directly on the optical bench.

### (3) Measurement of 2d:-

Now take a convex lens whose focal length less than one forth of the distances between bi prism and eye piece and that convex lens placed in between bi prism and eye piece. The lens is adjusted at that point where the sharp image of  $S_1$  and  $S_2$  are obtained in the field of view of telescope.



Now with the help of tangent screw the cross wire of eye piece are adjusted on the first image reading of screw noted. With the help of tangent screw the cross wire of the eye piece is adjusted on the second image and reading is to be noted. Now difference between both the two reading gives the image distance say it be ( $d_1$ ). Thus

$$\text{Magnification: } \frac{\text{Image distance}}{\text{Object distance}} = \frac{I}{O} = \frac{d_1}{2d} = \frac{V}{u} \quad \text{--- (1)}$$

After that the position of the lens are adjusted at that point when object distance is made of image distance and image distance is made of object distance after that sharp image of  $S_1$  and  $S_2$  are again view in the field of view of telescope

Thus with the help of tangent screw the cross wire of eye piece are adjusted on the first image and reading of screw is noted. With the help of tangent screw the cross wire of eye piece are adjusted on the second bright fringe and reading of screw is noted. Difference between both of the two reading give's image distance say ( $d_2$ )

Application: Image distance =  $\frac{1}{O} = \frac{d_2}{2d} = \frac{u}{v}$  ----- (2)

Multiply eq. (1) And (2) we get,

$$\frac{d_1}{2d} \times \frac{d_2}{2d} = \frac{u}{u} \times \frac{v}{v} = 1$$

$$\frac{d_1 d_2}{(2d)^2} = 1 \text{ or } d_1 d_2 = (2d)^2$$

$$2d = \sqrt{d_1 d_2}$$

### Application of Fresnel Bi Prism:-

#### Determination of thickness of a thin transparent sheet:-

Let us consider a thin sheet of mica having thickness  $t$  and its refractive index be  $\mu$  is placed in the path of any one of the interfering beam then the entire fringe pattern is displaced through a constant distance toward the path of the beam in which the mica sheet is introduced.

Now consider the path  $S_1P$ , The length of this path  $[S_1P-t]$  is traveled in air with velocity of light  $C$ . While the length  $t$  of this path is traveled in mica with the velocity  $C_g$ . Where  $C_g$  is the velocity of light in mica then total time taken by the light to cover the distance is given by:-

$$T = \frac{(S_1P-t)}{C} + \frac{t}{C_g} \quad [\text{Velocity} = \text{distance} / \text{time}] \dots \dots \dots (1)$$

By definition we know that  $\mu = C/C_g$  or  $C_g = C/\mu$

Putting value of  $C_g$  in eq. 1 we get

$$T = \frac{S_1P-t}{C} + \frac{\mu t}{C} = \frac{1}{C} [S_1P + (\mu - 1)t]$$

Thus the path  $S_1$  to  $P$  is equivalent to an air path.

Distance = vel. X time

$$= C \times \frac{1}{C} [S_1P + (\mu - 1)t] = [S_1P + (\mu - 1)t]$$

Now path difference between two rays will be:-

$$S_2P - [S_1P + (\mu - 1)t] = S_2P - S_1P - (\mu - 1)t \dots \dots \dots (2)$$

But  $S_2P - S_1P = \frac{2xd}{D}$  already calculated. Putting we get path difference between

two rays will be  $\frac{2xd}{D} - (\mu - 1)t \dots \dots \dots (3)$

Thus intensity of light will be maximum only when path difference equal to  $n\lambda$ .

thus  $\frac{2xd}{D} - (\mu - 1)t = n\lambda \Rightarrow \frac{2xd}{D} = [n\lambda + (\mu - 1)t]$

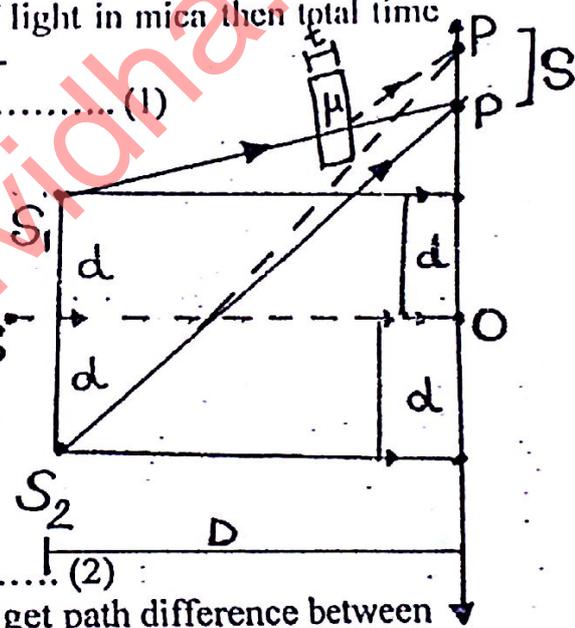
or value of  $x = \frac{D}{2d} [n\lambda + (\mu - 1)t] \dots \dots \dots (4)$

In the absence of mica sheet ( $t=0$ ) then  $n$ th maxima is obtained at a distance

$$x_1 = Dn\lambda / 2d \dots \dots \dots (5)$$

But if mica sheet is introduced then  $n$ th maxima is displayed through a distance

$$x_2 = D [n\lambda + (\mu - 1)t] / 2d \dots \dots \dots (6)$$



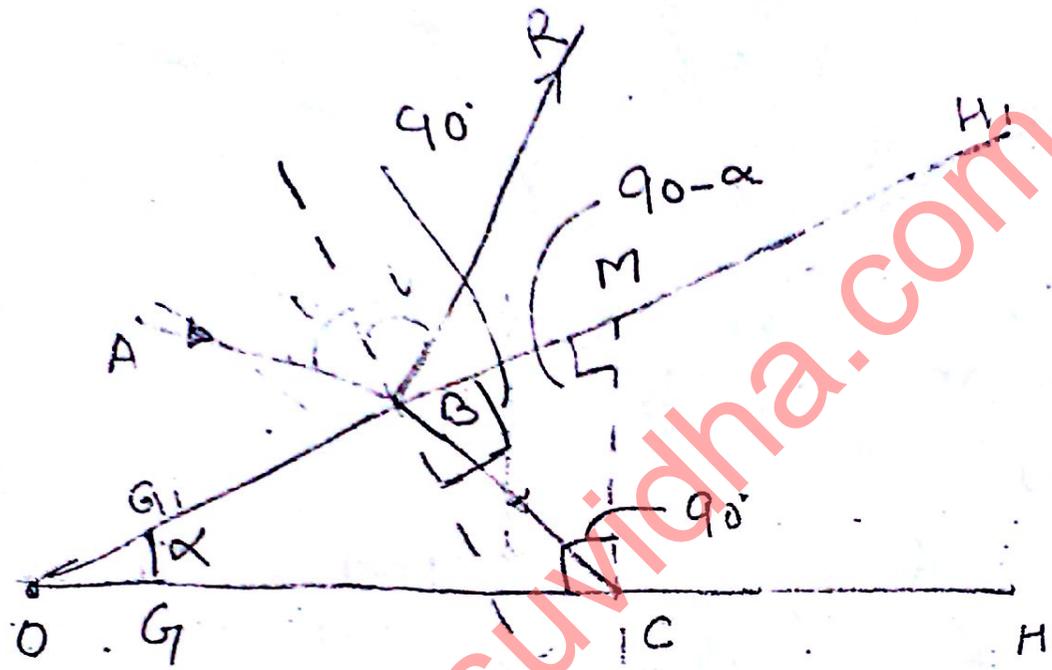
$S$  is the displacement of the  $n$ th maxima when mica is introduced then

$$\begin{aligned} S_2 - S_1 = x_2 - x_1 &= \frac{D}{2d} [nd + (\mu - 1)t] - \frac{Dnd}{2d} \\ &= \frac{D(\mu - 1)t}{2d} \dots \dots \dots (7) \end{aligned}$$

or value of  $t$  comes out to be

$$t = \frac{S \times 2d}{D(\mu - 1)}$$

Thus by using Fresnel bi prism we can calculate thickness of mica sheet.



In  $\Delta OMC$

$$\angle MCO = 90^\circ \text{ \& } \angle OMC = 90^\circ - \alpha$$

In  $\Delta BQM$

$$\angle B = 90^\circ \quad \angle M = 90^\circ - \alpha \quad \angle Q = 180^\circ - (90^\circ + 90^\circ - \alpha) = \alpha$$