

PHYSICS - II

Dec 2009

Paper Code: PHY-102-E

Figures in the bracket indicate full marks.

Note: Attempt any five questions, selecting at least two questions from each part.

PART-A

Q.1 (a) Explain clearly the concept of Miller indices. Deduce formula for the distance between two adjacent planes of a simple cubic lattice. (10)

Ans. Miller indices: In crystal structure analysis it is more useful to specify the orientation of a plane by another set of three numbers called "miller indices" which are obtained as follows:

The lattice points forming a space lattice may be thought of as occupying various sets of parallel planes.

(i) Take any atom as the origin in the crystal and construct co-ordinate axes from this atom in the direction of crystal axes a, b, c. The axes may be primitive or non-primitive.

(ii) Find the intercepts on the axes a, b, c in terms of lattice constants.

(iii) Take a reciprocals of these numbers. If fractions result, reduce them to smallest three integers, by multiplying with lowest common denominator. The result is enclosed in brackets (h k l) and called Miller indices.

numbers are $\frac{1}{3}, \frac{1}{2}, \frac{1}{2}$. The smallest three integers having the same ratio are 2, 3, 3 and thus the miller indices of the plane are (233)

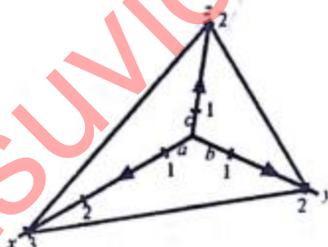


Fig. Plane for determining "Miller indice".

miller indices is a system of notation that denotes the orientation of the faces of a crystal the planes and directions of atoms within that crystal.

Simple Cubic : In this structure the atoms are situated at the corners of cube touching each other along the edges. The co-ordination number of cubic structure is 6. Each atom at the corner is shared by eight cubes so that number of atoms per unit cell will be one. The volume occupied by the atom in the unit

cell is $\frac{4}{3}\pi r^3$, where r is the atomic radius and is equal to a/2, a being the cube edge. Volume of the unit cell is a³. The packing fraction

$$f = \frac{4/3\pi(a/2)^3}{a^3} = \pi/6 = 52\%$$

Q.1 (b) Show that the number of Frenkel defects in equilibrium at a given temperature is proportional to $NN_i^{1/2}$, where N and N_i are number of atoms and interstitials respectively. (10)

Ans. Refer Q.1 (b) of paper Dec 2008.

Q.2 (a) What is the difference between phase and group velocities? Show that the group velocity associated with wave packet is equal to the velocity of relativistic particle. (10)

Ans. Difference between phase and group velocity phase velocity : When a single wave of a definite wave lengths travels in a medium, its velocity of propagation in the medium is called the wave velocity or the phase velocity.

$$\text{Phase velocity}(v) = \frac{\omega}{k}$$

Group Velocity: When a number of wave of different wavelengths are moving with different velocity in a medium, and this is the velocity of wave packet formed by the waves, this is called the group velocity. Group velocity is less then the phase velocity.

$$\text{Group velocity}(v) = \frac{d\omega}{dk}$$

Relation between phase velocity and group velocity:

put $k = \frac{2\pi}{\lambda}$ in group velocity, we get

$$v = \frac{d\omega}{d\left(\frac{2\pi}{\lambda}\right)}$$

$$= -\frac{\lambda^2}{2\pi} \frac{d\omega}{d\lambda}$$

putting $\omega = 2\pi\nu$, we get

$$v = \frac{-\lambda^2 dv}{d\lambda}$$

we know that,

$$\text{the phase velocity}(u) = \frac{\omega}{k}$$

$$\omega = ku$$

$$= \frac{2\pi}{\lambda} \times u$$

put $\omega = \frac{2\pi}{\lambda} \times u$ in equation (i)

$$v = \frac{-\lambda^2}{2\pi} \frac{d}{d\lambda} \left(\frac{2\pi u}{\lambda} \right)$$

$$= -\lambda^2 \left(\frac{-u}{\lambda^2} + \frac{1}{\lambda} \frac{du}{d\lambda} \right)$$

$$v = u - \lambda \frac{du}{d\lambda}$$

The above equation shows that group velocity v is smaller than the phase velocity u . In the absence of dispersion in the medium we have $\frac{du}{d\lambda} = 0$

so $v = u$

Thus for a non-dispersive medium the group velocity is equal to the particle velocity.

Q.3 (a) Derive Richardson's thermoionic emission equation.

(15)

Ans. Richardson Equation : In Richardson Equation first to calculate $n(p)$. This quantity is given by the product of the number of possible states of momentum p , per unit volume and the probability that an electron is in each state. The probability factors is given by the Fermi Dirac distribution function, and the former factor is calculated as follows :

Now it is shown that number of states per unit volume with energy between E and $E + dE$ is

$$D(E)dE = \frac{1}{4\pi^2} \left(\frac{2m}{\hbar^2} \right)^{3/2} E^{1/2} dE$$

using $E = \frac{p^2}{2m}$ and $dE = \frac{p}{m} dp$, we can write the number of states per unit volume with momentum between p and $p + dp$ as

$$D(p) = \frac{1}{4\pi^2} \left(\frac{2m}{\hbar^2} \right)^{3/2} \left(\frac{p^2}{2m} \right)^{1/2} \frac{p}{m} dp$$

or
$$D(p) = \frac{8\pi}{h^3} p^2 dp \quad \dots(i)$$

This is the momentum values of p in any direction while we want momenta in x -direction only. We want to calculate the number of electrons $n(p_x)$ which have the momentum lying in range dp_x at p_x when p_x and p_x are arbitrary. Thus we construct a plot in 'momentum space', such that each point represents a particular combination of the momentum components p_x , p_y , and p_z with

$$p = \sqrt{p_x^2 + p_y^2 + p_z^2}$$

This will be a spherical shell with radius p and thickness dp . This shell would contain all the states with momentum between p and $p + dp$. In other words, the number of states in volume element $4\pi p^2 dp$ (volume of the shell) of momentum space is $D(p)dp$, as expressed by equation (i). The number of states in the volume elements dp_x , dp_y , dp_z is given by

$$\begin{aligned} D(p_x, p_y, p_z) dp_x dp_y dp_z &= \frac{D(p)dp}{4\pi p^2 dp} dp_x dp_y dp_z \\ &= \frac{8\pi p^2 dp}{h^3 4\pi p^2 dp} dp_x dp_y dp_z \end{aligned}$$

$$D(p_x, p_y, p_z) dp_x dp_y dp_z = \frac{2}{h^3} dp_x dp_y dp_z \quad \dots(ii)$$

This gives the number of states per unit volume with momentum component in the x -direction lying between the values p_x and $p_x + dp_x$ and similarly for p_y and p_z . The number of these states which is occupied by electrons is given by

$$n(p_x, p_y, p_z) dp_x dp_y dp_z = \frac{2}{h^3} \frac{dp_x dp_y dp_z}{\exp\left\{ \frac{(E - E_f)}{k_B T} \right\} + 1}$$

where $\frac{1}{\exp\left\{ \frac{(E - E_f)}{k_B T} \right\} + 1}$ is the Fermi - Dirac function.

This is the number of electrons occupying states with momenta between p_x and $p_x + dp_x$, p_y and $p_y + dp_y$, p_z and $p_z + dp_z$. The number of electrons per unit volume having momentum values p_x and $p_x + dp_x$ in x -direction is given by

$$n(p_x) dp_x = \frac{2}{h^3} dp_x \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dp_y dp_z}{\exp\{(E - E_F)/k_B T\} + 1} \quad \text{---(iii)}$$

Now, since only those electrons which have energies greater than E_F can be emitted, we should consider the integrand $E > E_F$, i.e. $E_n - E_F = \phi \gg k_B T$ for all metals at temperature below melting point, the term unity in the denominator of the integrand may be neglected. Putting,

$$E_n = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2)$$

Equation (iii) becomes

$$\begin{aligned} n(p_x) dp_x &= \frac{2}{h^3} dp_x \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{E_n}{k_B T}} e^{-\frac{E_F}{k_B T}} dp_y dp_z \\ &= \frac{2}{h^3} dp_x \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\frac{(p_x^2 + p_y^2 + p_z^2)}{2mk_B T}} \exp\left(\frac{E_F}{k_B T}\right) dp_y dp_z \\ &= \frac{2}{h^3} dp_x e^{\frac{E_F}{k_B T}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left(-\frac{p_y^2 + p_z^2}{2mk_B T}\right) dp_y dp_z \end{aligned}$$

The integrals are standard integrals and have values $\sqrt{2\pi mk_B T}$ each.

$$\begin{aligned} n(p_x) dp_x &= \frac{2}{h^3} dp_x e^{\frac{E_F}{k_B T}} e^{-\frac{p_x^2}{2mk_B T}} \left(\sqrt{2\pi mk_B T}\right)^2 dp_x \\ &= \frac{4\pi mk_B T}{h^3} e^{\frac{E_F}{k_B T}} e^{-\frac{p_x^2}{2mk_B T}} dp_x \end{aligned}$$

Now we know that

$$j = \frac{e}{m} \int_{p_{min}}^{\infty} p_x n(p_x) dp_x$$

Putting this value in equation (iv), we get the emission current density

$$j = \frac{e}{m} \int_{p_{min}}^{\infty} \frac{4\pi mk_B T}{h^3} e^{\frac{E_F}{k_B T}} e^{-\frac{p_x^2}{2mk_B T}} p_x dp_x$$

also writing $\frac{p_x^2}{2m} = E_F + \phi$ and $p_x dp_x = m d\phi$

$$j = \frac{e}{m} \int_{p_{min}}^{\infty} \frac{4\pi mk_B T}{h^3} e^{\frac{E_F}{k_B T}} e^{-\frac{E_F + \phi}{k_B T}} e^{-\frac{p_x^2}{2mk_B T}} m d\phi$$

or

$$= \frac{e}{m} \frac{4\pi mk_B T}{h^3} m \int_{\phi}^{\infty} e^{-\frac{\phi}{k_B T}} d\phi$$

$$j = \frac{4\pi me k_B^2}{h^3} T^2 e^{-\frac{\phi}{k_B T}}$$

$$j = A T^2 e^{-\frac{\phi}{k_B T}}$$

Equation(v) is called Richardson-Dushman equation. The quantity A is the emission coefficient

$$A = \frac{4\pi m e k_B^2}{h^3} = 1.20 \times 10^8 \text{ amp/m}^2 - \text{deg}^2\text{K}$$

Q.3(b) At what temperature we can expect a 10% probability that electrons in silver posses an energy which is 1% above the Fermi-energy. Fermi-energy of silver is 5.5 eV. (5)

Ans. Given data: $f(\epsilon) = 10\% = \frac{10}{100} = 0.1, \epsilon_F = 5.5 \text{ eV}$

$$\epsilon = \epsilon_F + \frac{1}{100} \epsilon_F$$

$$\Rightarrow \epsilon - \epsilon_F = \frac{1}{100} \times 5.5 \text{ eV} = 0.055 \text{ eV} = 0.055 \times 1.6 \times 10^{-19} \text{ J.}$$

Substituting these values in Fermi-function expression

$$f(\epsilon) = \frac{1}{1 + e^{(\epsilon - \epsilon_F) / k_B T}}$$

$$\Rightarrow 0.1 = \frac{1}{1 + \exp\left(\frac{0.055 \times 1.6 \times 10^{-19}}{1.35 \times 10^{-23} T}\right)}$$

or $1 + \exp\left(\frac{637.7}{T}\right) = 10 \Rightarrow \exp\left(\frac{637.7}{T}\right) = 9$

$$\frac{637.7}{T} = \ln 9$$

$$T = \frac{637.7}{\ln 9} = \frac{637.7}{2.197} = 290.3 \text{ K.}$$

Q.4 (a) Discuss Laue method for crystal structure determination. (10)

Ans. Refer Q.1(a) of paper Dec 2008.

Q.4 (b) Derive the expression for conductivity of metal on the basis of Drude-Lorentz theory. (10)

Ans. Refer Q.1(iii) of paper May 2008.

Part - B

Q.5 (a) Discuss band theory of solids and explain the formation of band. (15)

Ans. **Band theory of solids:** The theory certainly explains several electronic properties of metals e.g., specific heat, paramagnetism etc., yet there are several other properties of solids for which the free electron model helps in no way. For example it does not help to distinguish between a metal and an insulator; the resistivity of a good conductor at low temperatures may be of the order of 10^{-19} ohm-metre and that of a good insulator may be as high as 10^{20} ohm-metre. Further the theory does not explain the behaviour of semi-conductors. Therefore the free electron theory must be modified.

We know that the electrons revolving around the nucleus of an isolated atom posses energy states 1s, 2s, 2p, and they occupy the quantum states available in these energy levels. Due to periodicity of lattice, each atom is in the electrostatic field of the neighbouring atoms. Consequently the

energy levels of an individual atom lose their validity. If the crystal contains N -atoms, then due to interaction between the atoms, each discrete energy level in an individual atom splits into N -close sub-levels. As N is very large ($N = 10^{23}/\text{cm}^3$), the separation between sub-levels is very small so that these discrete levels are almost continuous and are said to form an energy band. These energy bands are in general, separated by regions which have no allowed energy levels. These regions are called forbidden bands or energy gaps or band gaps. In some crystals, the adjacent energy bands overlap. The number of levels in such a merged band is equal to sum of the number of the levels into which both levels of the atoms split up. Moreover the amount of splitting is not the same for different levels. Those filled up by the valence electrons in an atom are disturbed most, while those filled by inner electrons are disturbed only slightly.

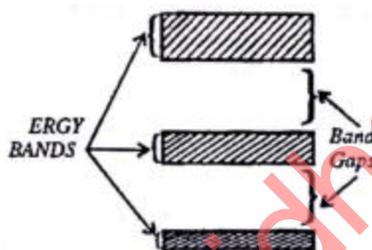


Fig.

The energy bands in a solid arise if free electron theory is extended to take into account the periodicity of the crystal lattice. These bands correspond to the energy levels of atomic electrons. An electron in a solid can have only those energies that fall within these energy bands. If the energy bands in a solid overlap, the electrons have a continuous distribution of allowed energies. The energy band structure of a solid and the occupation of energy bands by electrons determine the electrical properties of that solid.

Q.5 (b) Discuss effective mass of an electron and its physical significance. (5)

Ans. Refer Q.8(ii) of paper May 2008.

Q.6 (a) Derive the London equations and discuss how its solution explain Meissner effect. (15)

Ans. Refer Q.7 (a) of paper Dec 2008.

Q.6 (b) For a given specimen of a super conductor, the critical field are 4.2×10^5 and 1.2×10^5 amp/m for 14k and 13k respectively. Calculate the transition temperature. (5)

Ans. Given data: $T_1 = 14$ K, $T_2 = 13$ K

Critical field at temperature T_1 (H_{c1}) = 1.4×10^5 A/M

Critical field at temperature T_2 (H_{c2}) = 4.2×10^5 A/M

We know that,

$$H_c = H_n \left(1 - \left(\frac{T}{T_c} \right)^2 \right)$$

Thus the critical field at temp T_1 or T_2 are

$$Hc_1 = H_0 \left(\frac{T_c^2 - T_1^2}{T_c^2} \right)$$

$$Hc_2 = H_0 \left(\frac{T_c^2 - T_2^2}{T_c^2} \right) \quad \dots(ii)$$

Divide equation (i) by equation (ii) we get

$$\frac{Hc_1}{Hc_2} = \frac{T_c^2 - T_1^2}{T_c^2 - T_2^2} \quad \dots(iii)$$

Put the value of T_1 , T_2 , Hc_1 and Hc_2 in equation (iii)

$$\frac{1.4 \times 10^5}{4.2 \times 10^5} = \frac{T_c^2 - 14}{T_c^2 - 13}$$

Transition temperature (T_c) = 14.5K Ans.

Q.7 (a) Derive an expression for Bohr magneton. (5)

Ans. An atom consists of a positively charged nucleus at its centre and negatively charged electrons circulating around the nucleus. Each circulating electron produces electric current. Hence revolving electron is equivalent to a current loop which produces a magnetic field. Therefore, an atom is equivalent to a magnetic dipole. We now calculate the magnetic moment due to orbital motion of the electron.

Current produced due to an electron, $i = \frac{\text{Charge}}{\text{Time period}} = \frac{e}{T}$

If v is speed of electron in circular orbit of radius r , then

$$\text{Time period } T = \frac{2\pi r}{v}$$

$$\therefore \text{Current } i = \frac{e}{2\pi r/v} = \frac{ev}{2\pi r} \quad \dots(i)$$

Magnetic moment due to orbital electron

$$M = iA = \frac{ev}{2\pi r} \times (\pi r^2) = \frac{evr}{2} \quad \dots(ii)$$

From Bohr's condition of quantised orbits, angular momentum of electron,

$$L (= mvr) = \frac{nh}{2\pi}, n = 1, 2, 3, \dots$$

Where m is mass of electron,

$$\therefore vr = \frac{nh}{2\pi m}$$

Substituting this value in equation (ii), we get

$$M = \frac{e}{2} \cdot \frac{nh}{2\pi m} \text{ or } M = n \left(\frac{eh}{4\pi m} \right), n = 1, 2, 3, \dots$$

Thus magnetic moment due to orbital motion of electron is always an integral multiple of $\frac{eh}{4\pi m}$. The quantity $\frac{eh}{4\pi m}$ forms a natural unit for magnetic moment due to orbital motion and is called the Bohr magneton.

Q.7 (b) Show that magnetic susceptibility of diamagnetic substance is independent on temperature, on the basis of Langevin theory. (15)

Ans. Classical Theory of Diamagnetism : The usual derivation employs the Larmor theorem which "states that for an atom in a magnetic field the motion of the electrons is, to the first order in B, the same as a possible motion in the absence of B except for the super position of a precession of the electrons," with angular frequency

$$\omega_l = -\frac{eB}{2m} \quad \dots(i)$$

where ω_l is called Larmor frequency.

Larmor Precession : The precession of magnetic dipole moment μ_{cl} in a magnetic field is equivalent to the precession of angular momentum L about B. The equation of motion of vector L precessing about B, is

$$\frac{dL}{dt} = \tau$$

where τ is the torque $\mu_{cl} \times B$, which the field B exerts on the dipole

$$\therefore \frac{dL}{dt} = \mu_{cl} \times B$$

Now we know that $\mu_{cl} = -\left(\frac{e}{2m}\right)L$

$$\frac{dL}{dt} = -\left(\frac{e}{2m}\right)L \times B$$

This represents the precessional motion of L and B with an angular frequency

$$\omega_l = -\frac{eB}{2m}$$

known as the Larmor's frequency.

If the field is applied slowly, the motion in the rotation reference system will be the same as the original motion in the rest system before the application of the field. The Larmor precession of z electrons is equivalent to an electric current, given as

$$I = \text{charge} \times \text{revolution per unit time}$$

$$\text{or } I = (-ze) \left(\frac{1}{2\pi} \frac{eB}{2m} \right)$$

The magnetic moment μ of a current loop is given by the product of the current and area of the loop. If 'a' is the radius of the loop, then its area is πa^2 , thus

$$\mu = (-ze) \left(\frac{eB}{2\pi 2m} \right) \pi a^2 < a^2 >$$

$$\text{or } \mu = -\frac{ze^2 B}{4m} < a^2 >$$

where $\langle a^2 \rangle$ is the average radius of the electron from the field axis. If field is in z direction, then

$$\langle a^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle$$

The negative sign in equation (ii) indicates that the induced magnetic moment is always in opposite sense to that of the applied field. The mean square distance of the electrons from the nucleus is

$$\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + \langle z^2 \rangle$$

For a spherically symmetric charge distribution, we have

$$\langle x^2 \rangle = \langle y^2 \rangle = \langle z^2 \rangle, \text{ such that}$$

$$\langle r^2 \rangle = \frac{3}{2} \langle a^2 \rangle$$

Putting in equation (ii) we get

$$\mu = -\frac{ze^2 B}{4m} \frac{2}{3} \langle r^2 \rangle$$

or

$$\mu = -\frac{ze^2 B}{6m} \langle r^2 \rangle \quad \dots(iii)$$

If N is the number of atoms per unit volume, the $M = N\mu$ so equation (iii) becomes

$$M = -\frac{zNe^2 B}{6m} \langle r^2 \rangle \quad \text{where } B = \mu_0 H$$

\therefore

$$M = -\frac{\mu_0 Nze^2 H}{6m} \langle r^2 \rangle$$

or

$$\chi_{dia} = \frac{M}{H} = -\frac{\mu_0 Nze^2}{6m} \langle r^2 \rangle$$

This is diamagnetic susceptibility and is classical Langevin result. The above result shows that the diamagnetic susceptibility is directly proportional to the atomic number. It is independent of temperature. As the induced magnetic moment oppose the applied field, the diamagnetic susceptibility is negative. Further $\langle r^2 \rangle$ can be calculated by knowing the details of the wavefunction for all the electrons in the atom, but can be taken approximately of the order of the first Bohr radius. χ_{dia} for most materials is of the order of 10^{-6} and is a small effect.

Q.8 Write short note on the following:

(a) Hall effect,

(b) Concept of traps and its effect on photoconductivity.

(20)

Ans. (a) Hall effect : In a current carrying conductor (or semiconductor) is placed in a transverse magnetic field a voltage is produced perpendicular to both the direction of current and the direction of magnetic field. The voltage so developed is called Hall voltage and the phenomenon is called the Hall effect. The Hall voltage depends on the strength of the current and the magnetic field applied. The Hall effect may be explained by the motion of charged particles in electric and magnetic fields.

Consider a conductor having rectangular cross-section and carrying a current (I) along X-axis.

A uniform magnetic field B is applied along Z-axis, it is found that a voltage is developed along Y-axis in a direction both perpendicular to current (I) and magnetic field (\vec{B}). The voltage applied along X-axis causes electric field along positive X-axis and hence they gain a drift

velocity (v_x) along negative x -axis.

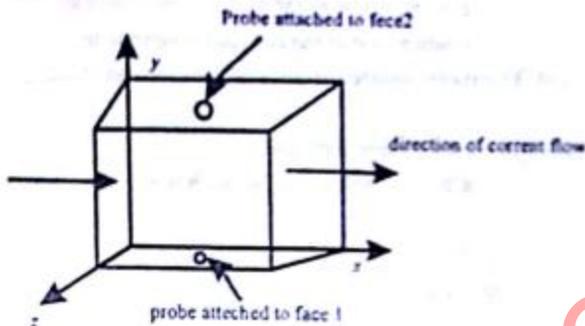


Fig: Geometry of Hall effect in a rectangular slab

(b) Concept of traps and its effect on photoconductivity: A trap is an impurity atom or other imperfection in the crystal capable of capturing an electron or hole; the captured carrier may be re-emitted at a subsequent time and may move to another trap. It is convenient to discuss models in which the holes are entirely trapped and a fraction of the electrons nominally in the conduction band are trapped at any instant. There are two types of traps, distinguished on the basis of their operation. One type helps electrons and holes to recombine and thereby assists in the restoration of thermal equilibrium. This type of traps are known as recombination centres. In the presence of traps recombination proceeds at much higher rate. Second type of traps does not contribute directly in an important way to recombination, but effects principally the freedom of motion of charge carriers of one sign or the other. This feature of traps is shown in Fig. below:

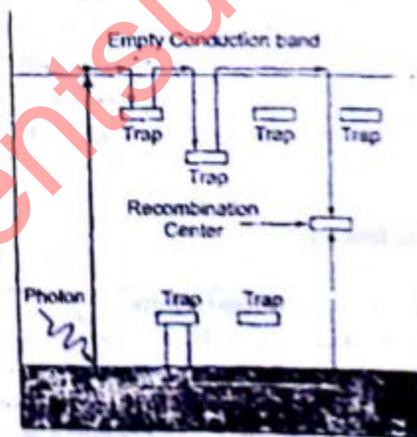


Fig. : Influence of traps in electron hole recombination

or
$$\bar{E}_s = 2 k_b T \log \left[\frac{(N-n)}{n} \right]$$

or
$$\frac{(N-n)}{n} = \exp \left(\frac{\bar{E}_s}{2k_b T} \right)$$

if $n \ll N$, then $N - n \cong N$, so that

$$n = N \exp \left(- \frac{\bar{E}_s}{2k_b T} \right)$$

Q.2. (a) What is Plank's constant? Discuss its importance. (8)

Ans. **Plank's constant:** The energy of black body oscillator of frequency ν cannot vary continuously but is limited to discrete set of value called quantum. This quantum of radiation is called 'photon'. Each photon has energy proportional to the frequency of radiation.

i.e $e \times \nu$

Here $e =$ photon energy

$\nu =$ frequency of radiation

$e = h\nu$

Where h is universal constant. called plank's constant.

Importance: in plank's quantum hypothesis plank's constant led to the important conclusion that radiation is not emitted in continuous fashion but in discrete packets or energy equal to $h\nu$. These packets of energy are called quanta or photons. In other words, it states that exchanges of energy between radiation and matter can not take place continuously but in discrete set of values. It may be remembered that photon nature of radiation regards that radiation exhibits a wave phenomenon but its energy contents are given in the atoms in the form of quanta. If the photon is regarded as particle of radiation, then we should find out its energy, mass, momentum etc.

Q.2.(b) Derive time independent Schrodinger wave equation for a free particle. (12)

Ans. **Time independent schrodinger wave equation for a free particle:** The schrodinger equation can be expressed as the function of spatial co-ordinates and a function of time. Consider a system of stationary waves to be associated with the particle. Let $\psi(r, t)$ be the wave characteristic for the de-Broglie wave at any location $r = ix + jy + kz$ at time t . Then the differential equation of the wave motion in three dimensions in accordance with Maxwell's wave-equation can be written as

i.e.,
$$\nabla^2 \psi = \frac{1}{u^2} \frac{\partial^2 \psi}{\partial t^2} \text{ or } \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{u^2} \frac{\partial^2 \psi}{\partial t^2}, \quad \dots(i)$$

where u is the wave velocity.

The solution of equation (i) given ψ as a periodic displacement in terms of time, i.e.,

$$\psi(r, t) = \psi_0 e^{i\omega t} \quad \dots(ii)$$

where ψ_0 is the amplitude at the point considered. It is function of position r i.e., of co-ordinates (x, y, z) and not of time t .

The equation (ii) may be expressed as

$$\psi(r, t) = \psi_0(r) e^{-i\omega t} \quad \dots(iii)$$

Differentiating equation (iii), twice with respect to t , we get

$$\frac{\partial \psi}{\partial t} = -i\omega \psi_0(r) e^{-i\omega t}$$

and

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi_0(r) e^{-i\omega t}$$

Substituting this in equation (i), we get

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = -\frac{\omega^2}{u^2} \psi \quad \dots(iv)$$

But

$$\omega = 2\pi\nu \\ = \frac{2\pi u}{\lambda}$$

i.e.,

$$\frac{\omega}{u} = \frac{2\pi}{\lambda} \quad \dots(v)$$

Also

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \nabla^2 \psi \quad \dots(vi)$$

Using (v) and (vi), equation (iv) becomes

$$\nabla^2 \psi + \frac{4\pi^2}{\lambda^2} \psi = 0 \quad \dots(vii)$$

So far we have not introduced wave mechanical concept and so that treatment is general.

For introducing the concept of wave mechanics, we must put for de Broglie equation

$$\lambda = \frac{h}{mv} \quad \dots(viii)$$

Substituting in equation (vii), we get

$$\nabla^2 \psi + \frac{4\pi^2 m^2 v^2}{h^2} \psi = 0 \quad \dots(ix)$$

If E and V are the total and potential energies of the particle respectively, then its kinetic energy

$\frac{1}{2}mv^2$ is given by

$$\frac{1}{2}mv^2 = E - V,$$

which gives $m^2v^2 = 2m(E - V)$.

Substituting this in equation (ix), we get

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0 \quad \dots(x)$$

The above equation is called *Schrodinger time independent wave equation*. The quantity ψ is usually referred as *wave function*.

Let us now substitute in equation (x),

$$\hbar = \frac{h}{2\pi} \quad \dots(xi)$$

Then the Schrodinger time-independent wave equation, in usually used form, may be written as

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \dots(xii)$$

This is Schrodinger's time independent wave equation.

Q.3. (a) Discuss the motion of free electron in a periodic potential and discuss. Fermi-Dirac distribution function. (14)

Ans. Motion of free electron: The valence electrons of all atoms of the metal move about freely through the volume of the specimen. The valence electron of atoms are responsible for the conduction of electricity by the metal and for this reason these electrons are called conduction electrons. The conduction electrons move about inside the specimen without any collisions, much like the molecules of an ideal gas. The forces between the conduction electrons and ion cores are neglected in the free electron approximation; all calculations proceed as if the conduction electrons were free to move every where within the specimen. The total energy is all kinetic energy; the potential energy is neglected. However, the free electron gas in metals differs from ordinary gas in some respects. First the free electron gas is charged while the ordinary gas molecules are mostly neutral. Second the concentration of electrons in metals is large than the ordinary gas.

Fermi-Dirac distribution function: To discuss the effect of temperature, it is clear that at non-zero temperature, the distribution of filled states would not end abruptly, rather some states below E_f would be empty and some above it would be filled. E_f Fermi has shown that the probability that a particular quantum state of energy E is filled is given by the function, called Fermi-Dirac distribution function

$$F(E) = \frac{1}{\exp\left\{\frac{(E-E_f)}{k_B T}\right\} + 1}$$

where E_f is the Fermi energy. Fig. shows the plot of this function for several values of temperature including zero. Particles which obey Fermi-Dirac statistics, having half integral spin, are called Fermions.

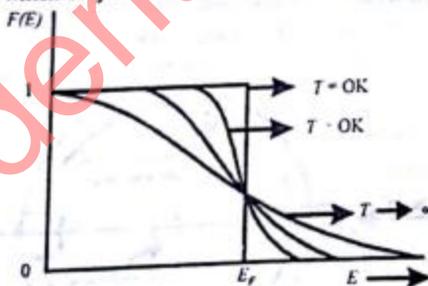


Fig. Fermi distribution function at different temperatures

Q. 3.(b) What is density of states? Discuss briefly. (6)

Ans. Density of states: The density of states is the electronic states per unit energy range. It is denoted by $D(E)$.