B.Tech. 1st Semester (F-Scheme) Examination, December–2011

MATHEMATICS–I

Paper–Math–101–F

Time allowed : 3 hours] *[Maximum marks : 100*

Note : *Attempt five questions in all, selecting one question from each section and question number 1 is compulsory.*

1. (Compulsory Question)

(a) If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$ then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

(b) Evaluate $\iint_R xy\,dx\,dy$, where R is the first quadrant of the circle $x^2 + y^2 = a^2$ where $x, y \geq 0$.

(c) Find the rank of the matrix :

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

(d) Using Cayley-Hamilton Theorem, find A^2, where

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

(e) If $x = r \sin\theta \cos\phi, \quad y = r \sin\theta \sin\phi,$

$z = r \cos\theta$, then find $\dfrac{\partial (r, \theta, \phi)}{\partial (x, y, z)}$

(f) Test for convergence of the series, where nth term is $\dfrac{n^2}{2^n}$

(g) Discuss the convergence of the series

$$1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{3^3} + \frac{1}{2^2} - \frac{1}{35} + \ldots..$$

(h) Prove that

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

Section–A

2. Discuss the convergence of the series below :

(i) $\frac{x^2}{2\log 2} + \frac{x^3}{3\log 3} + \frac{x^4}{4\log 4} + \dots$ 12

(ii) $1 + \frac{3}{2!} + \frac{5}{3!} + \frac{7}{4!} + \dots$ 8

3. Discuss the convergence of the following series :

(i) $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}$ 10

(ii) $1 + \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} - \frac{1}{7^2} - \frac{1}{8^2} + \dots$ 10

Section–B

4. (a) Find non-singular matrices P and Q such that PAQ is in the normal form ; where

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 1 \end{bmatrix}$$

(b) Find the values of λ and μ so that the equations

$$2x + 3y + 5z = 9$$

$$7x + 3y - 2z = 10$$

$$2x + 3y + \lambda z = \mu$$

have no solution, a unique solution or an infinite no. of solutions.

5. (a) Show that for any square matrix A, the product of all the eigen values of A is equal to let (A), and the sum of all eigen values of A is equal to the sum of the diagonal elements.

(b) Using Cayley-Hamilton Theorem find the inverse of the matrix :

$$A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 3 \\ 3 & 1 & -2 \end{bmatrix}$$

Section–C

6. (a) If $y = \cos(m \log x)$, then show that

$$x^2 y_{n+2} + (2n+1) x y_{n+1} + (m^2 + n^2) y_n = 0$$

(b) Expand $\log x$ in powers of $(x - 1)$ by Taylor's Theorem and hence find the value of $\log_e^{1.1}$

7. (a) Find the radius of curvature of

$$y = x^2 + 2xy + y^2 + x \quad \text{at the origin.} \qquad 5$$

(b) Find all the asymptotes of the curve

$$x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y = 0$$ 5

(c) Prove that if the perimeter of a triangle is constant then its area is maximum when the triangle is equilateral. 10

Section–D

8. (a) Evaluate the following integrals : 10

(i) $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$

(ii) $\int_0^{\pi/2} \sin^2\theta . \cos^4\theta \, d\theta$

(b) Change the order of integration and hence evaluate the integral :

$$\int_0^2 \int_{\sqrt{2y}}^2 \frac{x^2 \, dx \, dy}{\sqrt{x^4 - 4y^2}}$$ 10

9\. (a) Evaluate $\iiint\limits_R (x^2 + y^2 + z^2)\, dx\, dy\, dz$, where R denotes the region bounded by $x = 0, y = 0, z = 0$ and $x + y + z = a, (a > 0)$. 10

(b) Find the area bounded by $y^2 = 4 - x$ and $y^2 = x$. 10