Unit-VIII: Compressibility and Consolidation

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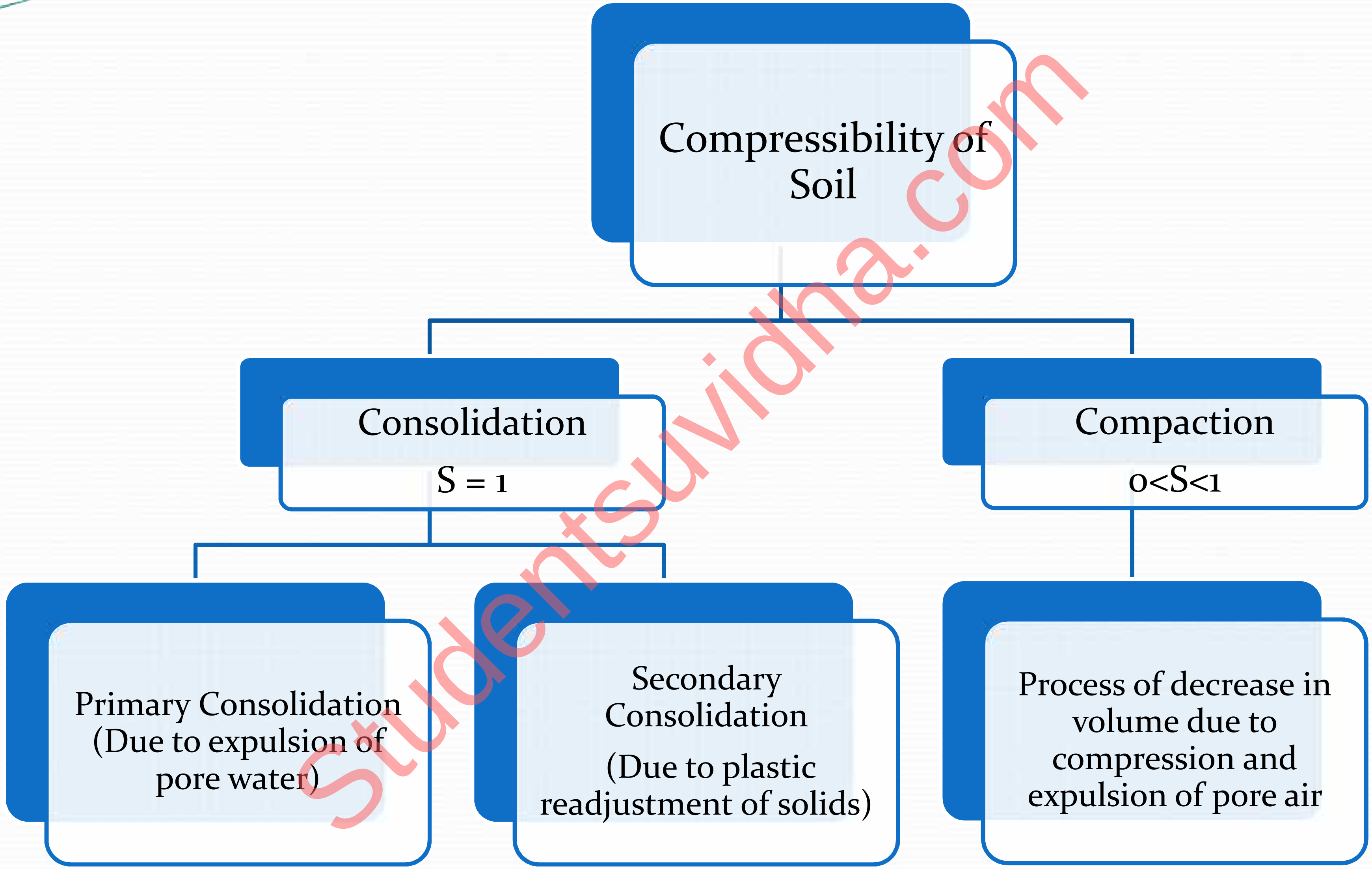
Unit-VIII: Compressibility and Consolidation

- Introduction, components of total settlement, consolidation process, one-dimensional consolidation test, typical void ratio-pressure relationships for sands and clays, normally consolidated and over consolidated clays, Casagrande`s graphical method of estimating pre-consolidation pressure, Terzaghi`s theory of one-dimensional primary consolidation, determination of coefficients of consolidation, consolidation settlement, Construction period settlement, secondary consolidation.


Compressibility of Soil
Consolidation
S = 1
Compaction
0<S<1
Primary Consolidation (Due to expulsion of pore water)
Secondary Consolidation
(Due to plastic readjustment of solids)
Process of decrease in volume due to compression and expulsion of pore air

COMPRESSIBILITY

- The process of volume decrease due to increase in vertical load in a soil mass is called compressibility
- The soil mass is considered semi infinite (Infinite Area and finite depth), hence the volume change is primarily due to change in depth
- The volume change may be due to the following reason
  - Compression of pore air -----Compaction
  - Expulsion of pore air -------Compaction
  - Expulsion of pore water ------Primary Consolidation
  - Change in orientation of molecules or plastic readjustment of solids --------Secondary Consolidation
  - Compression of water molecules ----Negligible
  - Compression of Solids --------Negligible

CONSOLIDATION

- Primary Consolidation
  - This process begins when soil is fully saturated. Due to increase in effective stress over a saturated soil mass pore pressure increases and if expulsion of pore water is permitted then gradually pore pressure decreases. Hence effective stress increases and settlement occurs
  - The settlement caused by expulsion of pore water is called primary consolidation, during the entire process of primary consolidation soil remains saturated
  - It is time dependent phenomena and the time of primary consolidation depends upon
    - Rate of application of load
    - Coefficient of permeability of soil
    - Drainage facility in the soil
    - Length of drainage path

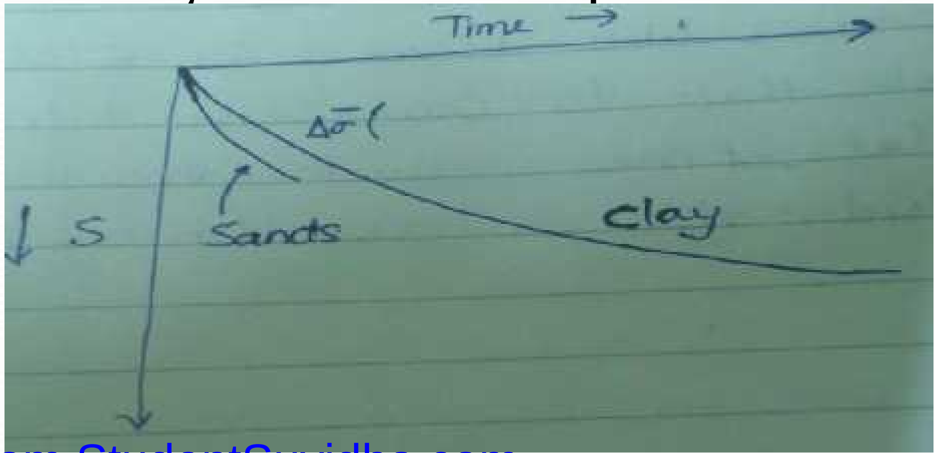
CONSOLIDATION

- Secondary Consolidation
  - After completion of primary consolidation when expulsion of pore water is stopped and load continues to act, then with time at very slow rate further volume decrease may be recorded which is due to plastic readjustment of solids and is called secondary consolidation
  - Secondary consolidation is negligible for granular soils (gravels and sands) but it may be 10 to 20% of total settlement in fine clays such as highly plastic soils

NORMALLY CONSOLIDATED SOIL AND OVER CONSOLIDATED SOIL

- Normally consolidated soils are those which are loaded first time, the past applied effective stress in the history of soil is less than present applied effective stress.
- Over consolidated soils are those which have been subjected to the effective stress in the past greater than the present applied effective stress
- Over consolidated soils are less compressible and show negligible volume change due to increase in effective stress
- A heavily over consolidated clay behave like a dense sand and it will have high value of OCR (Over Consolidation Ratio) OCR>>1

- Over Consolidation Ratio
  - It is defined as the ratio of maximum applied effective stress in the past to the present applied effective stress
    - For Normally consolidated soils OCR ≤ 1
    - For Over consolidated soils OCR > 1
    - Over consolidated soils are also called pre-compressed soils
- Time Vs Settlement curve for a given increase in effective stress
  - Note that the total settlement and time required to complete the settlement is less in sand but it is greater in clays due to high void ratio
  - Settlement of clays is more hence clays are more compressible than sands

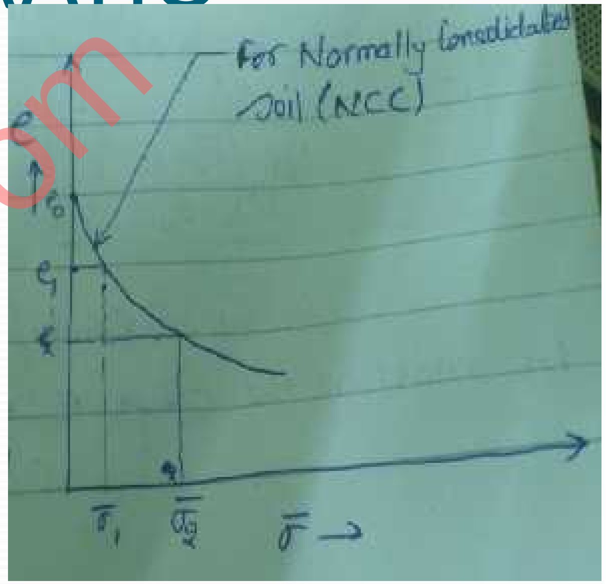


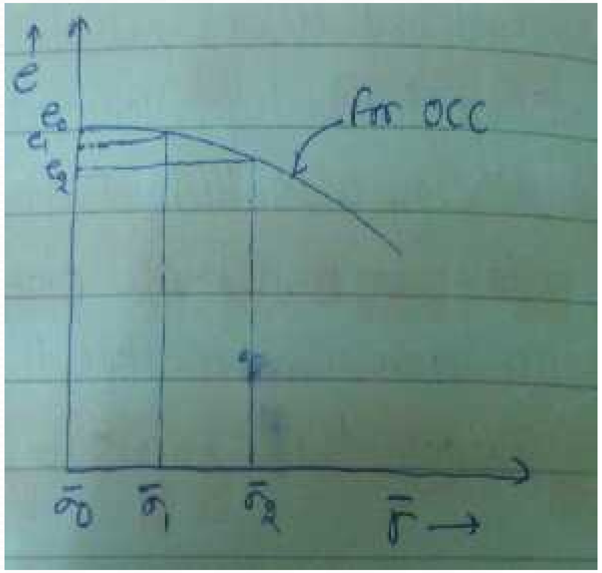
EFFECTIVE STRESS VS VOID RATIO

- For Normally Consolidated soil –
  - Rate of compressibility gradually decreases
  - The slope of effective stress Vs void ratio curve for NCC is called coefficient of compressibility

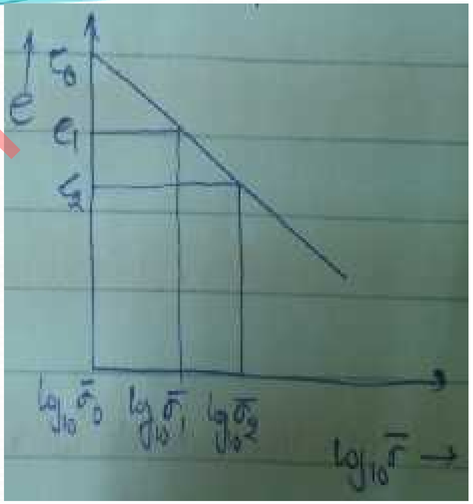
$$a_v = -\frac{\Delta e}{\Delta \bar{\sigma}}$$

  - a_v decreases with increase in effective stress
- For Over consolidated soils
  - The curve is found convex hence coefficient of compressibility increases with increase in effective stress
  - Note that for OCC soils curve is more flat it means due to increase in effective stress decrease in void ratio is less. Hence such soils are less compressible



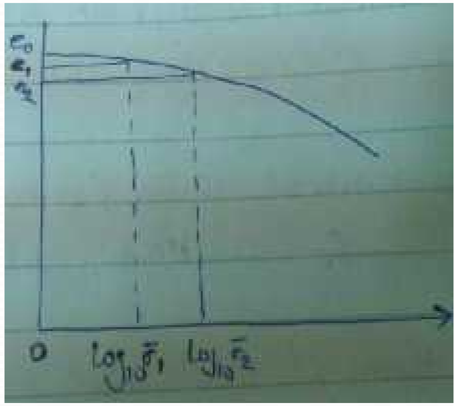


EFFECTIVE STRESS Vs VOID RATIO CURVE ON SEMI LOG SCALE



- In semi log scale for NCC, this curve is found straight line and the slope of this curve is constant which is called coefficient of compression/compression index

$$c_c = -\frac{\Delta e}{\log_{10}\left(\frac{\pm_o + \Delta\pm_o}{\pm_o}\right)} = \frac{\Delta e}{\log_{10}\left(\frac{\pm_2}{\pm_1}\right)}$$



- If the above curve is plotted for OCC, then curve will not be straight line but it will be little curved (Convex) and the slope of void ratio Vs effective stress in log scale is called coefficient of recompression or recompression Index
- Note that c_c and a_v both have similar significance but cc is constant and av changes with effective stress hence for all practical purpose c_c will be used

DETERMINATION OF C_C

- For Undisturbed clays of medium sensitivity c_c is

$$c_c = 0.009(w_l - 10)$$

- For remolded clays of medium to low sensitivity c_c is

$$c_c = 0.007(w_l - 10)$$

- c_c in terms of water content

$$c_c = 0.0115 w_n$$

- c_c in terms of void ratio

$$c_c = 0.04(e_o - 0.25)$$

KARL TERZAGHI THEORY OF 1D CONSOLIDATION

- Assumptions –
  - Soil is homogeneous, Isotropic
  - Darcy's law is valid
  - The strain produces due to applied stress is small it means there is no change in soil structure
  - The soil is fully saturated and remains saturated throughout the process of consolidation
  - The flow is essentially 1D (vertical) and no change in area occurs due to volume change due to expulsion of pore water, only depth changes not the area
  - The hydrodynamic lag is considered but plastic lag is ignored. It means that this theory is valid only for primary consolidation and not for secondary consolidation

KARL TERZAGHI THEORY

- For two way drainage, the length of drainage path $d = \frac{H_o}{2}$
- For one way drainage, the length of drainage path $d = H_o$
- 1D Consolidation Equation of Terzaghi

$$\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2}$$

  - Where "u" is excess pore pressure which is developed due to applied effective stress Δσ
  - ∂u/∂t is rate of change of pore pressure
  - $\partial^2 u/\partial z^2$ is rate of pore pressure change with depth
  - C_v is coefficient of consolidation which depends upon type of soil

$$C_v = \frac{k}{m_v \gamma_w}$$

$$m_v = \frac{a_v}{1+e_o} = \frac{-(\frac{\Delta V}{V})}{\Delta \sigma}$$

  - m_v is coefficient of volume compressibility

CONSOLIDATION

- Degree of Consolidation (U)-
  - Degree of consolidation refers to the % completion of primary consolidation at any instant of time. Theoretically primary consolidation is completed in Infinite time but practically it will be completed in certain finite time. It may be expressed in following three forms
    - In terms of settlement
      - Let ΔH is ultimate primary consolidation settlement corresponding to 100% degree of consolidation
      - Let Δh is consolidation settlement at any intermediate time at which degree of consolidation is U

$$\%U = \frac{\Delta h}{\Delta H} \times 100$$

CONSOLIDATION

- Degree of Consolidation (U)-
  - In terms of void ratio
    - Let e_o is initial void ratio at the beginning of consolidation
    - Let e_{100} is void ratio after completion of 100% primary consolidation
    - Let e is void ratio at any intermediate time when degree of consolidation is U then

$$\%U = \frac{e_o - e}{e_o - e_{100}} \times 100$$

  - In terms of excess pore pressure
    - Let u_i is initial excess pore pressure at the beginning of consolidation and u is excess pore pressure at intermediate time then degree of consolidation is defined as

$$\%U = \frac{u_i - u}{u_i} \times 100$$

TIME FACTOR Tv

- This parameter relates to the degree of consolidation and time required for that consolidation and is defined as

$$T_v = \frac{C_v t}{d^2}$$

  - Where C_v is coefficient of consolidation and t is time required for consolidation to reach at degree of consolidation
  - d is length of drainage path
  - When U≤0.6 $T_v = \frac{\pi}{4} U^2$
  - When U>0.6 $T_v = -0.9332 \log_{10}(1-U) - 0.0851$
  - T50 = 0.197 and T90 = 0.848

DETERMINATION OF C_v

- It depends on type of Soil and its value decrease with increase in liquid limit
- There are two methods to determine c_v which are based on time fitting curve approach
  - Taylor's Method (Square root of time fitting method)

$$C_v = \frac{T_{90} d^2}{t_{90}}$$

Where d = length of drainage path
t_{90} = time required for 90% consolidation
T_{90} = 0.848 time factor for 90% consolidation

  - A-Casagrande method (Logarithm time fitting method)

$$C_v = \frac{T_{50} d^2}{t_{50}}$$

Where d = length of drainage path
t_{50} = time required for 50% consolidation
T_{50} = 0.197 time factor for 50% consolidation

SETTLEMENT ANALYSIS

- The total settlement is sum of the following three

  $S = S_i + S_c + S_s$ Where S_i = Immediate settlement/elastic settlement
  S_c = Primary consolidation settlement
  S_s = Secondary consolidation settlement

- Immediate settlement/Elastic settlement
  - It is due to compression and expulsion of pore air or due to elastic deformation of molecules
    - For Cohesionless soils (Sands)

$$S_i = \frac{H_o}{C_s} \log_{10}\left(\frac{\sigma_o + \Delta\sigma}{\sigma_o}\right) \qquad C_s = 1.5\frac{C_r}{\sigma_o}$$

Where C_r = Static Cone resistant in KN/m^2
H_o = Initial thickness of soil
σ_o = Initial effective overburden pressure at the centre of compressible layer
$\Delta\sigma_o$ = Increase in effective stress due to applied load at the centre of compressible layer

SETTLEMENT ANALYSIS

- Elastic Settlement
  - For Cohesive Soils (Clays)
    - In case of saturated clays the immediate settlement is insignificant however small elastic deformation may occur due to squeezing of water
    - The immediate elastic settlement below the corner of a rectangular flexible foundation is given by

$$S_i = \frac{qB(1-\mu^2)I_t}{E_s}$$

Where q = pressure at the base of foundation

B = Width of Foundation

μ = Poisson's ratio of soil

I_t = Shape factor/Influence factor of footing which depends on shape of footing

E_s = Young's Modulus of elasticity of soil

SETTLEMENT ANALYSIS

- Primary Consolidation Settlement (S_c)
  - It is due to expulsion of pore water. is ultimate consolidation settlement corresponding to 100% degree of consolidation
  - It can be determined in following three ways
    - Using change in Void ratio

$$S_c = \Delta H = H_o \frac{\Delta e}{1 + e_o}$$

Where Δe = Change in void ratio due to application of effective stress $\Delta\sigma$ at the centre of compressible layer

e_o = Initial void ratio at the centre of compressible layer

H_o = Initial thickness of compressible layer

- In terms of coefficient of compression

$$S_c = \Delta H = \frac{C_c H_o}{1 + e_o} \log_{10}\left(\frac{\sigma_o + \Delta\sigma}{\sigma_o}\right)$$

Where C_c = Coefficient of compression

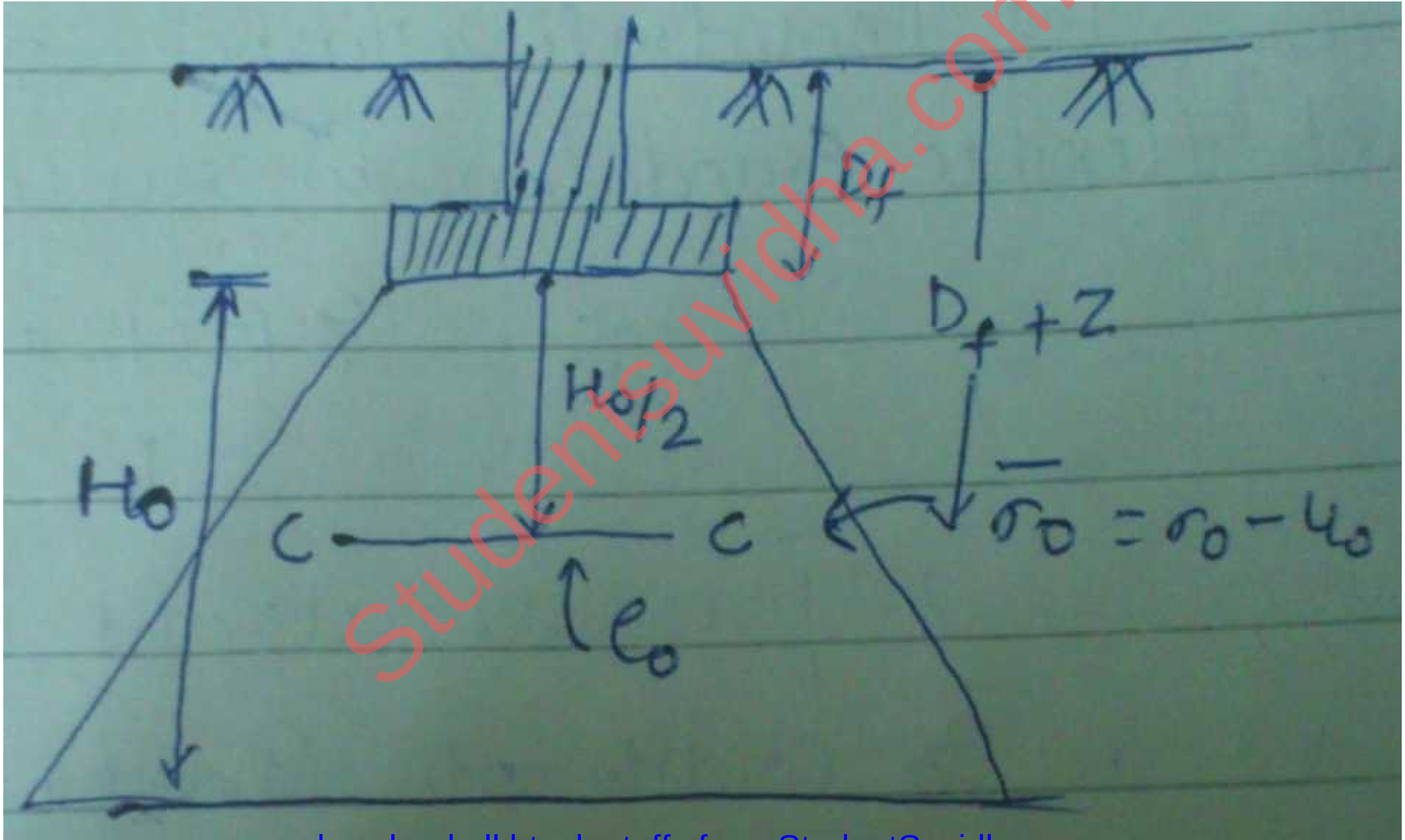
H_o = Initial thickness of soil

e_o = Initial void ratio at the centre of compressible layer

σ_o = Initial effective overburden pressure at the centre of compressible layer

$\Delta\sigma_o$ = Increase in effective stress due to applied load at the centre of compressible layer

CENTRE OF COMPRESSIBLE LAYER



SETTLEMENT ANALYSIS

- Primary Consolidation Settlement
  - Using coefficient of volume compressibility
    - In case of clays m_v changes with depth (m_v changes with changes in effective stress) therefore mv should be taken at the centre of clay layer
    - The ultimate consolidation is given by

$$S_c = \Delta H = H_0 m_v \Delta \bar{\sigma}$$

    - Δσ Increases in effective stress at the centre of compressible layer
    - If there are more than one no. of layer than settlement of each layer is found separately and add algebraically
- Note:- Ultimate consolidation settlement will be same under a given increase of effective stress for 2 way or 1 way drainage condition both however time required for completion will be different

SETTLEMENT ANALYSIS

- Secondary Consolidation Settlement (S_s)
  - It is due to plastic readjustment of solids ie due to change in orientation of molecules
  - It occurs at very slow rate and it may take several years for granular soils (gravels & Sands)
  - Secondary settlement is negligible but for highly plastic clays it may be 10 to 20% of total
  - The secondary settlement after time t from the completion of primary settlement is given as

$$S_s = \frac{C_s H_{100}}{1+e_{100}} \log_{10}\left(\frac{t}{t_{100}}\right)$$

Where t_{100} = time required for primary consolidation it may be taken equal to t_{90}

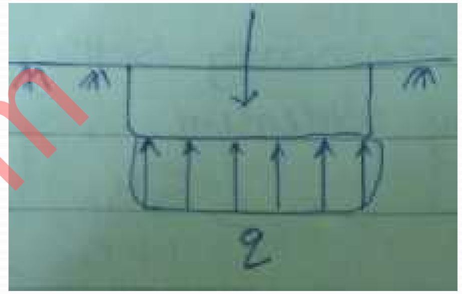
t = time of secondary consolidation at which Ss is required, It is measured after completion of primary consolidation

C_s = Secondary compression Index

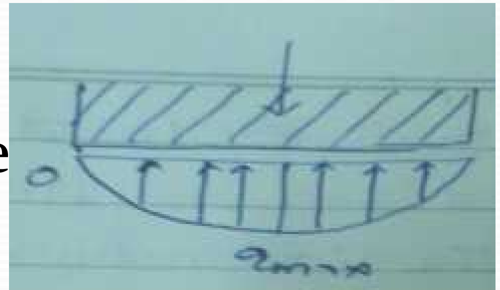
H_{100} = Thickness of soil layer after primary consolidation, In absence of H_{100},it ma be taken equal to H_o

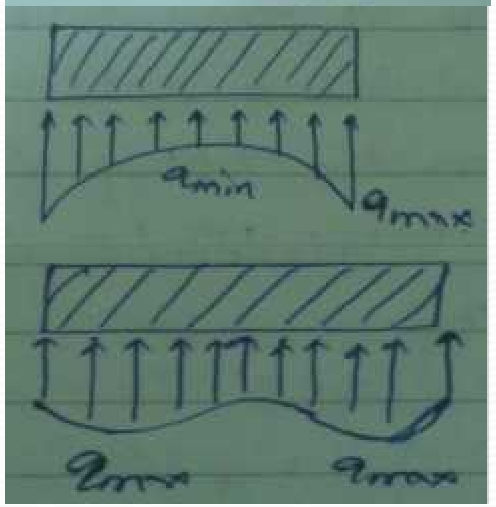
e_{100} = Void ratio after completion of primary consolidation

CONTACT PRESSURE AT THE BASE OF FOUNDATION



- Case 1) Flexible Foundation –
  - The settlement of flexible foundation may be non uniform but pressure at the base will be uniform irrespective of type of soil
- Case 2) Rigid Foundation
  - In rigid foundation settlement is uniform and pressure distribution depends on type of soil
    - For Sands – In sands, the maximum pressure is at the centre and minimum at the edges
    - For Clays – In clays the maximum pressure is at the edges and minimum at the centre
    - For Silts – Maximum pressure occurs between centre and edges





PROBLEMS

Ques. In a laboratory test on a clay sample of thickness 25mm drained at top only, 50% consolidation occurred in 11 mins. Find the time required for same clay layer in the field 2m thick which is drained at top and bottom both to undergo 70% degree of consolidation. Given that time factor for 50% consolidation 0.197 and 70% consolidation is 0.405

Ans

Lab → $T_{v_1} = \frac{C_v t_1}{d_1^2}$ $\qquad T_{v_2} = \frac{C_v t_2}{d_2^2}$ ← field

Lab $\quad 0.197 = \frac{C_v \times 11}{H_{o_1}^2}$ $\qquad d_1 = H_{o_1}$ one way drainage $= 25mm$

$$C_v = \frac{0.197\, H_{o_1}^2}{11}$$

$T_{v_2} = \frac{C_v t_2}{d_2^2}$ $\qquad d_2 = \frac{H_{o_2}}{2} = \frac{2}{2} = 1m. = 1000$

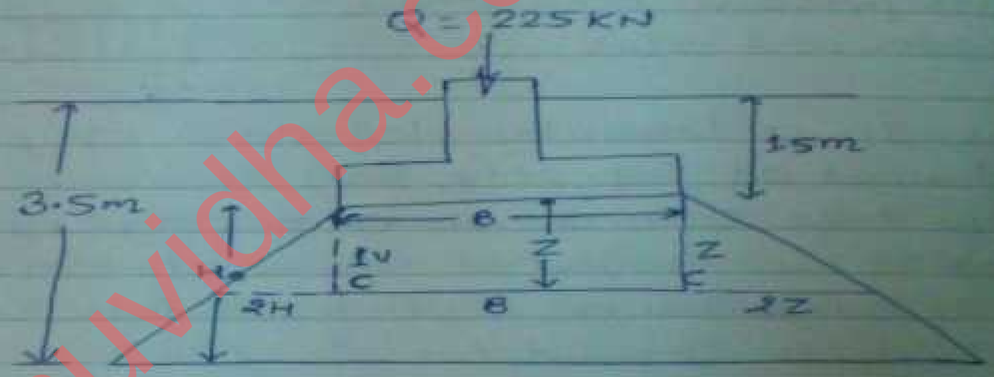
(2 way drainage)

$$\frac{0.197 \times 25^2}{11} = \frac{0.405 \times 1000^2}{t_2}$$

$$t_2 = 11 \times \left(\frac{0.405}{0.197}\right) \times \left(\frac{1000}{25}\right)^2 \text{ min.}$$

$$t_2 = 25.13 \text{ days}$$

Ques A square footing of size 1.5m is located at depth of 1.5m below the Ground level. The soil is uniform deposit of clay with unit wt. $20 kN/m^3$. The void ratio of soil is 0.8 and its compression Index is 0.07. The total thickness of clay is 3.5m below which a firm strata is located. Compute total primary consolidation settlement of footing, if 225 kN load is applied on the footing. Assume trapezoidal distribution of load below the footing at 1V : 2H. Consider water table is beyond the zone of influence of the stress due to loading and soil to be saturated.



Ans

$C_c = 0.07$

$e_0 = 0.8$

$H_0 = 2m$

$\gamma = 20 kN/m^3$

The compressible layer is below the footing.

Center of compressible layer is C–C

initial effective stress at C–C.

$$\bar{\sigma}_0 = \sigma_0 - u_0 \quad (u_0 \to 0)$$

$$= 2.5 \times 20 = 50 \, kN/m^2$$

$\Delta\bar{\sigma}$ = increase in Eff stress at C–C

$$= \frac{Q}{\text{Area of soil at C–C}} = \frac{Q}{(B+4Z)(B+4Z)}$$

$$= \frac{225}{(1.5+4\times 1)^2} = \frac{225}{(5.5)^2} = 7.438 \, kN/m^2$$

$$\Delta H = \frac{C_c H_0}{1+e_0} \log_{10}\left(\frac{\bar{\sigma}_0 + \Delta\bar{\sigma}}{\bar{\sigma}_0}\right)$$

$\Delta H = 4.68$ mm