Unit-VII: Vertical Stress Below Applied Loads

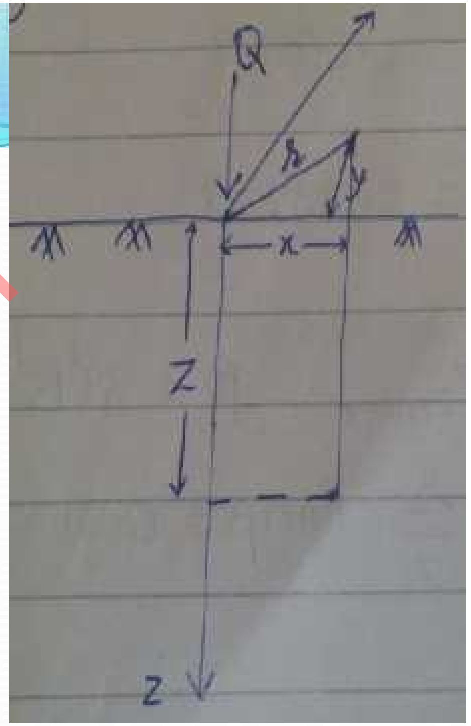
By Tanuj Gupta

Unit-VII: Vertical Stress Below Applied Loads

Hours – 1 Hour

- Topics –
  - Introduction, Boussinesq`s equation,
  - vertical stress distribution diagrams, vertical stress beneath loaded areas,
  - Newmark`s influence chart, approximate stress distribution methods for loaded areas,
  - Westergaard`s analysis, contact pressure.

BOUSSINESQUE'S THEORY



- Assumptions –
  - Soil mass is considered semi Infinite, elastic, homogeneous and Isotropic
  - The medium obeys hooke's Law
  - The self weight of soil is ignored
  - The soil is initially unstressed
  - The change in volume of soil due to applied load is negligible
  - The top surface of soil is free from shear stresses and subjected to point load only
  - The stresses are distributed symmetrically with respect to vertical axis

$$\pm_z = K_B \frac{Q}{z^2} \qquad K_B = \frac{3}{2\circledR}\left[\frac{1}{(1+\frac{r^2}{z^2})}\right]^{\frac{5}{2}}$$

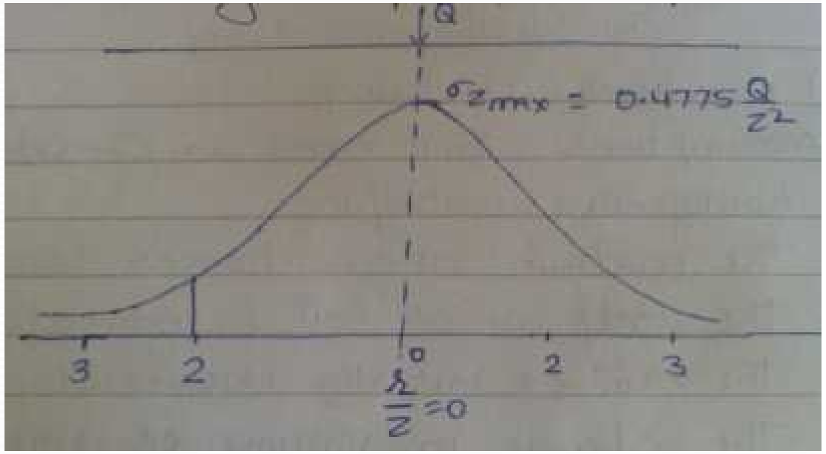
BOUSSINESQUE'S THEORY

- Special Case :- If Point P is located vertically below to the Load at depth z then r=0 then

$$K_{B\,\max} = \frac{3}{2\pi} = 0.4775$$

$$\sigma_z = 0.4775\frac{Q}{z^2}$$

- Case 1 – variation of σ_z in a horizontal plane at depth z
- It is found that when r/z = 2, then σ_z is only 1.8% of σ_{max} and when r/z = 3 then is σ_z only 0.3% of σ_{max}



WESTERGARD'S THEORY

- Assumption –
  - The material is semi-infinite, elastic, homogeneous but non-isotropic.
    - The actual soil deposit has layered structure
  - The material is laterally reinforced with numerous closely spaced particles of negligible thickness. It means soil is considered rigid in horizontal direction and elastic in vertical direction
  - The ratio of young's modulus of soil in horizontal direction to the vertical direction is infinite
  - The poisson ratio of soil which is defined as –(Lateral /longitudinal) strains = 0

$$\pm_z = K_w \frac{Q}{z^2}$$

$$K_w = \frac{1}{®}\left[\frac{1}{\left(1+\frac{2r^2}{z^2}\right)}\right]^{\frac{3}{2}}$$

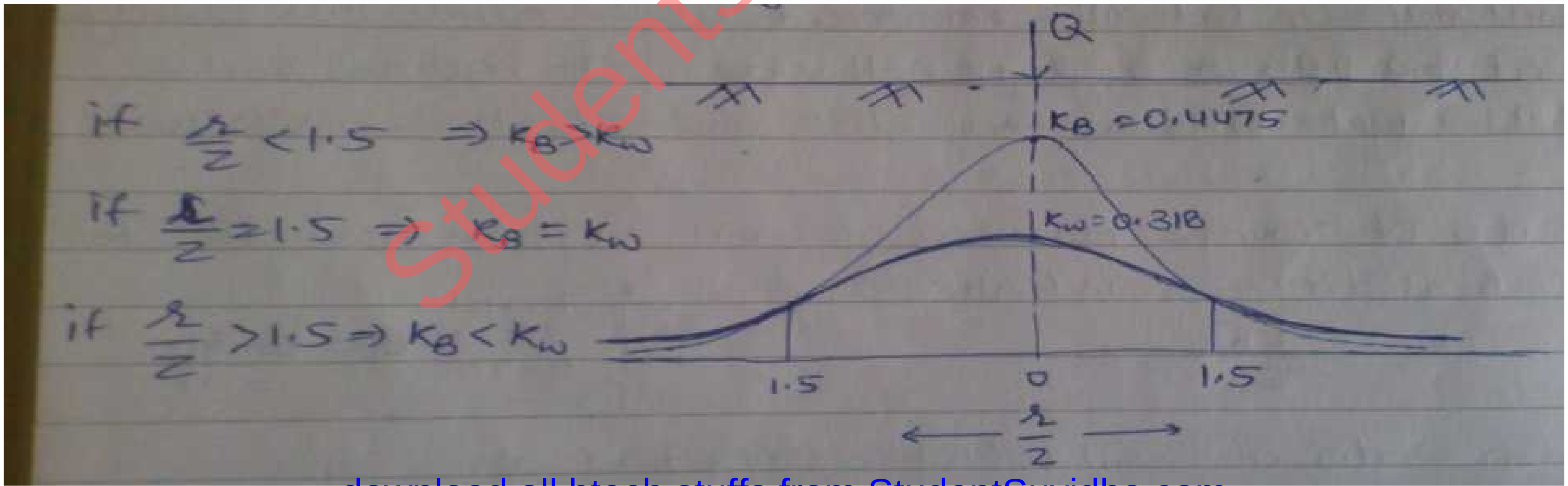
WESTERGARD'S THEORY

- Special Case :- If point p is located at depth z vertically below the Load Q

$$K_{w\,\max} = \frac{1}{\circledR} = 0.318$$

$$\pm_z = 0.318\frac{Q}{z^2}$$

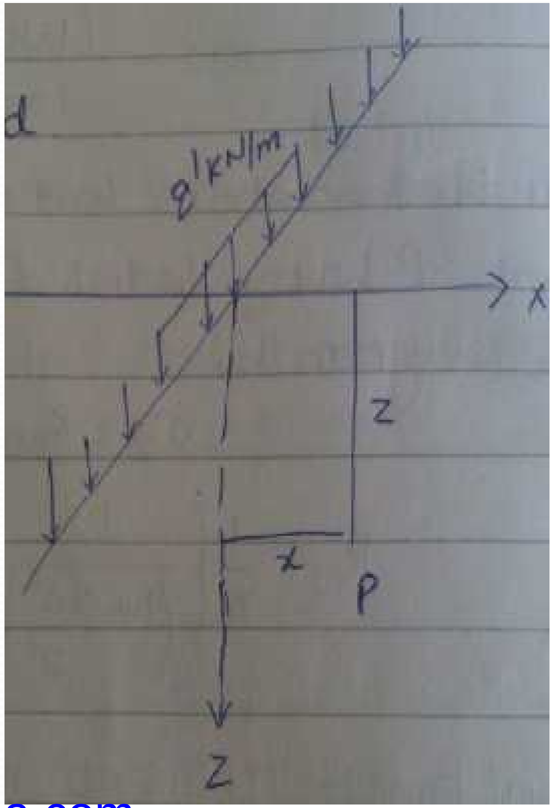
- Comparison b/w Westergard and Boussinesques Theory



VERTICAL STRESS DUE TO LINE LOAD

- Let the intensity of line load is q' KN/m which acts in z direction having length along Y direction
- Vertical stress at point P at a distance x and z from a line load is given as

$$\sigma_z = \frac{2q'}{\pi z}\left[\frac{1}{1+\frac{x^2}{z^2}}\right]^2$$

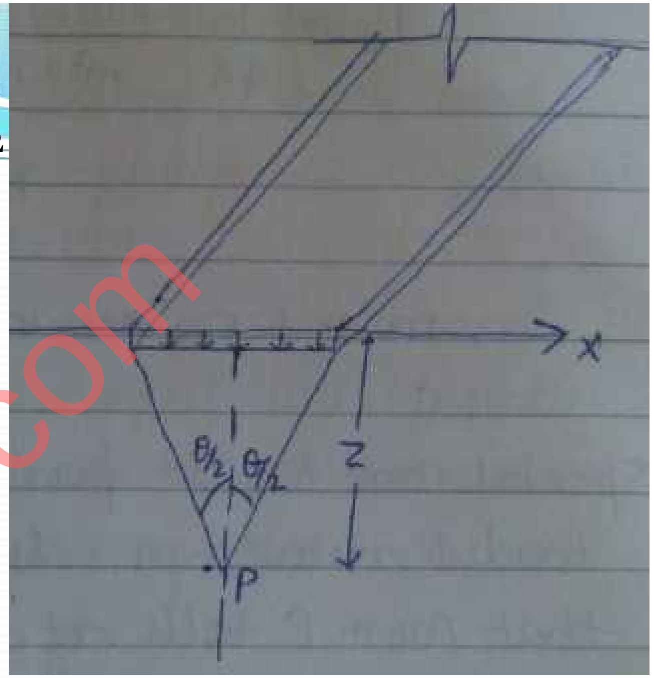


- Special Case :- If point p is located at depth z below the line of action of load then

$$\sigma_z = \frac{2q'}{\pi z}$$

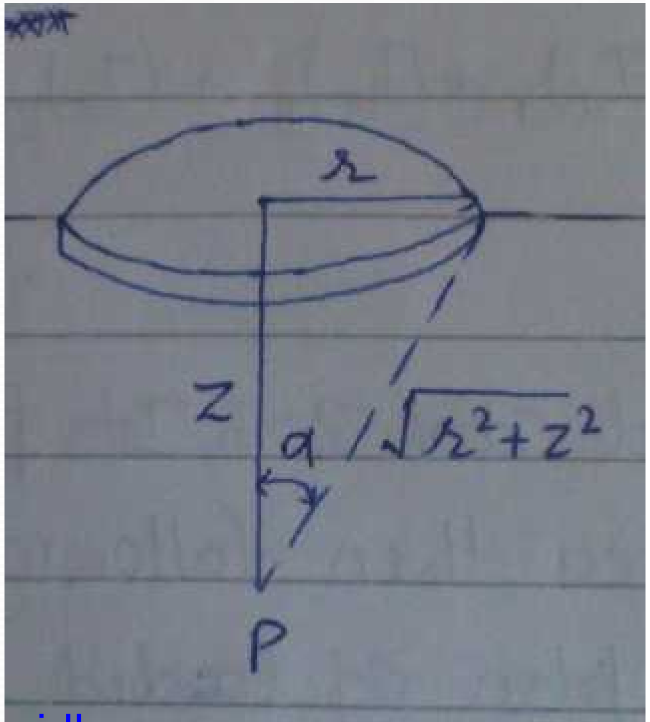
- Vertical stress due to strip loading
  - Let intensity of strip load is Q KN/m² the vertical stress at point p below the centre of strip load at which the angle subtended by strip load is θ then σ_z at point p is

$$\pm_z = \frac{Q}{®}(® + \sin^{-})$$

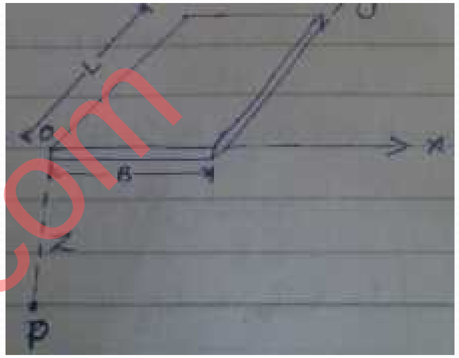


- Vertical stress due to circular loaded area with UDL
  - Let r is radius of circular loaded area and vertical stress required at point p at depth z below the centre line of the load $\pm_z = q(1-\cos^3 \sim)$

$$Cos\sim = \frac{z}{\sqrt{(r^2 + z^2)}}$$



- Vertical stress below the corner of a rectangular loaded area with UDL
  - Let load Intensity q L/z = m
    $\pm_z = qI_\pm$ B/z = n
  - I_σ is Influence factor of load at P



$$I_\sigma = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1+m^2n^2}\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right) + \sin^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1+m^2n^2}\right)\right]$$

m and n are interchangable

- Special Case 1 – If point P is located inside the plan of loaded area
- Special Case 2 – if point P is located outside the plan of loaded area

NEWMARK'S CHART METHOD/ INFLUENCE CHART METHOD

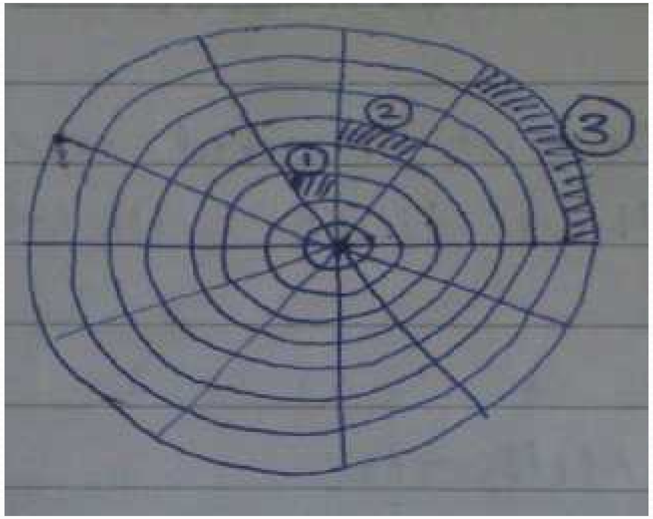
- It is applicable for homogeneous, Isotropic, elastic and semi-infinite soil mass. It can be applied for any shape of area loaded with UDL
- Newmark's Chart consists of concentric circles and radial lines (let m circles and n radial lines). It means chart contains m*n No. of area units. The influence of each area at the centre of chart is equal
- Influence factor of each area = 1/(m*n)

  σ_z=q*(1/mn)* total no. of area Units of chart occupied by plan of loaded area
- The plan of loaded area is prepared on a tracing paper with scale z=OP, OP is marked in Newmark's Chart \
- The vertical stress at point P having depth z can be found as

  $\sigma_z=(1/mn)*N_A*q$

- The tracing paper is placed over the Newmark's chart in such a way that point o (vertcal projection of point p) concides with the centre of chart
- The No. of area units (N_A) which are occupied by Plan of loaded area on the chart are manually counted which includes fractional areas also
- If the plan of loaded area lies outside the Newmark's chart then there will be no influence of loaded area at point P.
- Let 10 areas fully occupied, 5 areas occupied 50% and 4 areas occupied 25% then

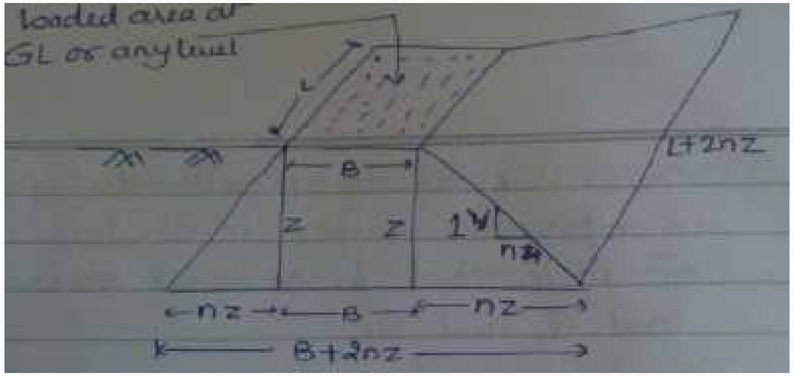
$N_A = 10*1 + 5*0.5 + 4*0.25$



APPROXIMATE METHOD

- Equivalent point load Method
  - In this method the loaded area with UDL is divided into a convenient no. of smaller area units and load of each area is assumed to act at the centroid of that area and point load method can be applied given by Boussinequs and westergard theory
- Trapezoidal or point spread method
  - Let load is applied on the footing of size BxL, let vertical stress is required at depth z below the footing then

$$\pm_z = \frac{q*(B*L)}{(B+2nz)(L+2nz)}$$



- Stress Isobars or pressure bulbs
  - Let at the base of footing, pressure is q KN/m2. Let the width of footing is B. The pressure/vertical stress below the footing decreases. Stress isobars are the lines joining with equal vertical stress. Generally effect of vertical stress is neglected beyond the zone of 0.2q