Unit-V: Effective Stress Concept

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Unit-V: Effective Stress Concept

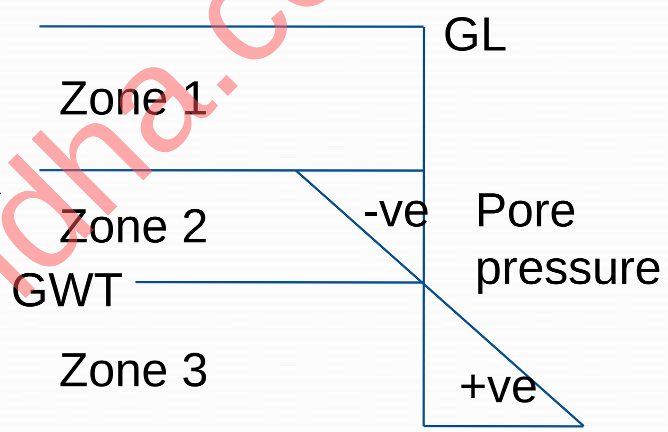
Total 3Hrs

Topics-

- Principle of effective stress, effective stress under hydrostatic conditions, capillary rise in soils, effective stress in the zone of capillary rise, effective stress under steady state hydro-dynamic conditions,
- seepage force, quick condition, critical hydraulic gradient,
- two dimensional flow, Laplace`s equation, properties and utilities of flow net, graphical method of construction of flow nets, piping, protective filter.

Effective Stress Concept

- Capillary Action
  - Zone 1 – It is in Hygroscopic condition. If degree of saturation is given then Bulk unit weight should be used and is degree of saturation is not mentioned then this zone may be considered dry
  - Zone 2 – Capillary zone – due to suction effect of soil this zone is in capillary water. The pore water pressure is negative ie Hydrostatic tension or capillary tension. If degree of saturation is not given then this zone may be considered fully saturated



Zone 3- Submerged Zone – Below GWT pore pressure is hydrostatic and at the GWT pore pressure is atmospheric and at any depth h below GWT pore pressure is $u = h\gamma_w$

Effective stress Concept

- Total Stress
  - Under geostatic condition total vertical stress due to self weight of soil is equal to total weight/total area of c/s
    - If uniform surcharge "q" acts over top surface of soil mass having infinitely large area then total stress at depth z will be

$$\pm_{x} = q + ¥z$$

- Pore Pressure
  - If soil is flush by water then at any depth z below GWT, pore pressure is hydrostatic. Under static condition pore pressure is equal in all directions hence vertical pore pressure is

$$u_{x} = ¥_{w}z$$

  - In this case pores are fully filled by water

- Effective Stress
  - It is grain to grain contact pressure which is transmitted from one layer to the other layer. If soil is submerged then due to +ve pore pressure grain to grain contact pressure is reduced $\pm'_x = \pm_x - u_x$
- Note –
  - If uniformly distributed surcharge is applied on a soil mass in which water table is at Ground level then there will be two effects
    - In short term condition, the applied surcharge q will be taken by pore water hence total pressure and pore pressure both will increase by q but effective stress will be Unchanged
    - In long term condition in which drainage is permitted water will flow out of the pores hence excess pore pressure will reduce therefore due to applied surcharge total pressure and effective pressure both will increase by equal amount

Seepage Analysis

- Seepage force is the drag force exerted by the water on the soil molecules which always acts in the direction of flow. It is due to viscous friction offered by the water into soil, seepage force at any section is directly proportional to Hydraulic head which is the head loss through the length of soil to the downstream of that section
- If flow is vertical, then it will either increase or decrease the effective stress between particles. Hence net effective stress will be

$$\overline{\pm} \pm p_s$$

+ when seepage flow is downward

- When seepage flow is upward

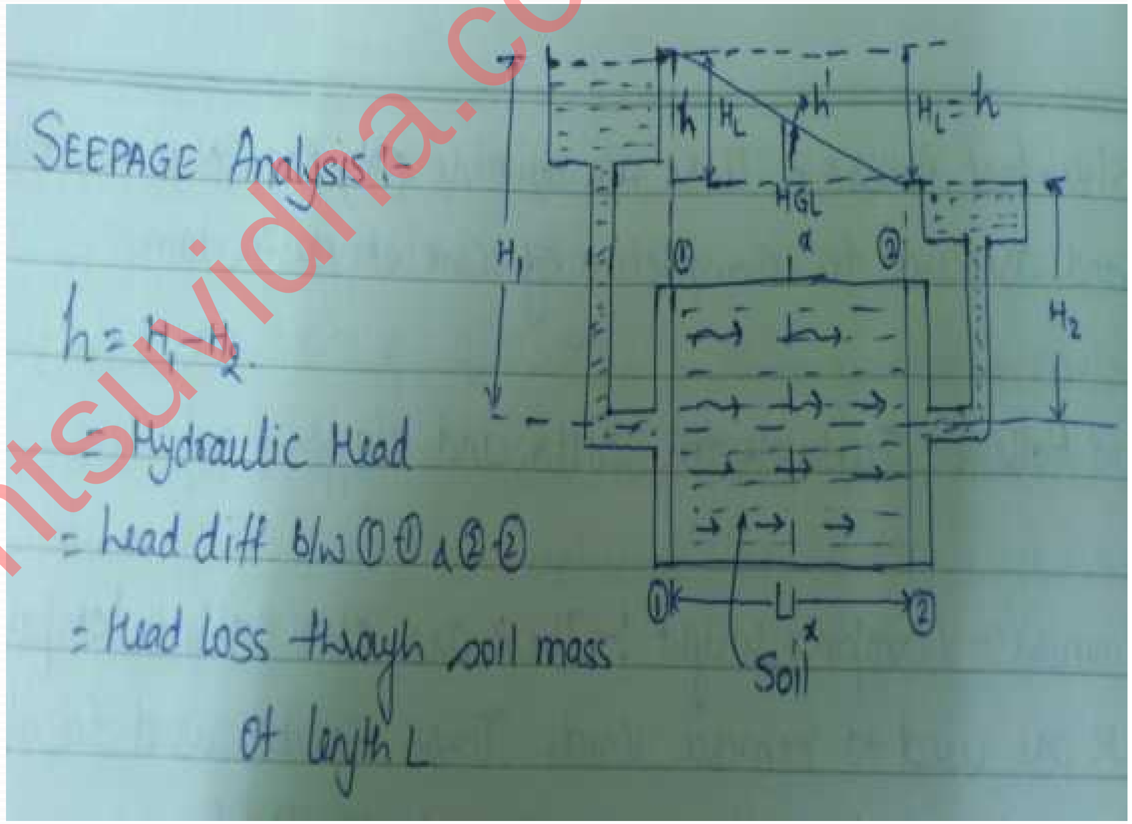
- Seepage pressure at 1-1 is $p_s = h\gamma_w$
- Seepage pressure at x-x is $p_s = h'\gamma_w$
- Seepage pressure at 2-2 is $p_s = 0$
- Let area of cross section at 1-1 is A

$$p_s = h\gamma_w$$

$$p_s = iL\gamma_w$$

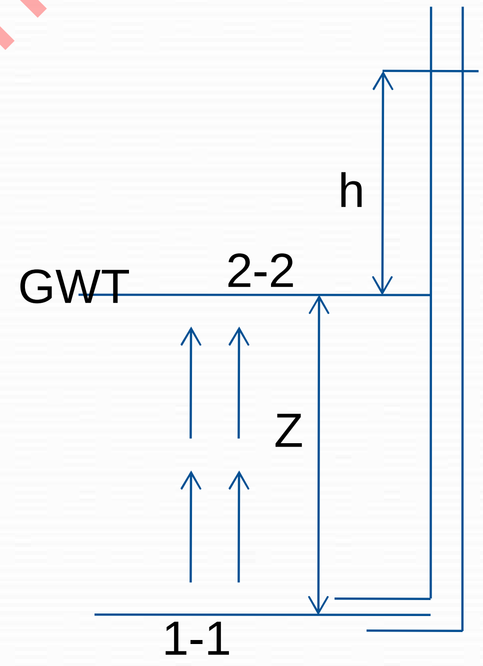
- Seepage force per Unit volume

$$f_s = i\gamma_w$$



Quick Sand Condition

- Piezometric head at 1-1 = z+h
- Piezometric head at 2-2 = z
- Piezometric head difference b/w 1 & 2 is h=hydraulic head, due to hydraulic head seepage flow will occur in vertical upward direction.
- The net effective stress at 1-1 $\bar{\sigma}' = \gamma' z - iz\gamma_w$
- If net effective stress is reduced to Zero then hydraulic gradient is called critical hydraulic gradient $i_c = \frac{\gamma'}{\gamma_w} = \frac{G-1}{1+e}$
- For sands $i_c = 0$



Quick Sand Condition

- If seepage flow occurs in cohesionless soils (sands & fine gravels) in vertically upward direction then a stage may occur at which net effective stress b/w particles may become zero under such condition cohesionless particles will start floating and may flow with the water in upward direction such a condition is called Quick sand condition.
- If quick sand failure is to be prevented then the Hydraulic gradient should be restricted below critical hydraulic gradient

Methods to prevent or minimize piping failure

- By increasing the length of flow
  - If length of flow is increased then hydraulic gradient will be reduced
- Provision of inverted filter/graded filter
  - Protective filter will permit movement of water, but no of solids
  - The design of protective filter is done according to the following guidelines

$$\frac{(D_{15})_{filter}}{(D_{85})_{soil}} < 5$$

    - This condition ensures that no significant invasion of particles will take place through the pores of filter

Methods to prevent or minimize piping failure

$$4 < \frac{(D_{15})_{filter}}{(D_{15})_{soil}} < 20$$

- This criteria ensures that though there is significant head loss through the filter but without measure seepage force. It also fixes the lower limit size of filter material

$$\frac{(D_{50})_{filter}}{(D_{50})_{soil}} < 25$$

- This is additional guideline to design the voids to prevent the entry of solid

- If a single filter material does not satisfy above condition then multilayered graded material may be used

Flow net

- Flow net is graphical solution of Laplace equation. It represents the graphical representation of energy flow through a flow field. Flow net consists of equipotential lines and streamlines which intersect orthogonally
- Assumption in the laplace equation
  - Darcy's Law is valid
  - Soil is homogeneous and saturated
  - Flow is 2D
  - Flow is laminar
  - Water and solid molecules are incompressible
  - Due to seepage flow there is no change in volume of soil mass

Properties of Flow Net

- The equipotential lines and streamlines always intersect orthogonally to each other
- The 2 equipotential lines or 2 streamlines can not intersect each other
- The flow fields are approximate square when soil is isotropic but if soil is non isotropic then flow fields may be rectangular
  - The square may be Linear or curvilinear
- The head loss through each flow field is equal which is called equipotential drop

N_d =No. of equipotential drop

$$\Delta h = \frac{h}{N_d}$$

- The discharge through each flow channel is equal which is equal to difference in stream function

$$\Delta q = \frac{q}{N_f}$$

Properties of Flow Net

- The flow through any flow channel remains constant hence continuity principal for a steady state flow can be applied between 2 points in a flow channel
  - Consider unit depth of flow

$$v_1b_1 = v_2b_2 = v_3b_3 = v_nb_n = const$$

- In convergent flow fields the hydraulic gradient is maximum at exit (the size of the flow is min) hence the hydraulic gradient at the last flow field is called exit gradient which should be smaller than critical hydraulic gradient
- For a given flow field, flow net remains same if direction of flow is reversed. It means flow net doesnot depend on head difference but it depend on boundary condition

Application of Flow Net

- Determination of seepage Discharge

$$q = kh\left(\frac{N_f}{N_d}\right)$$

- Determination of exit gradient

$$i_e = \frac{\Delta h}{b_n} = \left(\frac{h}{b_n N_d}\right)$$

- Determination of seepage pressure

$$p_s = h'\gamma_w = (h - \Delta h)\gamma_w = \left(h - \frac{h}{N_d}\right)\gamma_w$$

PHREATIC LINES

- Case I- when drainage filter is provided
  - Phreatic line is top flow line at which pore pressure is atmospheric. Below phreatic line soil is submerged under seepage flow. The upstream wetted surface is called 100% equipotential line because there is no head loss at this line. The flow lines/stream lines will be perpendicular to upstream wetted surface
  - The top flow line follows path of base parabola and it meets the filter orthogonally
  - If filter is provided then uplift pressure can be minimized which will ensure stability

BASE PARABOLA

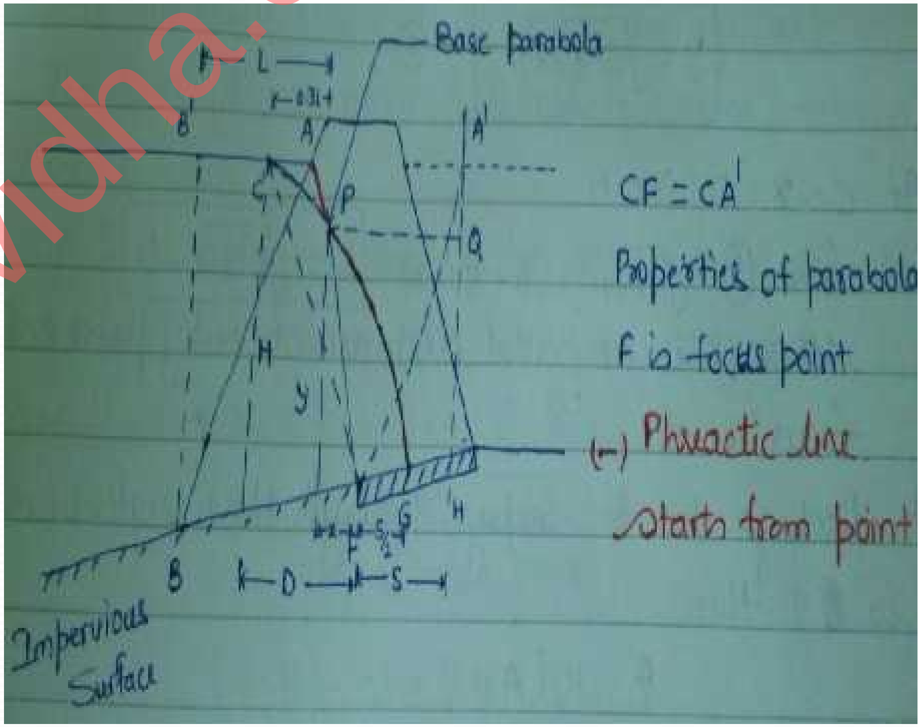
- The focus of base parabola is location at junction of permeable and impermeable surface (F)

$$\sqrt{(X^2+Y^2)} = X + S$$

$$\sqrt{(D^2+H^2)} = D + S$$

$$q = kS$$

- Where S focal length and q discharge due to seepage through Unit length of the Dam



PHREATIC LINES

- Case II- when drainage filter is not provided
  - Case I- when α<30°

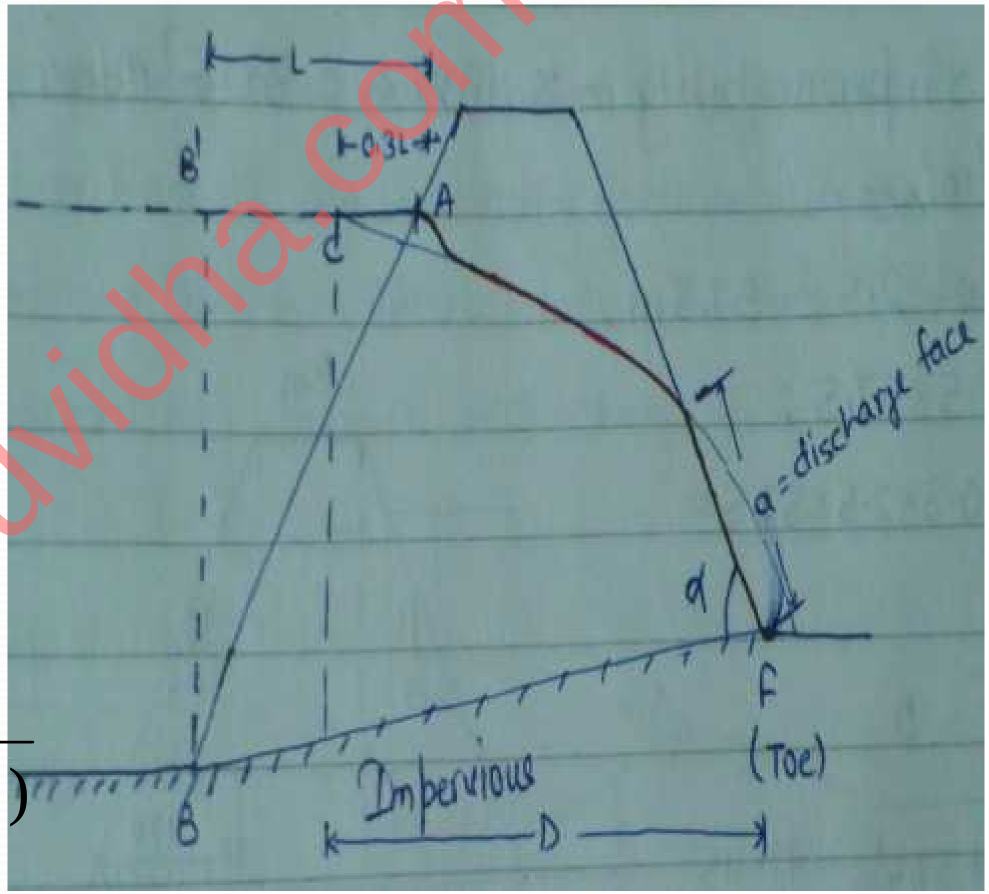
$$a = \frac{D}{\cos\sim} - \sqrt{\left(\frac{D^2}{\cos^2\sim} - \frac{H^2}{\sin^2\sim}\right)}$$

$$q = ka\sin\sim\,\tan\sim$$

  - Case II – when α>30°

$$a = \sqrt{(D^2 + H^2)} - \sqrt{(D^2 - H^2 Cot^2\sim)}$$

$$q = ka\sin^2\sim$$



Questions

Example 9.8. *A 3 m thick soil stratum has coefficient of permeability of 3×10^{-7} m/sec. A separate test gave porosity 40% and bulk unit weight 21 kN/m^3 at a moisture content of 31%.*

Determine the head at which upward seepage will cause quick sand condition. What is the flow required to maintain critical condition?

(Civil Services Exam. 1990)

Solution :

$$e = \frac{n}{1-n} = \frac{0.4}{1-0.4} = 0.667$$

$$G = \frac{\gamma_d}{\gamma_w}(1+e) = \frac{\gamma}{(1+w)\,\gamma_w}(1+e) = \frac{21}{(1+0.31)\,9.81}(1+0.667)$$

$= 2.724$

$$i_c = \frac{G-1}{1+e} = \frac{2.724-1}{1+0.667} = 1.0342 = \frac{H_c}{L}$$

$\therefore$ $H_c = i_c \,.\, L = 1.0342 \times 3 = 3.103$ **m**

Now $Q_c = k\, i_c\, A = 3 \times 10^{-7} \times 1.0342\, A = 3.103 \times 10^{-7} A$ m^3/sec

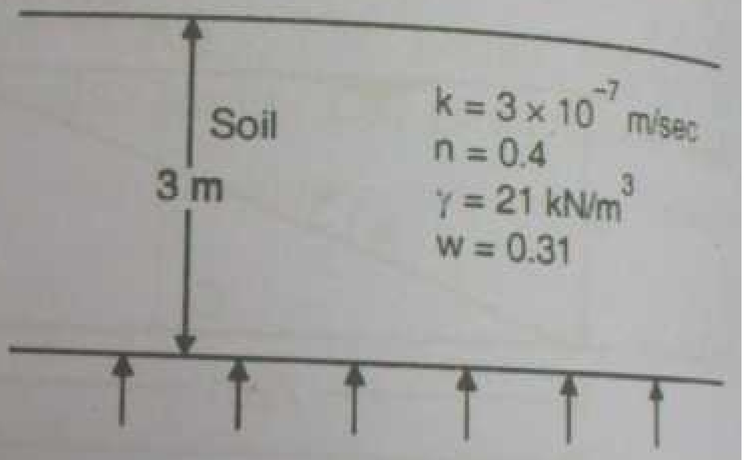


FIG. 9.22

Example 9.10. *Calculate the seepage through an earth dam resting on an impervious foundation. The relevant data are given below :*

Height of a dam = 60.0 m

Upstream slope = 2.75 :1 (H : V) ; Downstream slope = 2.50 : 1 (H : V)

Free board = 2.5 m ; Crest width = 8.0 m ; Length of drainage blanket = 120.0 m

Co-efficient of permeability of the embankment material in

x-direction $= 8 \times 10^{-7}$ *m/s ; y-direction* $= 2 \times 10^{-7}$ *m/s.* **(Engg. Services Exam., 1996)**

Solution Discharges $q = k \cdot s$

Here $k = \sqrt{k_x \cdot k_y} = \sqrt{(8 \times 10^{-7})(2 \times 10^{-7})} = 4 \times 10^{-7}$ m/s.

s = focal distance $= (\sqrt{D^2 + H^2} - D)$

$H = 60 - 2.5 = 57.5$ m ; $L = 2.75 \times 57.5 = 158.13$ m

$0.3\,L = 0.3 \times 158.13 = 47.44$ m;

$D = 47.44 + (2.75 \times 2.5) + 8 + (2.5 \times 60) - 120 = 92.32$ m

$\therefore \quad s = \sqrt{(92.32)^2 + (57.5)^2} - 92.32 = 16.44$ m

$\therefore \quad q = 4 \times 10^{-7} \times 16.44 = 65.77 \times 10^{-7}$ m³/sec/m.

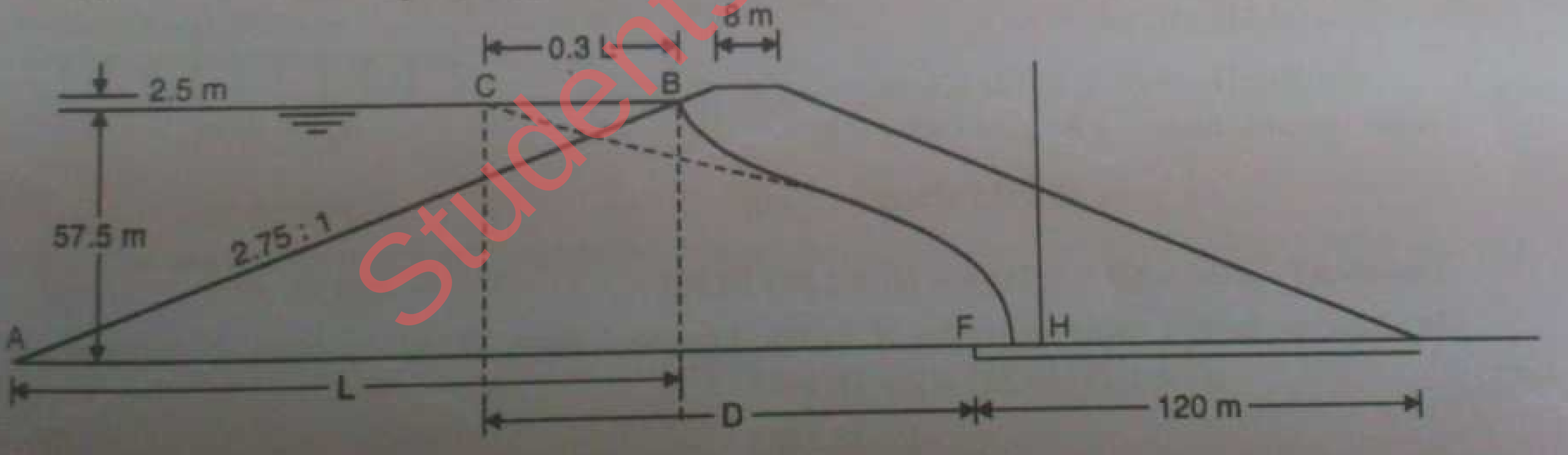


FIG. 9.23

Questions

Example 9.11. *A homogeneous anisotropic earth dam, which is 20 m high is constructed on an impermeable foundation. The coefficients of permeability of soil used for the construction of the dam, in the horizontal and vertical direction are 4.8×10^{-8} m/s and 1.6×10^{-8} m/s respectively. The water level on the reservoir side is 18 m from the base of the dam : downstream side is dry. It is seen that there are 4 flow channels and 18 equipotential drops is a square flownet drawn in the transformed dam section. Estimate the quantity of seepage per unit length in m^3/s through the dam.*

(*Gate Exam; 1994*)

Solution :

$$q = k \cdot H \cdot \frac{N_f}{N_d}$$

Here,

$$k = \sqrt{k_H \cdot k_V} = \sqrt{4.8 \times 1.6} \times 10^{-8} = 2.7713 \times 10^{-8} \text{ m/s}$$

$\therefore$

$$q = 2.7713 \times 10^{-8} \times 18 \times \frac{4}{18} = 11.085 \times 10^{-8} \text{ m}^3/\text{s/m run}$$