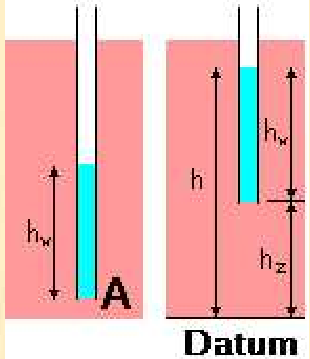
Unit-IV: Permeability of Soils

TOPICS TO BE COVERED:

Introduction
Darcy's law and its validity
discharge velocity and seepage velocity
factors affecting permeability
laboratory determination of coefficient of permeability
determination of field permeability
permeability of stratified deposits.

Pressure, Elevation and Total Heads

In soils, the interconnected pores provide passage for water. A large number of such flow paths act together, and the average rate of flow is termed the coefficient of permeability, or just permeability. It is a measure of the ease that the soil provides to the flow of water through its pores.



At point **A,** the pore water pressure (**u**) can be measured from the height of water in a standpipe located at that point.

The height of the water column is the **pressure head** (**h_w**).

h_w = u/g_w

To identify any difference in pore water pressure at different points, it is necessary to eliminate the effect of the points of measurement. With this in view, a datum is required from which locations are measured.

The **elevation head** (**h_z**) of any point is its height above the datum line. The height of water level in the standpipe above the datum is the **piezometric head** (**h**).

$h = h_z + h_w$

Total head consists of ***three components:*** elevation head, pressure head, and velocity head. As seepage velocity in soils is normally low, velocity head is ignored, and total head becomes equal to the piezometric head. Due to the low seepage velocity and small size of pores, the flow of water in the pores is steady and laminar in most cases. Water flow takes place between two points in soil due to the difference in total heads.

PERMEABILITY

Permeability is the passage or seepage of water into the soil through its interconnecting voids

The flow of water can be laminar or turbulent but the flow of water into soil is mostly laminar

The unit of Permeability is “cm/s”

It has a dominating influence on the total engg. behavior of soil

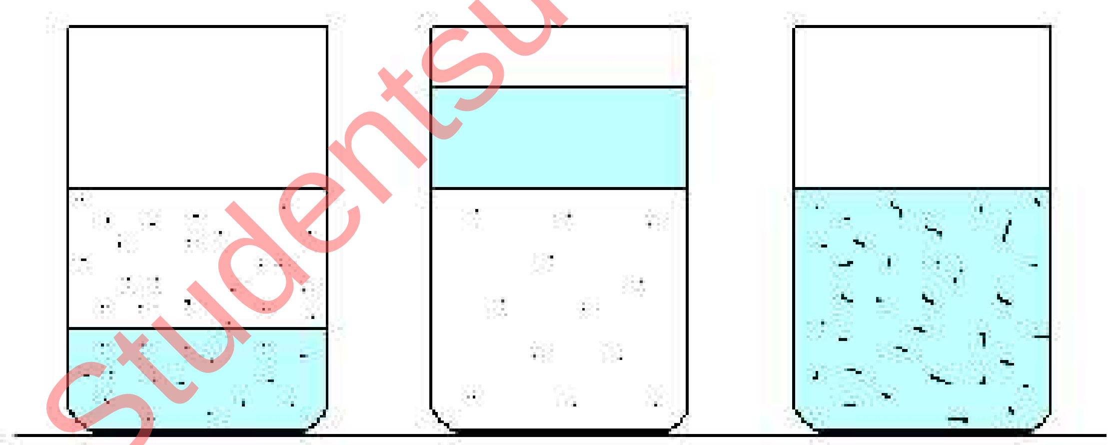
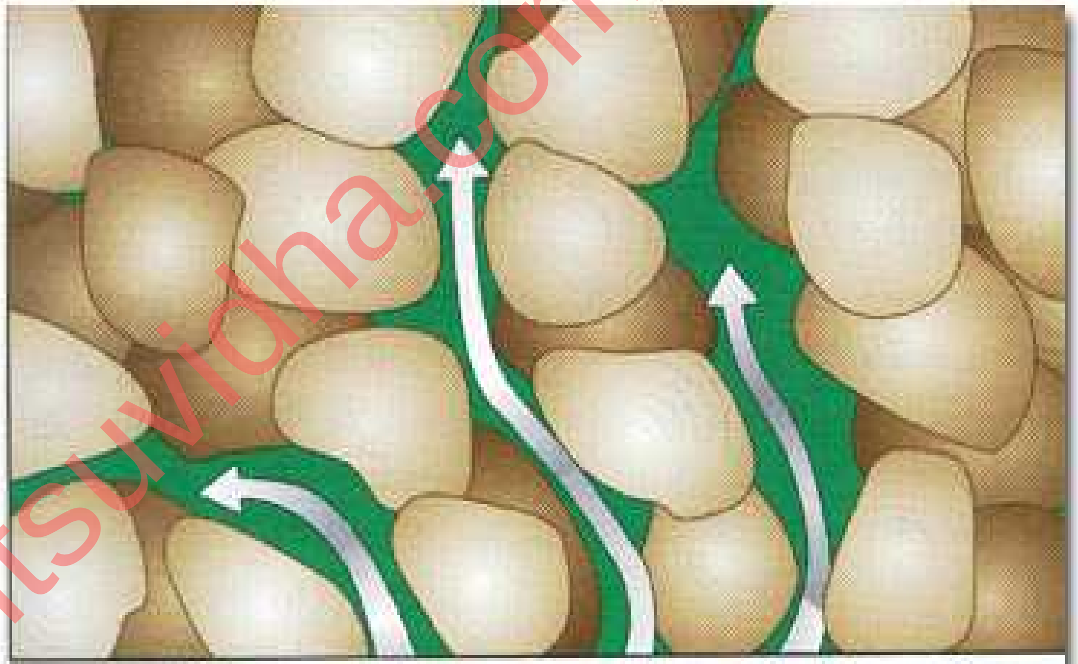


Figure 2 - Permeability of different types of soil components. In blue the distribution of water.

WHY PERMEABILITY STUDY IS ESSENTIAL?



Connected pores give a rock permeability.

To calculate the following:

•rate of settlement

•seepage though body of earth dam

•uplift pressure under hydraulic structures

•ground water flow towards wells and drainage of soils

Coefficient of Permeability (k)

Coefficient of permeability (k) of a soil is defined as the average velocity of flow through the total cross sectional area of soil under unit hydraulic gradient

Coefficient of permeability (*k*) of a soil is proportional to square of the particle size (D)

The value of coefficient of permeability of coarse grained soil may be one million times more than that of clay

$$k = \frac{QL}{Aht}$$

Where
Q = total volume of water
t = time period
h = head causing flow
L = length of the specimen
A = cross sectional area of soil

Darcy's law

For laminar flow conditions in a saturated soil the rate of flow or discharge per unit time (q) is proportional to hydraulic gradient (i)

$q = k\ i\ A$ where,

q = discharge per unit time

A= Total cross sectional area of soil perpendicular to the direction of flow

k = coefficient of permeability

i = hydraulic gradient

DARCY'S LAW

The rate of flow or discharge per unit time is proportional to hydraulic gradient.

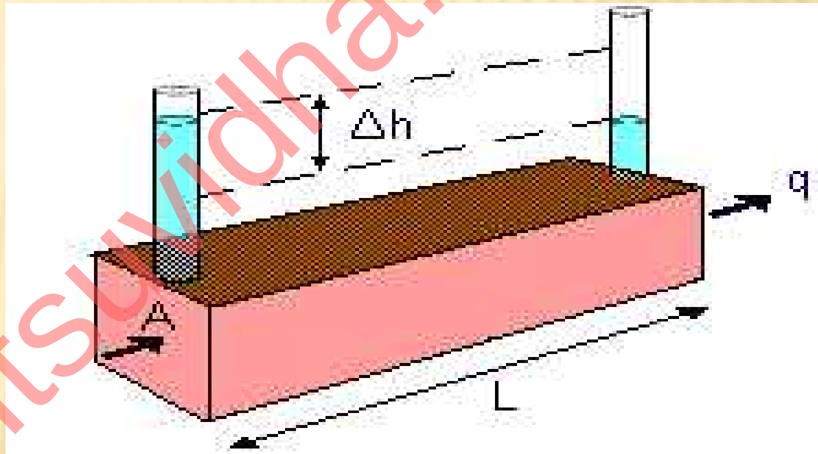
Or **Darcy's law** states that there is a linear relationship between flow velocity (**v**) and hydraulic gradient (**i**) for any given saturated soil under steady laminar flow conditions.

$$q \propto iA$$

$$q = kiA$$

$$\frac{q}{A} = ki = v$$

i = Δh /L



q = Discharge per unit time

k = Darcy's coefficient of permeability

i = Hydraulic gradient

A = C/S area of soil mass perpendicular to the direction of flow

v= velocity of flow or average discharge velocity

Validity of Darcy's law

Darcy's law is valid only for laminar flow conditions of flow of water through soil.

Reynolds found that the flow is laminar as long the velocity of flow is less than lower critical velocity (v_c) expressed in terms of Reynold's number.

$$\frac{v_c \, d \, \gamma_w}{\eta \, g} = 2000$$

Darcy's law is valid as long the flow through sand is laminar, for which the Reynolds number given in the following form must be less than unity.

$$\frac{vD_a \gamma_w}{\mu g} \leq 1$$

Typical values of coefficient of permeability

Soil type	Co-efficient of permeability (mm/s)	Drainage properties
Clean gravel	10^{1} - 10^{2}	Very good
Coarse and medium sand	10^{-2} - 10^{1}	Good
Fine sand, loose silt	10^{-4} - 10^{-2}	Fair
Dense silt, clayey silt	10^{-5} - 10^{-4}	Poor
Silty clay, clay	10^{-8} - 10^{-5}	Very poor

Discharge velocity and seepage velocity

Actual velocity of flow through the soil takes place only through voids of the soil is also known as seepage velocity.

Seepage velocity (v_s)is defined as the rate of discharge of percolating water per unit cross sectional area of voids perpendicular to the direction of flow.

Hence the actual velocity of flow will be more than the discharge velocity.

Discharge velocity and seepage velocity contd…

$$q = vA = v_s A_v$$

$$v_s = v\frac{A}{A_v}$$

$$v_s = \frac{v}{n} = \frac{(1+e)}{e}v$$

$$\frac{v_s}{v} = \frac{1}{n}$$

The seepage velocity is proportional to the hydraulic gradient ***Vs*** ~ ***i***

$V_s = k_p\, i$ ***where, " k_p" is coefficient of percolation***

wkt, *v = ki*

$V_s /v = k_p/k = 1/n$ OR

$k_p = k/n$

Methods of determination of permeability of soil

LABORATORY METHODS

- Constant head permeability test
- Variable or Falling head permeability test

The constant head test method is used for relatively more permeable soil ($k>10^{-4}$ cm/s) and
Variable or falling head test is used for less permeable soils ($k<10^{-4}$ cm/s)

FIELD METHODS (To measure in situ permeability)

- Pumping out tests
- Pumping in tests

Pumping out test influences large area around the well and give an overall value of coefficient of permeability

Pumping in tests influences small area surrounding the hole and gives the value of coefficient of permeability of the soil surrounding the hole

METHODS OF DETERMINATION OF PERMEABILITY OF SOIL

LABORATORY METHODS

1) Constant head permeability test

Constant head permeameter is recommended for coarse-grained soils only since for such soils, flow rate is measurable with adequate precision. As water flows through a sample of cross-section area **A**, steady total head drop **h** is measured across length **L**.

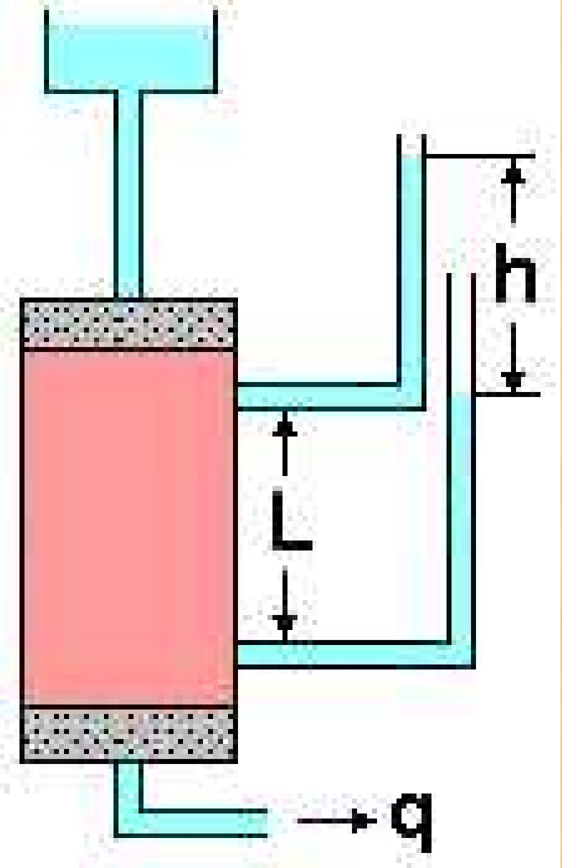
Permeability **k** is obtained from:

$$k = \frac{qL}{Ah}$$

Where, q = Q/t,

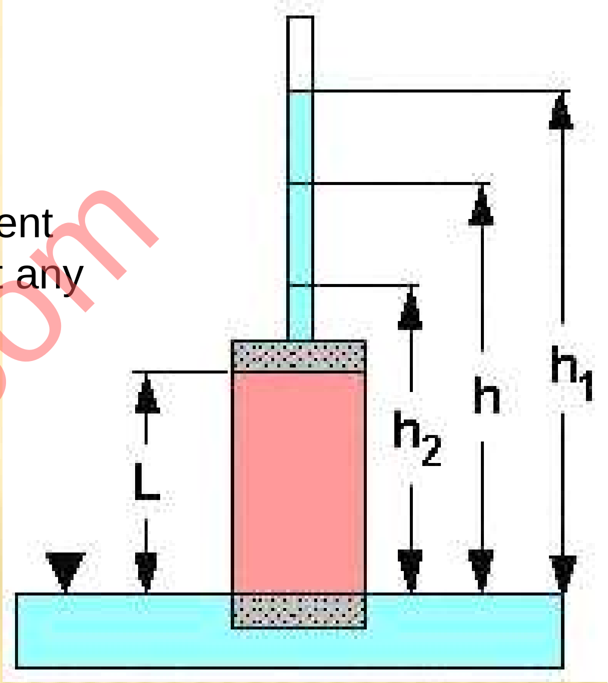
so k = (Q/t)*(L/h)* (1/A)

The constant head test method is used for relatively more permeable soil ($k>10^{-4}$ cm/s)



2) Variable or falling head permeability test

It is used for less permeable soils ($k<10^{-4}$ cm/s)



Falling head permeameter is recommended for fine-grained soils.
Total head **h** in standpipe of area **a** is allowed to fall. Hydraulic gradient varies with time. Heads $\mathbf{h_1}$ and $\mathbf{h_2}$ are measured at times $\mathbf{t_1}$ and $\mathbf{t_2}$. At any time **t**, flow through the soil sample of cross-sectional area **A** is

$$q = k.h.\frac{A}{L}$$ ------------------ (1)

Flow in unit time through the standpipe of cross-sectional area **a** is

$$= a \times \left(-\frac{dh}{dt}\right)$$ ------------------ (2)

Equating **(1)** and **(2)** ,

$$-a.\frac{dh}{dt} = k.h\frac{A}{L}$$ or $$-\frac{dh}{h} = \left(\frac{kA}{La}\right)dt$$

Integrating between the limits,

$$\frac{Akdt}{aL} = -\frac{dh}{h}$$

$$\frac{Ak}{aL}\int_{t_1}^{t_2} dt = -\int_{h_1}^{h_2} \frac{dh}{h}$$

$$\frac{Ak}{aL}(t_2 - t_1) = \log_e\left(\frac{h_1}{h_2}\right)$$

$$k = \left(\frac{aL}{At}\right)\log e\left(h_1 / h_2\right)$$

where

$$k = \left(\frac{2.3\, aL}{At}\right)\log_{10} h_1 / h_2$$

Factors affecting permeability are:

•Grain size ($k = c\, D_{10}^{2}$)

"C" is a constant]100, when D_{10} is expressed in cm.

•Properties of the pore fluid

•Voids ratio of the soil

•Structural arrangement of soil particle

•Entrapped air and foreign matter

•Adsorbed water in the clayey soil

Factors affecting Permeability

In soils, the permeant or pore fluid is mostly water whose variation in property is generally very less. Permeability of all soils is strongly influenced by the density of packing of the soil particles, which can be represented by void ratio (**e**) or porosity (**n**).

For Sands

In sands, permeability can be empirically related to the square of some representative grain size from its grain-size distribution. For filter sands, Allen Hazen in 1911 found that **k** ù **100** $(\mathbf{D_{10}})^2$ cm/s where $\mathbf{D_{10}}$= effective grain size in cm. Different relationships have been attempted relating void ratio and permeability, such as **k** ó $\mathbf{e^3/(1+e)}$, and **k** ó $\mathbf{e^2}$. They have been obtained from the Kozeny-Carman equation for laminar flow in saturated soils.

$$k = \frac{1}{k_0 k_T S_S^2} \cdot \frac{e^3}{1+e} \cdot \frac{\gamma_w}{\eta}$$

where $\mathbf{k_o}$ and $\mathbf{k_T}$ are factors depending on the shape and tortuosity of the pores respectively, $\mathbf{S_S}$ is the surface area of the solid particles per unit volume of solid material, and g_w and h are unit weight and viscosity of the pore water. The equation can be reduced to a simpler form as

$$k = C.\frac{e^3}{1+e} \approx C.e^2$$

For Silts and Clays

For silts and clays, the Kozeny-Carman equation does not work well, and **log k** versus **e** plot has been found to indicate a linear relationship.

For clays, it is typically found that

$$\log_{10} k = \frac{e - e_k}{C_k}$$

where $\mathbf{C_k}$is the permeability change index and $\mathbf{e_k}$ is a reference void ratio.

Permeability of Stratified Deposits

When a soil deposit consists of a number of horizontal layers having different permeabilities, the average value of permeability can be obtained separately for both vertical flow and horizontal flow, as $\mathbf{k_z}$ and $\mathbf{k_x}$ respectively.
Consider a stratified soil having horizontal layers of thickness $\mathbf{Z_1, Z_2, Z_3}$, etc. with coefficients of permeability $\mathbf{k_1}$, $\mathbf{k_2}$,$\mathbf{k_3}$, etc.

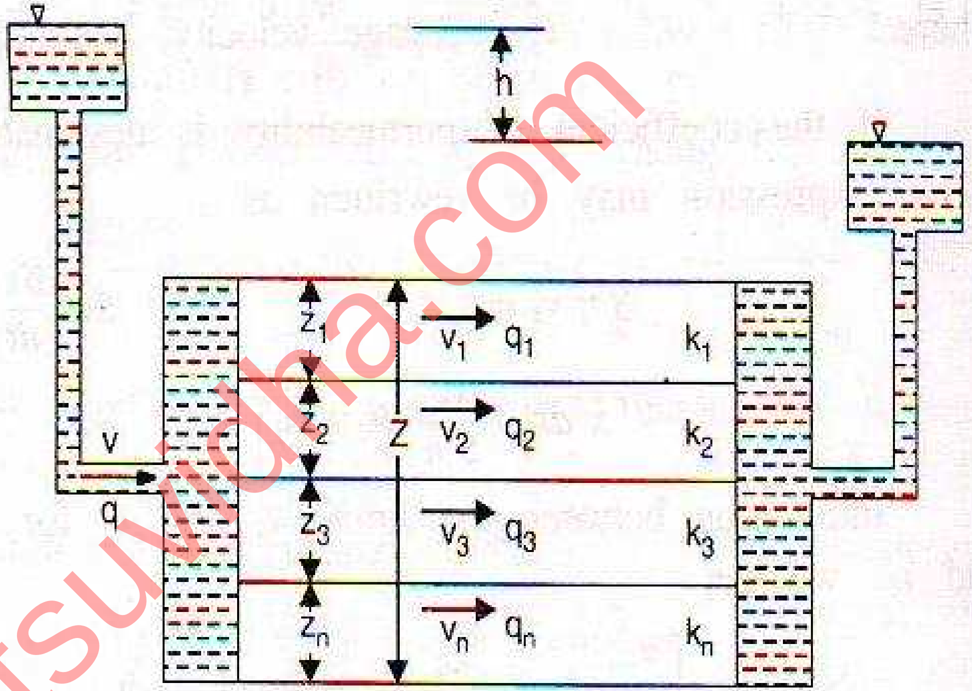
There are two cases:

Permeability parallel to bedding plane (stratification)
Permeability perpendicular to bedding plane (stratification)

1. Average permeability parallel to the bedding planes.

Let Z_1 , Z_2.....Z_n = thickness of layers and k_1 , k_2......k_n = permeabilities of the layers. For flow to be parallel to the bedding planes, the hydraulic gradient i will be the same for all the layers. However, since $v = ki$ and since k is different, the velocity of flow will be different in different layers.

Let k_x= average permeability of the soil deposit parallel to the bedding plane.



FLOW PARALLEL TO BEDDING PLANE.

Total disharge through the soil deposit = Sum of discharge throuh the individual layers

$\therefore$ $$q = q_1 + q_2 + ...q_n$$

or $$q = k_x\, i\, Z = k_1\, i\, Z_1 + k_2\, i\, Z_2 + ...k_n\, i\, Z_n$$

or $$k_x = \frac{k_1 Z_1 + k_2 Z_2 + ... + k_n Z_n}{Z} \quad \text{(where } Z = Z_1 + Z_2 + ...Z_n.) \quad ...(7.29)$$

2. Average permeability perpendicular to the bedding planes. In this case, the velocity of flow, and hence the unit discharge, will be the same through each layer. However, the hydraulic gradient, and hence the head loss through each layer will be different. Denoting the head loss through the layers by h_1 , h_2.......h_n and the total head loss as h, we have

$$h = h_1 - h_2 \ldots\ldots h_n$$

But $\quad h_1 = i_1 Z_1 \; ; \; h_2 = i_2 Z_2 \ldots\ldots \; ; \; h_n = i_n Z_n$

$$\therefore \quad h = i_1 Z_1 + i_2 Z_2 + \ldots\ldots i_n Z_n \qquad \ldots(i)$$

Now, if k_z = average permeability perpendicualr to the bedding plane, we have

$$v = k_z \, i = k_z \frac{h}{Z} \, , \qquad \text{or} \qquad h = \frac{vZ}{k_z}$$

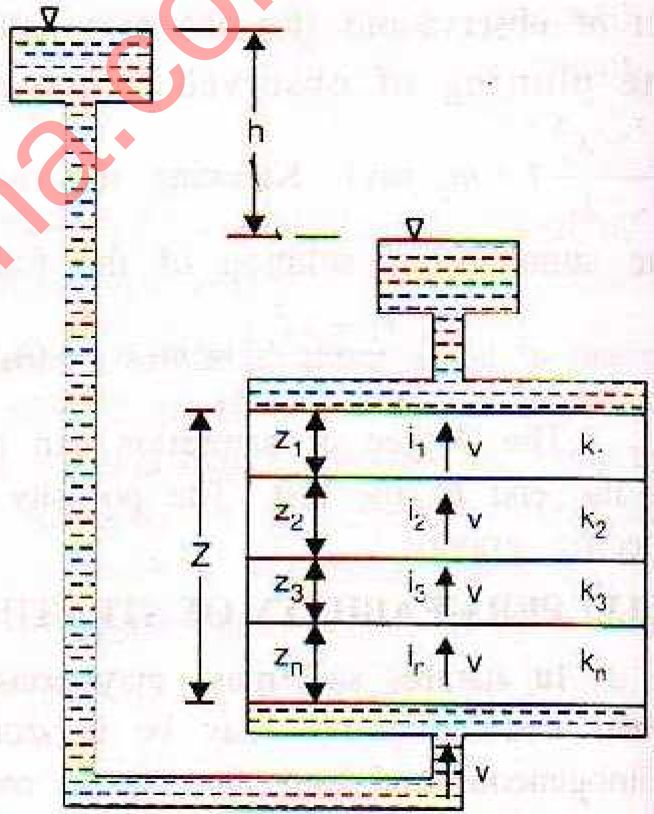
Also $\quad i_1 = \dfrac{v}{k_1} \; ; \; i_2 = \dfrac{v}{k_2} \; ; \quad i_n = \dfrac{v}{k_n}$

Substituting these values in (*i*), we get

$$\frac{vZ}{k_z} = \frac{vZ_1}{k_1} + \frac{vZ_2}{k_2} + \ldots\ldots \frac{vZ_n}{k_n}$$

or
$$k_z = \frac{Z}{\dfrac{Z_1}{k_1} + \dfrac{Z_2}{k_2} + \ldots\ldots \dfrac{Z_n}{k_n}} \qquad \ldots(7.30)$$

It can be shown that for any stratified soil mass k_x is always greater than k_z.



FLOW PERPENDICULAR BEDDING PLANE

Well hydraulics

- Aquifer
  - Aquifers are permeable formations having structures which permit appreciable quantity of water to move through them under ordinary field condition.

- Aquiclude
  - Aquiclude are the impermeable formations which contain water but are not capable of transmitting or supplying a significant quantity.

- Aquifuge
  - Aquifuge is an impermeable formation which neither contains water nor transmits any water.

Specific yield

The specific yield S_Y of an aquifer is defined as the ratio, expressed as percentage of the volume of water which, after being saturated, can be drained by gravity of the total volume of the aquifer.

S_y = Volume of water drained by gravity/Total volume

$S_y = (V_{wy}/V) * 100$

Specific retention:

The specific retention S_R of an aquifer is the ratio expressed water it will retain after saturation against the force of gravity to its own volume.

S_R = Volume of water retained/Total volume

$S_R = (V_{wr}/V) * 100$

Storage coefficient:

Storage coefficient is defined as the volume of water that an aquifer releases per unit surface area of aquifer per unit change in the component of head normal to that surface.

Co-efficient of transmissibility:

Co-efficient of transmissibility *T* is defined as the rate of flow of water (in m^3/day) through a vertical strip of aquifer of unit width (1m) and extending the full saturation height under unit hydraulic gradient. Thus the **Co-efficient of transmissibility *T*** equal the field **Co-efficient of permeability** multiplied by the aquifer thickness b.

$$T = b*k$$

STEADY RADIAL FLOW TO A WELL: DUPUIT'S THEORY

When a well is penetrated into an extensive homogeneous aquifer, the water table initially remains horizontal in the well. When the well is pumped, water is removed from the aquifer and the water table or the piezometric surface, depending upon the type of the aquifer, is lowered resulting in a parabolic depression in the water table or the piezometric surface. This depression is called the cone of depression or the drawdown curve. At any point, away from the well, the drawdown s is the vertical distance by which the water table or the piezometric surface is lowered.

Assumption of Dupuit's theory:

- The velocity of flow is proportional to the tangent of the hydraulic gradient instead of its sine.
- The flow is horizontal and uniform everywhere in the vertical section.
- Aquifer is homogeneous, isotropic and of infinite aerial extent.
- The well penetrates and receives water from the entire thickness of the aquifer.
- Co-efficient of transmissibility is constant at all places and times.
- Natural ground water regime affecting an aquifer remains constant with time.
- Flow is laminar and Darcy's law is valid.

FIELD METHODS (To measure in situ permeability)

Field or *in-situ* measurement of permeability avoids the difficulties involved in obtaining and setting up undisturbed samples in a permeameter. It also provides information about bulk permeability, rather than merely the permeability of a small sample.

A field permeability test consists of pumping out water from a main well and observing the resulting drawdown surface of the original horizontal water table from at least two observation wells. When a steady state of flow is reached, the flow quantity and the levels in the observation wells are noted.

Two important field tests for determining permeability are: Unconfined flow pumping test, and confined flow pumping test.

Unconfined Aquifer

1. Unconfined aquifer. Fig. 8.2. shows a well penetrating an unconfined or free aquifer to its full depth.

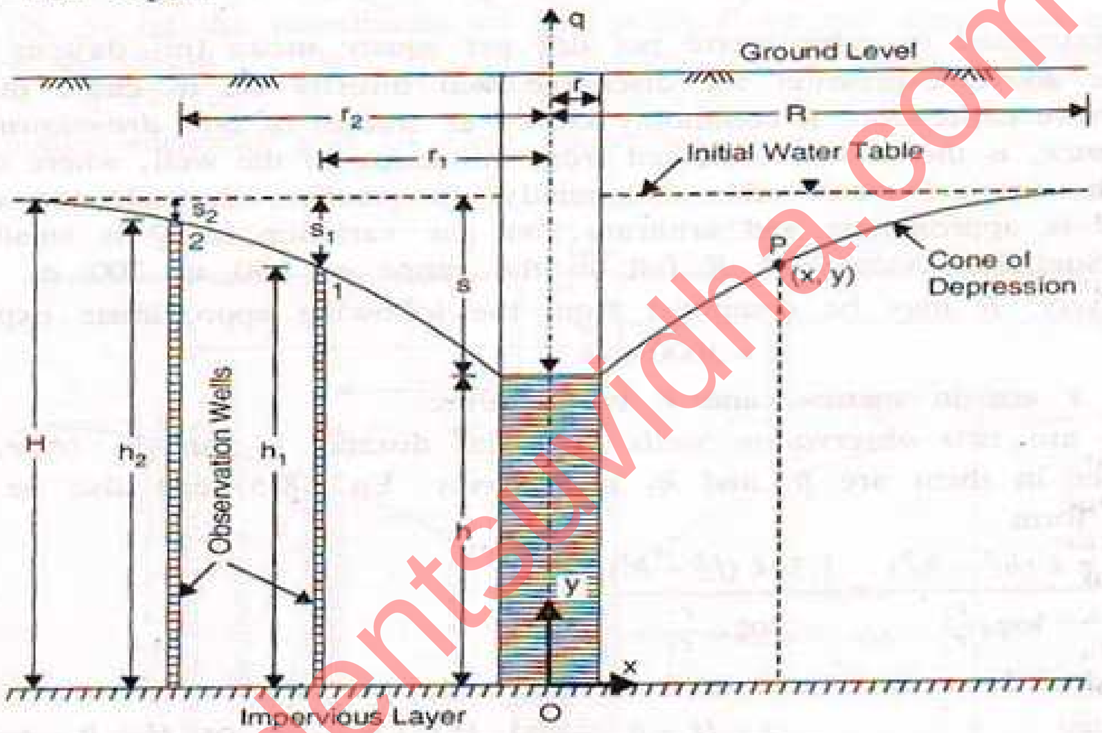


FIG. 8.2. UNCONFINED AQUIFER.

Let r = radius of the well

H = thickness of the aquifer, measured from the impermeable layer to the initial level of water table

s = drawdown at the well

h = depth of water in the well measured above the impermeable layer

Considering the origin of co-ordinates at a point O at the centre of the well at its bottom, let the co-ordinates of any point P on the drawdown curve be (x, y).

Then from Darcy's law, Discharge $q = k\, A_x\, i_x$

where A_x = area of cross-section of the saturated part of the aquifer at P

$= (2\pi\, x)\, y = 2\,\pi\, x\, y$

i_x = hydraulic gradient at $P = \dfrac{dy}{dx}$

Hence $q = k\,(2\pi\, x\, y)\,\dfrac{dy}{dx}$ or $q\,\dfrac{dx}{x} = 2\pi\, ky\, dy$

Integrating between the limits (R, r) for x and (H, h) for y, we get

$$q \int_r^R \frac{dx}{x} = 2\pi k \int_h^H y\, dy \qquad \therefore \quad q\,\left(\log_e x\right)_r^R = 2\pi\, k \left(\frac{y^2}{2}\right)_h^H$$

From which
$$q = \frac{\pi\, k\,(H^2 - h^2)}{\log_e \dfrac{R}{r}} = \frac{1.36k\,(H^2 - h^2)}{\log_{10} \dfrac{R}{r}} \qquad \ldots(8.5$$

If there are two *observation wells* at radial distance r_1 and r_2 $(r_2 > r_1)$ and if the depths of water in them are h_1 and h_2 respectively. Eq. (8.5) can also be expressed in the following form

$$q = \frac{\pi k (h_2^2 - h_1^2)}{\log_e \frac{r_2}{r_1}} = \frac{1.36k (h_2^2 - h_1^2)}{\log_{10} \frac{r_2}{r_1}} \quad \ldots(8.7)$$

If the drawdown s is measured at the well, we have

$$s = H - h \quad \text{and} \quad H = s + h \qquad \text{or} \quad H + h = s - 2h$$

Then, from Eq. 8.5, we get

$$q = \frac{\pi k(H - h)(H + h)}{\log_e \frac{R}{r}} = \frac{\pi ks(s + 2h)}{\log_e \frac{R}{r}} = \frac{\pi ks (s + 2L)}{\log_e \frac{R}{r}} = \frac{1.36ks(s + 2L)}{\log_{10} \frac{R}{r}} \quad \ldots(8.8)$$

where L = effective length of the strainer = h.

Confined Aquifer

2. Confined aquifer. Fig. 8.3 shows a well fully penetrating a confined or artesian aquifer. Let (x, y) be the coordinates of any point P on the drawdown curve, measured with respect to the origin O. Then, from Darcy's law, flow crossing a vertical plane through P is given by $q = k\, i_x\, A_x$.

where A_x = cross-sectional area of flow, measured at $P = 2\pi\, x\, b$

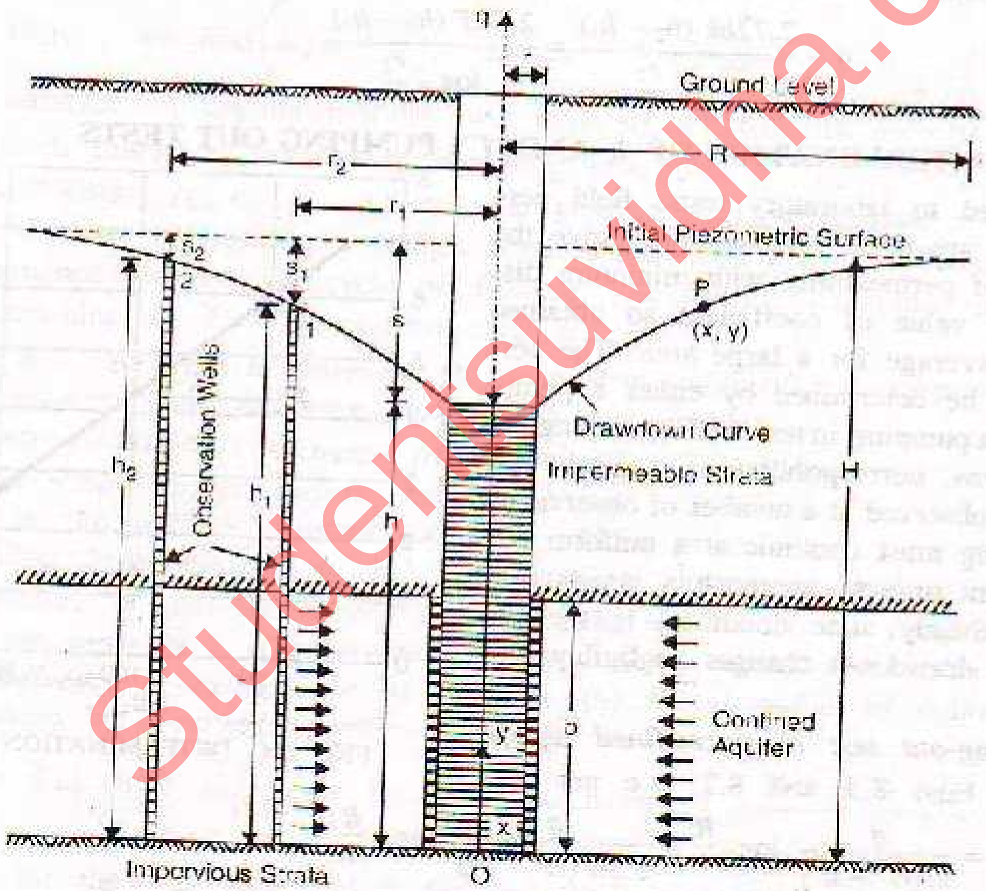


FIG. 8.3. CONFINED AQUIFER.

b = thickness of confined aquifer ; i_x = hydraulic gradient at $P = \frac{dy}{dx}$

$$\therefore \quad q = k\left(\frac{dy}{dx}\right)(2\pi\, x\, b) \quad \text{or} \quad q\,\frac{dx}{x} = 2\pi k\, b dy$$

Integrating between the limits (R , r) *for* x and (H , h) for y, we get

$$q\int_r^R \frac{dx}{x} = 2\pi kb \int_h^H dy \quad \text{or} \quad q\left[\log_e x\right]_r^R = 2\pi kb\left[y\right]_h^H$$

From which
$$q = \frac{2\pi\, k\, b\,(H-h)}{\log_e \frac{R}{r}} = \frac{2.72bk\,(H-h)}{\log_{10} \frac{R}{r}} \quad \ldots(8.9)$$

or
$$q = \frac{2\pi T\, s}{\log_e \frac{R}{r}} = \frac{2.72\; T\, s}{\log_{10} \frac{R}{r}} \quad \ldots(8.10)$$

where T = coefficient of transmissibility = bk and s = drawdown at the well

Eq. 8.9 is known as the *equilibrium equation* or the *Thiem equation.*

If h_1 and h_2 are the measured depths of water in two observation wells situated radially at distance r_1 and r_2 respectively, we get

$$q = \frac{2.72bk\,(h_2 - h_1)}{\log_{10} \frac{r_2}{r_1}} = \frac{2.72T\,(h_2 - h_1)}{\log_{10} \frac{r_2}{r_1}} \quad \ldots(8.11)$$

Field determination of *k* and *T*

- Pumping out tests in unconfined acquirer
- Pumping out tests in confined acquirer

Pumping out tests in unconfined acquirer

$$k = \frac{q}{1.36\left(H^2 - h^2\right)} \log_{10} \frac{R}{r}$$

$$k = \frac{q}{1.36\left({h_2}^2 - {h_1}^2\right)} \log_{10} \frac{r_2}{r_1}$$

Pumping out tests in confined acquirer

$$k = \frac{q}{2.72b\left(H - h\right)}\log_{10}\frac{R}{r}$$

$$k = \frac{q}{2.72b\left(h_2 - h_1\right)}\log_{10}\frac{r_2}{r_1}$$

From ground water point of view, it is the practice to determine coefficient of transmissibility (*T*) first then coefficient of permeability (*k*) is calculated by observing the drawdown from various observation wells. WRT figure (confined aquifer) let

S_1 = drawdown in observation well 1 = $(H-h_1)$.....a

S_2 = drawdown in observation well 2 = $(H-h_2)$.....b

a-b $\rightarrow (H-h_1) - (H-h_2) = (h_2 - h_1) = S_1 - S_2$, w k t ,

$$q = \frac{2.72T(h_2 - h_1)}{\log_{10}\frac{r_2}{r_1}} = \frac{2.72T(S_1 - S_2)}{\log_{10}\frac{r_2}{r_1}}$$

$$T = \frac{q}{2.72(S_1 - S_2)}\log_{10}\frac{r_2}{r_1}$$

Choosing $r_2 = 10\ r_1,$

$\log_{10} r_2/r_1 = 1$

$$T = \frac{q}{2.72(S_1 - S_2)} = \frac{q}{2.72\Delta s} \longrightarrow \textbf{A}$$

Where Δs is the difference in drawdown at the two observation wells so selected that $r_2 = 10\ r_1$

- Like this drawdown (S_1, S_2, S_3 and S_n) are measured for the various observation wells spaced at distance (r_1, r_2, r_3 and r_n) selected.

- Plot a graph between drawdown (s) as ordinate and distances (*r*) on $\log_{10}$ scale as abscissa. A straight line is obtained. From the graph Δs can be obtained for one log cycle distance and can be substituted in equation A to get *T*

- From *T* , *k* is calculated using , $k = T/b$

Further suggested reading

Types of pumping in tests (U S Bureau of reclamation)

- Open end test
- Packer test

According to penetration

Fully penetrating artesian well

Partially penetrating artesian well and

Recuperation (open well) test