Roll No. ....

Total No. of Questions : 09] **[Total No. of Pages : 03**

B. Tech. (Sem. - 3rd)

DISCRETE STRUCTURES

SUBJECT CODE : CS - 203

Paper ID : [A0452]

[Note : Please fill subject code and paper ID on OMR]

Time : 03 Hours **Maximum Marks : 60**

Instruction to Candidates:

1) Section - A is **Compulsory**.
2) Attempt any **Four** questions from Section - B.
3) Attempt any **Two** questions from Section - C.

Section - A

Q1) ***(10 × 2 = 20)***

a) Show that the sum of degree of all the vertices in a graph is even.

b) What is the difference between directed and undirected graph?

c) Give an example of a graph which is Hamiltonian but not Eulerian and vice versa.

d) Prove that $\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$.

e) How many positive integers not exceeding 500 are divisible by 7 or 11?

f) Consider the following relation on the set A = {1, 2, 3, 4}: R = {(1, 1), (2, 2), (2, 3), (3, 2), (4, 2), (4, 4)}. Draw its diagraph. Is R (i) reflexive (ii) antisymmetric and (iii) transitive?

g) Show that a semi group with two idempotent elements can not be a group.

h) Let G be a finite group with identity element e. Show that $a^n = e$ for any $a \in G$.

i) Consider the rings $(Z, +, .)$ and $(2Z, +, .)$ and define $f : Z \to 2Z$ by $f(n) = 2n$ for all $n \in Z$. Is f a ring isomorphism? Justify your answer.

j) Prove that the complement of every element of a Boolean algebra B is unique.

Section - B

(4 × 5 = 20)

Q2) (a) Show that the chromatic number of a graph C_n, where C_n is the cyclic with n vertices is either 2 or 3.

(b) Define rooted tree with example and show how it may be viewed as directed graph.

Q3) (a) Let P, Q and R be three finite sets. Prove that

$$|P \cup Q \cup R| = |P| + |Q| + |R| - |P \cap Q| - |P \cap R| - |Q \cap R| + |P \cap Q \cap R|.$$

(b) Solve the following recurrence relation using generating function :

$$S(k) - 6S(k-1) + 5S(k-2) = 0,\ k \geq 2, \text{ where } S(0) = 1,\ S(1) = 2.$$

Q4) What are the properties of relations? Explain with examples. Find the number of relation from the set $A = \{a, b, c\}$ to $B = \{1, 2\}$.

Q5) (a) Let (G, o) be a group. Show that (G, o) is an abelian group if and only if $(a \ o \ b)^2 = a^2 \ o \ b^2$.

(b) Let G be group of two by two invertible matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $ad - bc \neq 0$.

Let $H = \left\{ \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} : a \neq 0 \right\}$. Show that H is a normal subgroup of G.

Q6) (a) Let M be a ring of 2×2 matrices over integers. Consider the set $L = \left\{ \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} : a, b \in Z \right\}$. Show that L is a left ideal of M. Is L is right ideal of M?

(b) Show that the set of real numbers of the form $\{a + b\sqrt{2} : a, b \in Z\}$ is an integral domain. Is it a field?

Section - C

(2 × 10 = 20)

Q7) Let R be an equivalence relation on a set A. For $a, b \in A$ prove that

(a) $a \in [a]$

(b) $b \in [a]$ if and only if $[a] = [b]$.

(c) two equivalence classes are either identical or disjoint.

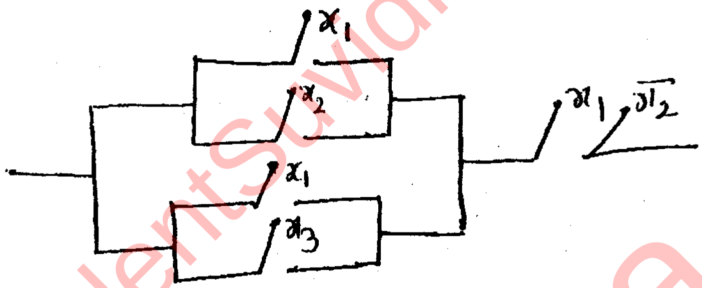
Q8) Consider the set Z together with binary operations $\oplus$ and $\otimes$ defined by $a \oplus b = a + b - 1$, $a \otimes b = a + b - ab$. Show that $(Z, \oplus, \otimes)$ is a ring.

Q9) Express the switching circuit shown in the figure through the logic or gate circuit.

(a) Write Boolean function.

(b) Simplify the function f algebraically.

(c) Find the minterm normal form by using Venn diagram and express it in gate diagram.



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