Roll No. $\square$ Total No. of Pages : 02
Total No. of Questions: 09

> B.Tech.(CSE/IT) (Sem.-3rd)
> DISCRETE STRUCTURES
> Subject Code: CS-203
> Paper ID: [A0452]

Time : 3 Hrs.
Max. Marks : 60

## INSTRUCTION TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B contains FIVE questions carrying FIVE marks each and students has to attempt any FOUR questions.
3. SECTION-C contains THREE questions carrying TEN marks each and students has to attempt any TWO questions.

## SECTION-A

1. Write short notes on :
(a) Define an equivalence relation on a set A .
(b) Let $f: \mathrm{R} \rightarrow \mathrm{R}$ and $g: \mathrm{R} \rightarrow \mathrm{R}$ be defined as $f(x)=\sin x, g(x)=x^{2}$. Find fog and gof.
(c) Prove that among any 13 people, there are at least 2 of them who were born in the same month.
(d) How many committees of four persons with a given chairman can be selected from 10 persons?
(e) Define a monoid.
(f) Prove that the intersection of two subgroups of a group $G$ is also a subgroup of G .
(g) Prove that a field F cannot have zero divisors.
(h) Draw logic circuit for $a b^{\prime}+a^{\prime} b$.
(i) Prove that in any graph, the number of vertices of odd degree can't be odd.
(j) Find the chromatic number of the complete graph $\mathrm{K}_{n}$ on $n$ vertices.

## SECTION-B

2. Construct a graph that has six vertices and five edges but is not a tree.
3. Prove that the set $A_{n}$ of all even permutations in $S_{n}$ is a normal subgroup.
4. Solve the recurrence relation
$a_{n}=a_{n-1}+2 a_{n-2}, n \geq 2$ with the initial condition $a_{0}=1, a_{1}=8$.
5. Prove that the relation $\{(a, b) \in \mathbb{N} \times \mathbb{N} / a=b \bmod 5\}$ is an equivalence relation on $\mathbb{N}$, the set of natural numbers.
6. Prove that every ideal A of a ring R is a kernel of some ring homomorphism.

## SECTION-C

7. (a) Does the graph shown below has a Hamiltonian circuit?

(b) Find the generating function of the sequence $1,2,3,4, \ldots$.
8. (a) Find the minimum distance of the encoding function $e: \mathrm{B}^{2} \rightarrow \mathrm{~B}^{3}$ defined by

$$
e\left(b_{1}, b_{2}\right)=\left(b_{1}, b_{2}, b_{1}+b_{2}\right)
$$

(b) Prove that in a graph having a vertex of odd degree, there is no Euler circuit.
9. (a) Prove that any two cyclic groups of the same order are isomorphic.
(b) State and prove the fundamental theorem of isomorphism for groups.

