

Roll No.

Total No. of Questions : 09]

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B.Tech. (Sem. - 3rd/4th)

MATHEMATICS - III

SUBJECT CODE : CS - 203/204 (2K2 Batch)

Paper ID : [A0457]

[Note : Please fill subject code and paper ID on OMR]

Time : 03 Hours

Maximum Marks : 60

Instruction to Candidates:

- 1) Section - A is **Compulsory**.
- 2) Attempt any **Four** questions from Section - B.
- 3) Attempt any **Two** questions from Section - C.

Section - A

Q1)

(10 × 2 = 20)

- a) Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 4$ in the interval $1 \leq x \leq 4$.
- b) Find the Laplace transform of $f(t) = \sqrt{t} e^{3t}$.
- c) Give an example of bilinear transformation.
- d) Prove that $\lim_{x \rightarrow a} (x - a) \sin \frac{1}{x - a} = 0$.
- e) State the Taylor's theorem with Lagrange's form of remainder.
- f) Give the statement of Cauchy Riemann equations.
- g) Using Picard's method find first approximation of $\frac{dy}{dx} = x + y^2, y(0) = 1$
- h) Write the necessary and sufficient conditions for Riemann's integration.
- i) Evaluate $\int_0^1 dx \int_0^x e^{\frac{y}{x}} dy$.
- j) Evaluate $\oint_C \frac{e^{-z}}{z+1} dz$, where C is the circle $|z| = 2$.

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P.T.O.

Section - B

(4 × 5 = 20)

Q2) Find the volume of tetrahedron bounded by the co-ordinate planes and the

$$\text{plane } \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

Q3) Find the sum of the residues of the function $f(z) = \frac{\sin z}{z \cos z}$ at its poles inside the circle $|z| = 2$.

Q4) Using convolution theorem, find the inverse of $\frac{s+2}{(s^2+4s+5)^2}$.

Q5) Using Fourier sine integral, show that

$$\int_0^{\infty} \frac{1 - \cos \pi \lambda}{\lambda} \sin x \lambda \, d\lambda = \begin{cases} \frac{1}{2} \pi, & \text{when } 0 < x < \pi \\ 0, & \text{when } x > \pi \end{cases}$$

Q6) Find the entire length of the cardioid $r = a(1 + \cos \theta)$.

Section - C

(2 × 10 = 20)

Q7) Using Runge Kutta method of order 4, find $y(0.1)$ and $y(0.2)$ given that

$$\frac{dy}{dx} = x^2 - y, \quad y(0) = 1. \quad (\text{Take } h = 0.2)$$

Q8) Find the Laurent series of $f(z) = \frac{1}{(z-1)(z-2)}$ for the following intervals

(a) $1 < |z| < 2$ (b) $|z| > 2$

Q9) (a) A tightly stretched string with fixed end points $x = 0$ and $x = L$ is initially in a position given by $f(x)$. The initial velocity is zero, where

$$f(x) = \begin{cases} \frac{2K}{L}x & 0 \leq x \leq \frac{L}{2} \\ \frac{2K}{L}(L-x) & \frac{L}{2} < x < L \end{cases}$$

Obtain the displacement at any point x and any time t . (use wave equation).

(b) Find the values of a, b, c, d so that the function $f(x)$ is analytic.

where $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$

