

Roll No.

Total No. of Questions : 09]

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B.Tech. (Sem. - 4th)
MATHEMATICS - III
SUBJECT CODE : CS - 204
Paper ID : [A0495]

[Note : Please fill subject code and paper ID on OMR]

Time : 03 Hours**Maximum Marks : 60****Instruction to Candidates:**

- 1) Section - A is **Compulsory**.
- 2) Attempt any **Four** questions from Section - B.
- 3) Attempt any **Two** questions from Section - C.

Section - A**Q1)****(10 × 2 = 20)**

- a) Show that the function $f(x) = \sin\left(\frac{1}{x}\right)$ is continuous & bounded on $\left(0, \frac{2}{\pi}\right)$ but is not uniformly continuous there.
- b) State necessary & sufficient condition for $f(z)$ to be analytic.
- c) Write Cauchy-Riemann equation in Polar co-ordinates.
- d) Evaluate $\oint_C (x^2 - y^2 + 2ixy) dz$, where C is the contour $|z| = 1$.
- e) Classify the partial differential = u
 $(x + 1) u_{xx} - 2(x + 2) u_{xy} + (x + 3) u_{yy} = \cos(x - 2y)$
- f) Find the Laplace transform of $f(t)$ defined as

$$f(t) = \begin{cases} \frac{t}{T}, & \text{when } 0 < t < T \\ 1, & \text{when } t > T. \end{cases}$$
- g) State convolution theorem for Fourier Transform.

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h) Discuss applicability for Rolle's theorem to the function $f(x)$ defined as follows :

$$f(x) = \begin{cases} x^2 - 4, & \text{if } x < 1 \\ 5x - 8, & \text{if } x \geq 1 \end{cases} \text{ in } \left[-2, \frac{8}{5} \right]$$

- i) Define unit Impulse function & find $\int_0^{\infty} t^3 \delta(t - 5) dt$.
- j) Define periodic function with examples.

Section - B

(4 × 5 = 20)

Q2) Use Cauchy's Integral Formula to evaluate $\oint_C \frac{e^{2z}}{(z+1)^4} dz$, where C is the circle $|z| = 2$.

Q3) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(1+x^2)^3} dx$.

Q4) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz dy dx}{\sqrt{1-x^2-y^2-z^2}}$, by changing to Spherical polar co-ordinates.

Q5) An electrostatic field in the xy-plane is given by the potential function $\phi = 3x^2y - y^3$, find the stream function.

Q6) Find a series of cosines of multiples of x which will represent xsinx in the interval $(0, \pi)$ & show that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots = \frac{\pi - 2}{4}$

Section - C

(2 × 10 = 20)

Q7) Expand $\frac{1}{(z^2 + 1)(z^2 + 2)}$ as a Laurent series valid for

(a) $0 < |z| < 1$

(b) $1 < |z| < \sqrt{2}$

(c) $|z| > \sqrt{2}$.

Q8) A string is stretched & pastened to two points l apart. Motion is started by displacing the string in the form $y = a \sin \frac{\pi x}{l}$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at the t is given by $y(x,t) = a \sin \frac{\pi x}{l} \cdot \cos \frac{\pi ct}{l}$.

Q9) Evaluate $y(0.2)$ by Runge-Kutta (4th order) method given that $y'' - xy'^2 + y^2 = 0$; $y(0) = 1, y'(0) = 0$

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