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**B.Tech. (Sem. - 3<sup>rd</sup>/4<sup>th</sup>)**

**MATHEMATICS - III**

**SUBJECT CODE : CS - 203/204 (2K2 Batch)**

**Paper ID : [A0457]**

[Note : Please fill subject code and paper ID on OMR]

Time : 03 Hours

Maximum Marks : 60

Instruction to Candidates:

- 1) Section - A is **Compulsory**.
- 2) Attempt any **Four** questions from Section - B.
- 3) Attempt any **Two** questions from Section - C.

**Section - A**

Q1)

(10 × 2 = 20)

- a) Verify Rolle's theorem for the function  $f(x) = x^2 - 5x + 4$  in the interval  $1 \leq x \leq 4$ .
- b) Find the Laplace transform of  $f(t) = \sqrt{t} e^{3t}$ .
- c) Give an example of bilinear transformation.
- d) Prove that  $\lim_{x \rightarrow a} (x - a) \sin \frac{1}{x - a} = 0$ .
- e) State the Taylor's theorem with Lagrange's form of remainder.
- f) Give the statement of Cauchy Riemann equations.
- g) Using Picard's method find first approximation of  $\frac{dy}{dx} = x + y^2, y(0) = 1$
- h) Write the necessary and sufficient conditions for Riemann's integration.
- i) Evaluate  $\int_0^1 dx \int_0^x e^{\frac{y}{x}} dy$ .
- j) Evaluate  $\oint_C \frac{e^{-z}}{z+1} dz$ , where C is the circle  $|z| = 2$ .

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Section - B

(4 × 5 = 20)

Q2) Find the volume of tetrahedron bounded by the co-ordinate planes and the

$$\text{plane } \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

Q3) Find the sum of the residues of the function  $f(z) = \frac{\sin z}{z \cos z}$  at its poles inside the circle  $|z| = 2$ .

Q4) Using convolution theorem, find the inverse of  $\frac{s+2}{(s^2+4s+5)^2}$ .

Q5) Using Fourier sine integral, show that

$$\int_0^\infty \frac{1 - \cos \pi \lambda}{\lambda} \sin x \lambda \, d\lambda = \begin{cases} \frac{1}{2} \pi, & \text{when } 0 < x < \pi \\ 0, & \text{when } x > \pi \end{cases}$$

Q6) Find the entire length of the cardioid  $r = a(1 + \cos \theta)$ .

Section - C

(2 × 10 = 20)

Q7) Using Runge Kutta method of order 4, find  $y(0.1)$  and  $y(0.2)$  given that

$$\frac{dy}{dx} = x^2 - y, y(0) = 1. \text{ (Take } h = 0.2)$$

Q8) Find the Laurent series of  $f(z) = \frac{1}{(z-1)(z-2)}$  for the following intervals

(a)  $1 < |z| < 2$  (b)  $|z| > 2$

Q9) (a) A tightly stretched string with fixed end points  $x = 0$  and  $x = L$  is initially in a position given by  $f(x)$ . The initial velocity is zero, where

$$f(x) = \begin{cases} \frac{2K}{L} x & 0 \leq x \leq \frac{L}{2} \\ \frac{2K}{L} (L-x) & \frac{L}{2} < x < L \end{cases}$$

Obtain the displacement at any point  $x$  and any time  $t$ . (use wave equation).

(b) Find the values of  $a, b, c, d$  so that the function  $f(x)$  is analytic.

$$\text{where } f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$$

