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Total No. of Questions : 09] **[Total No. of Pages : 02**

B.Tech.(Sem. – 3rd/4th)

MATHEMATICS III

SUBJECT CODE : CS - 204

Paper ID : [A0495]

Time : 03 Hours **Maximum Marks : 60**

Instructions to Candidates:

1) Section - **A** is **Compulsory**.
2) Attempt any **Four** questions from Section - **B**.
3) Attempt any **Two** questions from Section - **C**.

Section - A

Q1) ***(10 × 2 = 20)***

a) State Lagrange's mean value theorem and verify the same for $f(x) = x^2$, [1,5]

b) Evaluate $\int_0^2 \int_0^{\sqrt{2x}} xy\, dy\, dx$

c) Determine whether $f(z)=(x-y)^2 + 2i(x+y)$ analytic anywhere?

d) Find $\oint_c \frac{5z^2 - 4z + 3}{z - 2} dz$ where c is ellipse $16x^2+9y^2 = 144$

e) Determine the residue at the poles for $\frac{\sin z}{z^2}$

f) Write down one dimensional heat equation? Classify the differential equation in terms of i) Elliptic ii) Parabolic or iii) Hyperbolic

g) Write down the algebraic equation by taking Laplace transform of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = x \quad y(0) = 1, y'(0) = 0$

h) By Euler's method find y(0.2), $y' = x + y$ h= 0-1, y(0)= 1, where H is step length.

i) Explain Taylor's series method for solving the differential Eq $\frac{dy}{dx} = f(x,y)$, $y(n_0) = y_0$. Compute $y(0.2)$ if $f(x, y) = x + y$, $x_0=0$, $y_0=1$.

j) Find fourier transform of $f(x) = \begin{cases} 1 & |x| < 9 \\ 0 & |x| > 9 \end{cases}$

Section – B

(4 × 5 = 20)

Q2) Find the volume of tetrahedron bounded by coordinate planes and the plane $x+2y+3z=4$.

Q3) Show that for $f(z) = \begin{cases} \frac{2xy(x+iy)}{x^2+y^2} & \text{if } z \neq 0 \\ 0 & z = 0 \end{cases}$

Cauchy Riemann equations are satisfied at origin but derivative of f(z) does not exist at origin.

Q4) Prove that circles are mapped on to circles under the mapping $w=\frac{1}{z}$.

Q5) Use Runge-kutta method of order four to find Y at x=0.1 given that $x(dy+dx)=y(dx-dy)$, $y(0)=1$

Q6) Find the general solution of Laplace equation $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=0, v(x,\infty)=0$.

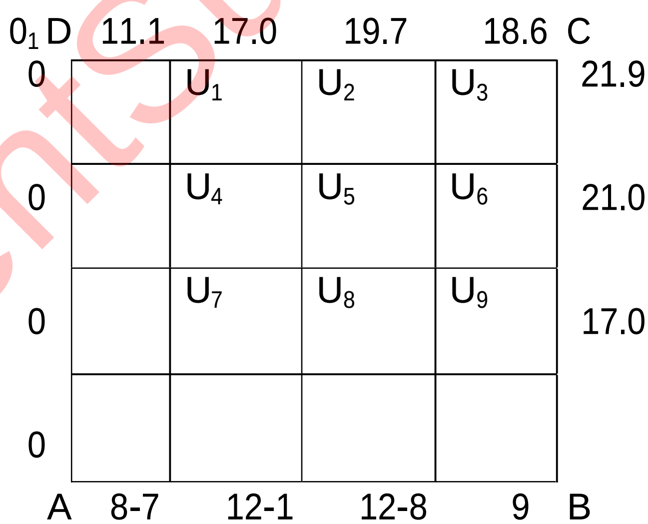
Section – C ***(2 × 10 = 20)***

Q7) a) Expand f(z) in lauss erits series, where f(z)= $\frac{1}{z^2-4z+3}$,for $1<|z|<3$.

b) Evaluate $\oint_C \frac{1}{(z^2+4)^2}$ dz, where is $|z-i|=2$

Q8) A tightly stretched string has its ends fixed at x=o and x=L. At time t=o, the string is given a shape defined by f(x)= $\mu x(l-x)$, where μ is a constant and then released. Find the displacement of any point x of the string at any time t>0 .

Q9) Find the value of u(x, y) satisfying the Laplace equation ∇^2u=0 at the pivotal points of a square region with boundary values as shown in figure.



Solve the problem up to 1st iteration after obtaining initial values.