Roll No.

**8616**

Printed Pages : 4

**BT–6 / M12**

**DIGITAL SIGNAL PROCESSING**

**Paper–ECE–306–E**

*Time allowed : 3 hours]* *[Maximum marks : 100*

**Unit–I**

**1.** (a) Determine the causal signal x (n) if its z-transform X (z) is given by :

$$X(z) = \frac{1}{4} \frac{\left(1 + 6\,z^{-1} + z^{-2}\right)}{\left(1 - 2\,z^{-1} + 2\,z^{-2}\right)\left(1 - \frac{1}{2}\,z^{-1}\right)} \quad 5$$

(b) If X (z) is the z-transform of x (n), show that if

$$x_k(n) = \begin{cases} x\left(\frac{n}{k}\right), & \text{if } \frac{n}{k} \text{ integer} \\ 0, & \text{otherwise} \end{cases}$$

then $X_k(Z) = X(Z^k)$ 8

(c) Determine the convolution of following pairs of signals by means of z-transform

$x_1(n) = nu(n)$

$x_2(n) = (2^n)\,u(n-1)$ 7

4. (a) Consider the system described by the difference equation

$$y(n) = a\,y(n-1) - a\,x(n) + x(n-1)$$

(i) Show that it is all-pass. 5

(ii) Obtain the direct form II realization of the system. 5

(b) Consider a causal IIR system with the system function

$$H(z) = \frac{1+2z^{-1}+3z^{-2}+2z^{-3}}{1+0.9z^{-1}-0.8z^{-2}+0.5z^{-3}}$$

Determine the equivalent lattice-ladder structure. 10

## Unit–III

5. (a) Design an FIR linear-phase, digital filter approximating the ideal frequency response

$$H_d(w) = \begin{cases} 1, & \text{for } |w| \le \frac{\pi}{6} \\ 0, & \text{for } \frac{\pi}{6} < |w| \le \pi \end{cases}$$

(i) Determine the coefficients of a 25-tap filter based on window method with a rectangular window. 5

(ii) Repeat part (i) using Hamming window. 5

(b) Explain the Gibbs phenomenon with example. 10

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6. (a) Write a short note on "Alternation theorem". 10

(b) Determine the unit sample response {h (n)} of a linear-phase FIR filter of length M = Y for which the frequency response at w = 0 and $w = \frac{\pi}{2}$ is specified as $H_r(0) = 1, H_r\left(\frac{\pi}{2}\right) = \frac{1}{2}$. 10

## Unit–IV

7. Explain the design of digital filters based on least square's methods. 20

8. A digital low-pass filter is required to meet the following specifications :

Passband Ripple : $\leq$ 1 dB

Passband Edge : 4 KHz

Stopband attenuation : $\geq$ 40 dB

Stopband edge : 6 KHz

Sample Rate : 24 KHz ; Assume t = 1

The filter is to be designed by performing a bilinear transformation on an analog system function. Determine what order Butterworth, Chebyshev and Elliptic analog designs must be used to meet the specifications in the digital implementation. 6+6+8