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# BT6/M11 8616

## Digital Signal Processing

## Paper—ECE-306E

Time : Three Hours] [Maximum Marks : 100

**Note** :—Attempt any **FIVE** questions in all, selecting at least **ONE** question from each unit.

### UNIT—I

1. (a) Let x(n) be a sequence with z-transform X(z). Determine, in terms of X(z), the z-transform of $x_1(n) = x\left(\frac{n}{2}\right)$, if n even. 5

(b) Use contour integration to determine the sequence x(n) where :

$$X(z) = \frac{\left(1-\frac{1}{4}z^{-1}\right)}{\left(1-\frac{1}{6}z^{-1}-\frac{1}{6}z^{-2}\right)},\ |z| > \frac{1}{2}.$$ 8

(c) Compute the quantity $\sum_{n=0}^{(N-1)} x_1(n)\,x_2(n)$ if :

$x_1(n) = \delta(n) + \delta(n-8)$

$x_2(n) = u(n) - u(n-N)$. 7

2. (a) Determine the signal x(n) with z-transform :

$X(z) = e^z + e^{1/z},\ |z| \neq 0.$ 8

(b) Determine if the following FIR system is minimum phase :—

$h(n) = [\underset{\uparrow}{5}, 4, -3, -4, 0, 2, 1].$ 5

(c) Prove the identity :

$$\sum_{\ell=-\infty}^{+\infty} \delta(n+\ell N) = \frac{1}{N}\sum_{k=0}^{(N-1)} e^{j\left(\frac{2\pi}{N}\right)kn}.$$ 7

## UNIT—II

3. (a) Consider a causal IIR system with system function :

$$H(z) = \frac{1+2z^{-1}+3z^{-2}+2z^{-3}}{1+0{\cdot}9z^{-1}-0{\cdot}8z^{-2}+0{\cdot}5z^{-3}}.$$

Determine the equivalent lattice ladder structure. 10

(b) Consider the system described by the difference equation

$$y(n) = ay(n-1) - ax(n) + x(n-1)$$

(i) Show that it is all-pass. 5

(ii) Obtain the direct form II realization of the system. 5

4. (a) Determine the state-space model for the system described by

$$y(n) = y(n-1) + 0{\cdot}11\, y(n-2) + x(n)$$

and sketch the type 1 and type 2 state-space realizations.

5+5+5

(b) Determine a direct form realization of following linear phase filters :

$$h(n) = \{\, \underset{\uparrow}{1}, 2, 3, 4, 3, 2, 1 \,\}.$$ 5

## UNIT—III

5. (a) Determine the unit sample response {h(n)} of a linear-phase FIR filter of length M = 4 for which the frequency response at $w = 0$ and $w = \frac{\pi}{2}$ is specified as $H_r(0) = 1$, $H_r\left(\frac{\pi}{2}\right) = \frac{1}{2}$. 10

(b) Write a short note on "Alternation Theorem". 10

6. (a) Explain the Gibbs phenomenon with example. 10

(b) Design an FIR linear-phase, digital filter approximating the ideal frequency response :

$$H_d(w) = \begin{cases} 1, & \text{for } |w| \le \pi/6 \\ 0, & \text{for } \frac{\pi}{6} < |w| \le \pi . \end{cases}$$

(i) Determine the coefficient of a 25-tap filter based on window method with a rectangular window. 5

(ii) Repeat part (i) using Hamming window. 5

## UNIT—IV

7. A digital low-pass filter is required to meet the following specifications :—

Passband ripple : ≤ 1 dB

Passband edge : 4 kHz

Stopband attenuation : ≥ 40 dB

Stopband edge : 6 kHz

Sample rate : 24 kHz; Assume t = 1.

The filter is to be designed by performing a bilinear transformation on an analog system function. Determine what order Butterworth, Chebyshev and Elliptic analog designs must be used to meet the specifications in the digital implementation. 6+6+8

8. Explain the design of digital filters based on Least-squares method. 20