

DIGITAL SIGNAL PROCESSING

Paper—ECE-306E

Time : Three Hours]

[Maximum Marks : 100

Note :—Attempt any FIVE questions.

1. (a) Determine the Z-transform of the signals and sketch their ROCs :

(i)  $x(n) = a^n (\cos \omega_0 n) u(n)$

(ii)  $x(n) = n(-1)^n u(n)$ . 4+4

- (b) Determine the inverse Z-transform of

$$X(z) = \frac{1 + 3z^{-1} + \frac{11}{6}z^{-2} + \frac{1}{3}z^{-3}}{1 + \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}}$$

by partial-fraction expansion. 8

- (c) A LTI system is characterized by the system function

$$H(z) = \frac{3 - 4z^{-1}}{1 - 3.5z^{-1} + 1.5z^{-2}}$$

Specify the ROC of H(z) and determine h(n) if

(i) system is causal

(ii) system is unstable. 4

2. (a) Determine, using Schür-Cohn stability test, if the system having the system function

$$H(z) = \frac{1}{1 - \frac{7}{4}z^{-1} - \frac{1}{2}z^{-2}} \text{ is stable.} \quad 10$$



(b) Determine DFT of data sequence

$$x(n) = \{1, 1, 2, 2, 3, 3\}$$

and compute corresponding amplitude and phase spectrum.

10

3. (a) Draw the Direct Form I, Direct Form II, Cascade and Parallel realisation structures for the system described by the system function:

$$(i) H(z) = \frac{5\left(1 - \frac{1}{4}z^{-1}\right)\left(1 - \frac{2}{3}z^{-1}\right)(1 + 2z^{-1})}{\left(1 - \frac{3}{4}z^{-1}\right)\left(1 - \frac{1}{5}z^{-1}\right)\left[1 - \left(\frac{1}{2} + j\frac{1}{2}\right)z^{-1}\right]}$$

$$(ii) H(z) = \frac{\frac{z}{6} + \frac{5}{24} + \frac{5}{24}z^{-1} + \frac{1}{24}z^{-2}}{1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}}$$

8+8

(b) Realise the following non causal linear phase FIR system function:—

$$H(z) = \frac{2}{3}z + 1 + \frac{2}{3}z^{-1}$$

4

4. (a) Realise a system defined by the following state space equations:—

$$\begin{bmatrix} r_1(n+1) \\ r_2(n+1) \end{bmatrix} = \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} r_1(n) \\ r_2(n) \end{bmatrix} + \begin{bmatrix} 4 \\ 6 \end{bmatrix} x(n)$$

10

(b) Realise an FIR filter with impulse response h(n) given by:

$$h(n) = \left(\frac{1}{2}\right)^n [4(n) - 4(n-5)]$$

10

5. (a) What is an FIR system. Compare an FIR system with an IIR system.

6



- (b) Define Phase delay and Group delay. 4
- (c) Determine the magnitude response and show that the phase and group delays are constant for the following transfer function which characterises an FIR filter ( $M = 11$ ):

$$H(z) = \sum_{n=0}^{M-1} h(n) z^{-n}. \quad 10$$

6. (a) A low-pass filter has the desired response

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & , \quad 0 \leq \omega \leq \pi/2 \\ 0 & , \quad \pi/2 \leq \omega \leq \pi. \end{cases}$$

Determine the filter coefficients  $h(n)$  for  $M = 7$ . 10

- (b) The desired response of a low-pass filter is :

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & , \quad -3\pi/4 \leq \omega \leq 3\pi/4 \\ 0 & , \quad 3\pi/4 < |\omega| \leq \pi. \end{cases}$$

Determine  $H(e^{j\omega})$  for  $M = 7$ , using Blackman window. 10

7. (a) Design a digital Butterworth filter to meet the constraint

$$\begin{aligned} 0.8 \leq |H(e^{j\omega})| \leq 1 & \quad 0 \leq \omega \leq 0.2\pi \\ |H(e^{j\omega})| \leq 0.2 & \quad 0.26\pi \leq \omega \leq \pi \end{aligned}$$

using bilinear transformation. 15

- (b) Explain Gibbs Phenomenon. 5

8. (a) Determine  $H(z)$  using the impulse invariant technique for the analog system function :

$$H(s) = \frac{1}{(s + 0.5)(s^2 + 0.5s + 2)}. \quad 10$$

- (b) Write short notes :

- (i) Design by approximation of derivatives
- (ii) Elliptical analog filters. 5+5