Signal and Systems

Paper : EE - 208 E

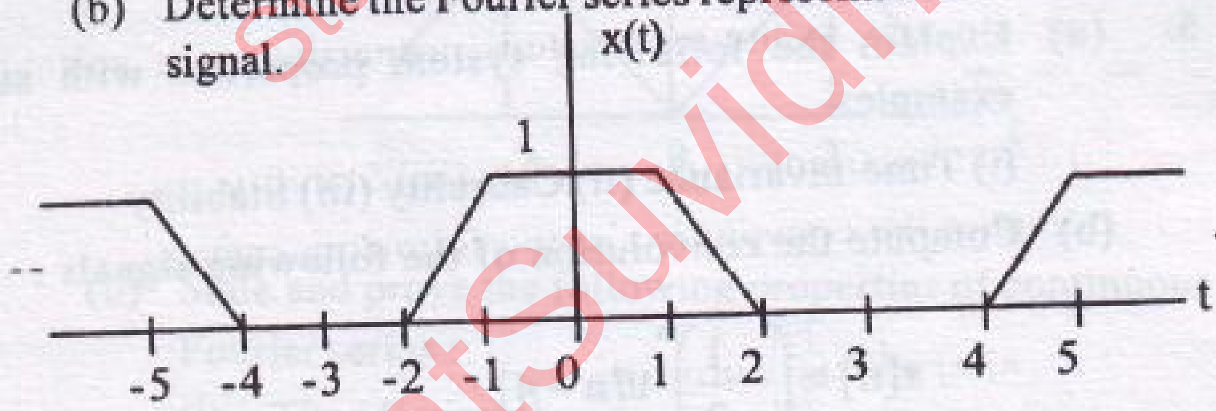
Time : Three Hours] [Maximum Marks : 100

Note :— Attempt any **FIVE** questions. Selecting at least **ONE** question from each unit.

UNIT–I

1. (a) Describe the Gram-Schmidt procedure for obtaining orthogonal signals required for representation of signals. 8

(b) Determine the Fourier series representation for the following signal. 12



2. (a) State and prove the following properties of Fourier transform

(i) Parseval's Relation (ii) Convolution 10

(b) Obtain the Fourier transform of the following signal

$$x(t) = t\left(\frac{\sin t}{\pi t}\right)^2.$$ 10

UNIT—II

3. (a) The noise voltage in a circuit is modelled as a Gaussian random variable with zero-mean and variance equal to 10^{-8}

Contd.

(i) What is the probability that the value of noise exceeds 10^{-4} ?

(ii) When this noise passes through a half-wave rectifier, find the PDF of the rectified noise. 10

(b) The random variable ϕ is uniformly distributed on the interval $[-\pi/2, \pi/2]$. Find the PDF of $x = \tan\phi$. Compute the mean and variance of x. 10

4. (a) State and prove the sampling theorem. 10

(b) Compute the z-transform of the following signal and obtain the pole-zero plot.

$$x(n) = 4^n \cos\left[\frac{2\pi}{6}n + \frac{\pi}{4}\right]u[-n+1]$$ 10

UNIT—III

5. (a) Explain the following system properties with suitable examples. 12

(i) Time-invariance (ii) Causality (iii) Stability

(b) Compute the convolution of the following signals :

$$x[n] = \left(-\frac{1}{2}\right)^n u[n-4]$$

and $h[n] = 4^n u[2-n]$. 8

6. (a) Explain the terms :

(i) Deterministic signals (ii) Stochastic signals

(iii) Power signals (iv) Energy signals. 8

(b) Write the technical notes on the following :

(i) Lumped and distributed parameter systems

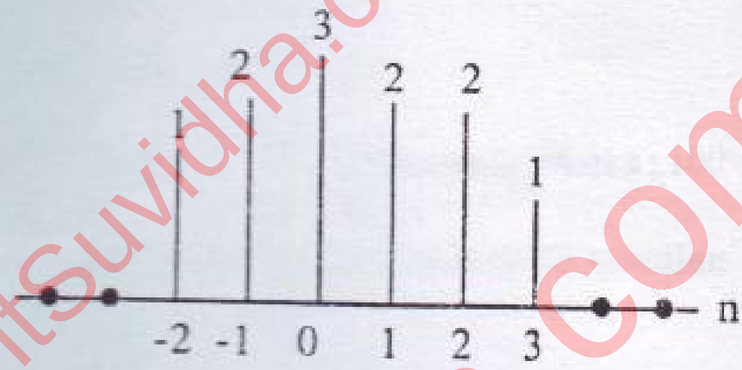
(ii) MIMO systems. 12

UNIT—IV

7. (a) An LTI system, initially at rest, is described by the difference equation

$$y[n] + 2y[n-1] = x[n] + 2x[n-2]$$

Find the response of this system to the following input x[n].



10

(b) For the given system

$$\dot{x}(t) = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

and the output $y(t) = [1 \;\; 0]\, x(t)$ with initial conditions

$x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and u(t) as a unit-step function applied at t=0,

find the output y(t). 10

8. (a) Consider the system characterized by the differential equations :

$$\frac{d^3y(t)}{dt^3} + 6\frac{d^2y(t)}{dt^2} + 11\frac{dy(t)}{dt} + 6y(t) = x(t)$$

Determine the zero-state response of this system for the input $x(t) = e^{-4t}\, u(t)$. 10

(b) An LTI system is described by the following information.

$$x(s) = \frac{s+2}{s-1};\; x(t) = 0 \quad \text{for} \quad t > 0 \text{ and}$$

$$y(t) = -\frac{2}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(t)$$

Determine the impulse response. 10