BT-2/M-15

8201

MATHEMATICS-II

(2006 Onwards)

Paper–MATH-102 (E)

Option–II

Time Allowed : 3 Hours]   [Maximum Marks : 100

Note : Attempt **five** questions in all, selecting at least **one** question from each Unit. All questions carry equal marks.

UNIT-I

1. (a) Find the Inverse of the matrix, using elementary transformation method.

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}.$$ 10

(b) Find the values of a and b for which the equations $x + ay + 3 = 3$; $x + 2y + 2z = b$; $x + 5y + 3z = 9$, are consistent. When will these equations have a unique solution?   10

2. (a) For a Symmetrical square matrix, show that the eigen vectors corresponding to two in equal eigen values are orthogonal.   10

(b) If $S = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$, where $a = e^{\frac{2\pi i}{3}}$. Prove that,

$$S^{-1} = \frac{1}{3}\overline{S}.$$ 10

UNIT-II

3. (a) Solve the differential equation :

$$y(2xy + e^x) dx - e^x dy = 0.$$ 10

(b) An object whose temperature is 75°C cools in an atmosphere of constant temperature 25°C at the rate of kT, T being the excess temperature of the body over that of the temperature. If after 10 min. the temperature of the object falls to 65°C, find the temperature after 20 min. Also find the time required to cool down to 55°C.

10

4. (a) Find the solution of the differential equation :

$(D^2 + 2) y = e^x + 2$, by method of undetemined coefficients. 10

(b) Solve $(2x - 1)^3 \dfrac{d^3y}{dx^3} + (2x - 1)\dfrac{dy}{dx} - 2y = x.$ 10

UNIT–III

5. (a) Find the Laplace transform :

    (i) $f(t) = |t - 3| + |t + 3|, t \geq 0$

    (ii) $f(t) = \int_0^t t\, e^t \sin t\, dt$.   10

(b) Find the inverse Laplace transform :

    (i) $\cot^{-1}\left(\dfrac{s}{\pi}\right)$

    (ii) $\dfrac{1}{s(s^2 + 4)}$.   10

6. (a) State and prove Convolution theorem and hence

    evaluate : $\dfrac{1}{s(s^2 + 1)}$.   10

(b) Using Laplace transform, solve

    $(D^2 + 1)\, x = t \cos 2t$

    given $x(0) = x'(0) = 0$.   10

UNIT–IV

7. (a) Solve the following PDE :

    $y^2 p - xyq = x(z - 2y)$.   10

(b) Using Charpit's method, solve

    $Z = p^2 x + q^2 y$.   10

8. (a) Solve the Diffrential equation :

$$(D^2 - DD')\, z = \cos 2y\, (\sin x + \cos x).$$ 10

(b) A Square plate is bounded by the lines $x = 0$, $y = 0$, $x = 20$ and $y = 20$. Its faces are insulated. The temperature along the upper horizontal edge is given by

$u(x, 20) = x(20 - x)$, when $0 < x < 20$, while other three edges are kept at 0°C. Find the Steady state temperature in the Plate. 10