BT–1 / D–14

MATHEMATICS–I

Paper–MATH–101 (E)

Time allowed : 3 hours] *[Maximum marks : 100*

Note : Attempt ***five*** *questions, selecting at least* ***one*** *question from each unit.*

Unit–I

1. (a) Find the asymptotes of the curve

$$x^3 + y^3 - xy^2 - x^2y + x^2 - y^2 = 0$$

(b) Show that the radius of curvature at any point of the cardioid $r = a(1 + \cos\theta)$ varies as $\sqrt{r}$.

2. (a) Expand $\log_e x$ in power of $(x - 1)$ and hence evaluate $\log_e 1.1$ correct to 4 decimal places. 0.095075

(b) Trace the curve $x^3 + y^3 = 3a\,xy$.

Unit–II

3. (a) If $u = x^y$, show that $\dfrac{\partial^3 u}{\partial x^2\,\partial y} = \dfrac{\partial^3 u}{\partial x\,\partial y\,\partial x}$

(b) Find the maximum and minimum distance of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1$.

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4. (a) If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, prove that

$$\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$$

(b) Using the method of differentiation under the integral sign, prove that

$$\int_0^{\pi} \frac{\log(1 + \cos\theta \sin\alpha)}{\cos\theta} d\theta = \pi\alpha$$

Unit–III

5. (a) Change the order of integration and hence evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 \, dy \, dx$$

(b) Express $\int_0^1 x^m (1 - x^n)^p \, dx$, in terms of gamma function and evaluate $\int_0^1 x^5 (1 - x^3)^{10} \, dx$.

6. (a) Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z \, dz \, dx \, dy$.

(b) Find, by double integral, the area lying between the parabola $y = 4x - x^2$ and the line $y = x$.

12.5

Unit–IV

7. (a) Define the gradient of a scalar point function and give its geometrical significance.

(b) Find the value of n, if $f = (x^2 + y^2 + z^2)^{-4}$ and div grad $f = 0$.

8. (a) Verify Green's theorem for

$$\int_C [(3x - 8y^2)\,dx + (4y - 6xy)\,dy]$$

where C is the boundary of the region bounded by $x = 0$, $y = 0$ and $x + y = 1$.

(b) Verify Divergence theorem for $\overline{F} = 4xz\,i - y^2\,j + yz\,k$, taken over the cube bounded by $x = 0, x = 1$; $y = 0, y = 1$, and $z = 0, z = 1$.