MATHEMATICS

MATH-101-E

Time : Three Hours] [Maximum Marks : 100

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Compute to four decimal places, the value of cos 32° by use of Taylor's series.

(b) Define asymptotes of a polar curve. Find asymptotes of the curve $r\sin\theta = 2\cos 2\theta$.

2. (a) Find the radius of curvature for the parabola $\frac{2a}{r} = 1 + \cos\theta$.

(b) Trace the curve :

$$y^2(a-x) = x^2(a+x)$$

Unit II

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3\. (a) Define Homogeneous function of degree n. State and prove Euler's Theorem for a homogeneous function of degree n in x, y and z.

(b) If $x = e^u \cos v$, $y = e^u \sin v$, find

$$J = \frac{\partial(u, v)}{\partial(x, y)}, \quad J' = \frac{\partial(x, y)}{\partial(u, v)}$$ and hence show that :

$$JJ' = 1.$$

4\. (a) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube.

(b) Evaluate $\int_0^\pi \log(1 + a\cos x)\, dx$ using the method of differentiation under the sign of integration.

Unit III

5\. (a) Find by double integration, the area lying inside the circle $r = a\sin\theta$ and outside the cardioids $r = a(1 - \cos\theta)$.

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(b) Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)} dx\, dy$ by changing into polar co-ordinates. Hence show that :

$$\int_0^{\infty} e^{-x^2} dx = \sqrt{\frac{\pi}{2}}.$$

6. (a) Find the volume of the tetrahedron bounded by the co-ordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

(b) Define Beta and Gamma functions. Prove that :

$$\beta\left(m, \frac{1}{2}\right) = 2^{2m-1}\beta(m, m)$$

Unit IV

7. (a) Define Gradient of a scalar point function and hence give its geometrical interpretation.

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(b) With usual notations, prove that :

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$$\nabla^2(r^n) = n(n+1)r^{n-2}$$

8. (a) Verify Green Theorem for $\int_C \left[\left(3x - 8y^2\right) dx + \left(4y - 6xy\right) dy\right]$ over C, the boundary of the region bounded by $x = 0$, $y = 0$ and $x + y = 1$.

(b) Evaluate :

$$\int \overline{\text{F}} . d\overline{\text{S}}$$

over the surface S bounding by the region $x^2 + y^2 = 4, z = 0$ and $z = 3$ by taking $\text{F} = 4xi - 2y^2 j + z^2 k$.