Roll No. .... Total Pages : 3

BT-1/D-12 **8001**

MATHEMATICS–I

Paper–MAT-101E

Time Allowed : 3 Hours] [Maximum Marks : 100

Note : Attempt **five** questions in all, selecting at least **one** question from each Unit. All questions carry equal marks.

UNIT–I

1. (a) Define curvature of a curve and using Maclaurin's series prove that

$$\frac{x}{\sin x} = 1 + \frac{x^2}{6} + \frac{7x^4}{360} + ...$$

(b) Define asymptotes and find the equation of the cubic which has the same as asymptotes the curve

$$x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y - 1 = 0$$

and which touch the axis of y at the origin and pass through the point (3, 2).

2. (a) Trace the curve $r\,\theta = a$.

(b) Trace the curve $x^3 + y^3 = 3axy$.

UNIT–II

3. (a) If $x^x y^y z^z = c$, show that at

$$x = y = z, \frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}.$$

(b) Define homogenous function of degree n and if

$$z = x^n f_1\left(\frac{y}{x}\right) + y^{-n} f_2\left(\frac{x}{y}\right),\ \text{then prove that}$$

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} + x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = n^2 z.$$

4. (a) If $pV^2 = k$ and the respective errors in p and V are respectively 0.05 and 0.25, show that the error in k is 10%.

(b) Evaluate the following integral by differentiating under integral sign

$$\int_0^\infty e^{-\left(x^2 + \frac{a^2}{x^2}\right)} dx.$$

UNIT–III

5. (a) Evaluate the following integral by changing the order of integration

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2 + y^2}}\, dy\, dx.$$

(b) Calculate the area of the region bounded by the curves

$$y = \frac{3x}{x^2 + 2} \text{ and } 4y = x^2.$$

6. (a) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the plane $y + z = 4$ and $z = 0$.

(b) Prove that

$$\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.$$

UNIT–IV

7. (a) Describe the physical interpretation of Divergence.

(b) Show that div (grad r^n) = $n(n+1)r^{n-2}$, where $r^2 = x^2 + y^2 + z^2$.

8. (a) Evaluate

$$\int_c \vec{F}\cdot d\vec{R}, \text{ where } \vec{F}=(2z, x, -y) \text{ and}$$

C is $\vec{R}=(\cos t, \sin t, 2t)$ from $(1, 0, 0)$ to $(1, 0, 4\pi)$.

(b) Evaluate

$$\int_S \vec{F}\cdot dS, \text{ where } \vec{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$$

and S is the surface bounding the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.