(Please write your Exam Roll No.) Exam Roll No. 60225602814

END TERM EXAMINATION

THIRD SEMESTER [B.TECH] DECEMBER-2015

Paper Code: ETMA-201 (Batch 2013 onwards) **Subject: Applied Mathematics-III**

Time: 3 Hours **Maximum Marks: 75**

Note: Attempt any five questions including Q.no.1 which is compulsory. Select one question from each unit. Use of scientific calculator is allowed.

Q1 (a) State Dirichlet's conditions for convergence of Fourier series and check whether the function $f(x) = \frac{1}{3-x}, 0 < x < 2\pi$ satisfy Dirichlet's conditions or not? **(5)**

(b) Obtain the Fourier transform of the function *f(x)* given by $f(x) = \begin{cases} 1-x^2; & |x| \le 1 \\ 0 & ; \text{otherwise} \end{cases}$ and hence evaluate $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right)\cos\frac{x}{2}dx$. **(5)**

(c) Derive the difference equation by eliminating the arbitrary constants ***A*** and ***B*** from $y_n = A.3^n + B.5^n$. **(5)**

(d) Prove the following relations: **(5)**

(i) $\Delta\nabla = \delta^2$

(ii) $\mu\delta = \frac{1}{2}(\Delta + \nabla) = \frac{1}{2}(E - E^{-1})$

(e) Solve the differential equation $\frac{dy}{dx} = \frac{y-x}{y+x}$ by Euler's method with the initial condition $y(0) = 1$ for $x = 0.06$ taking interval of differencing $h = 0.02$. **(5)**

Unit-I

Q2 (a) Find the Fourier series for $f(x) = x^2$ in (0, 4) and deduce that **(6.5)**

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots = \frac{\pi^2}{6}.$$

(b) Find a series of cosines of multiples of x which will represent $x \sin x$ in the interval $(0, \pi)$ and show that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \ldots = \frac{\pi-2}{4}$. **(6)**

Q3 (a) Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}; 0 < x < \infty, t > 0$ subject to the conditions.

(i) $u(0,t) = 0, t > 0$ (ii) $u(x,0) = \begin{cases} 1; & 0 < x < 1 \\ 0; & x > 1 \end{cases}$ (iii) $u(x,t)$ is bounded. **(6.5)**

(b) Obtain the Fourier sine series for *f(x)* containing three non-zero terms where *f(x)* is given in the following table: **(6)**

x	0	1	2	3	4	5
$f(x)$	0	10	15	8	5	3

Unit-II

Q4 (a) Solve the simultaneous difference equations. **(6.5)**

$u_{x+1} + v_x - 3u_x = x, \quad 3u_x + v_{x+1} - 5v_x + 4^x$

subject to the conditions $u_1 = 2, v_1 = 0$.

(b) Solve $y_{n+2} - 2\cos\alpha . y_{n+1} + y_n = \cos\alpha n$. **(6)**

Q5 (a) Using the Z-transform, solve $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$ with $u_0 = 0, u_1 = 1$. **(6.5)**

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(b) Find the inverse Z-transform of $\frac{2z}{(z-1)(z^2+1)}$. **(6)**

Unit-III

Q6 (a) Starting with $(x_0, y_0, z_0) = (0, 0, 0)$ and using Jacobi's method, find the next five iterations upto four decimal places for the system of equations $5x - y + z = 10$, $2x + 8y - z = 11$, $-x + y + 4z = 3$, and find where the iterations converges. **(6.5)**

(b) Use Newton-Raphson method to solve the equation $3x - \cos x - 1 = 0$ correct to four decimal places. **(6)**

Q7 (a) Find the number of men getting wages between Rs. 100 and 150 from the following data: **(6.5)**

Wages in Rs.	0-100	100-200	200-300	300-400
Frequency	9	30	35	42

(b) Find the interpolating polynomial for (0, 2), (1, 3), (2, 12) and (5, 147) using Lagrange's interpolation formula. **(6)**

Unit-IV

Q8 (a) From the data, $y(0.60) = 0.6221$, $y(0.65) = 0.6155$, $y(0.70) = 0.6138$ and $y(0.75) = 0.6170$ find the maximum or minimum value of y. **(6.5)**

(b) Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ by using Simpson's 1/3 rule taking $h = \frac{1}{4}$ and by using Simpson's $\frac{3}{8}$ rule taking $h = \frac{1}{6}$ where h is the interval of differencing. **(6)**

Q9 (a) Solve the differential equation $\frac{dy}{dx} = x + y$, $y(0) = 1$ for $y(0.1)$, $y(0.2)$ and $y(0.3)$ by using modified Euler's method correct to four decimal places. **(6.5)**

(b) Apply the fourth order Runge-Kutta method to solve $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$. Take step size h = 0.1 and determine approximation to y(0.1) and y(0.2) correct to four decimal places. **(6)**

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