

(Please write your Exam Roll No.)

Exam Roll No.

END TERM EXAMINATION

SECOND SEMESTER [B.TECH] MAY-JUNE 2017

Paper Code: ETMA 102

Subject: Applied Mathematics-II

(Batch 2013 onwards)

Time: 3 Hours

Maximum Marks: 75

Note: Attempt any five questions including Q.No. 1 which is compulsory.
Select one question from each unit.

- Q1. a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that
- $$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2} \quad (5)$$
- b) Using Cauchy-Riemann equations, show that the function $f(z) = z^3$ is analytic in entire z -plane. (5)
- c) Define impulse function and hence obtain its Laplace transform. (5)
- d) Calculate the volume of the solid bounded by the surface $x=0$, $y=0$, $lx+my+nz=1$ and $z=0$. (5)
- e) Solve the partial differential equation by Charpit's method. (5)
- $$(p^2 + q^2)y = qz.$$

Unit-I

- Q2. a) Expand $f(x,y) = x^2y + 3y - 2$ in powers of $x-1$ and $y+2$ using Taylor's expansion. (6.5)
- b) If $x+y+z=u$, $y+z=uv$, $z=uvw$, show that (6)
- $$\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v.$$
- Q3. a) A rectangular box, which is open at the top has a capacity of 256 cubic feet. Applying Lagrange's method of undetermined multipliers determine the dimensions of the box such that the least material is required for the construction of the box. (6.5)
- b) If $z = f(x,y)$ where $x = e^u \cos v$ and $y = e^u \sin v$, show that (6)
- $$y \frac{\partial z}{\partial u} + x \frac{\partial z}{\partial v} = e^{2u} \frac{\partial z}{\partial y}.$$

Unit-II

- Q4. a) Find the inverse Laplace transform of $\frac{s+4}{s(s-1)(s^2+4)}$. (6.5)
- b) Find the Laplace transform of unit step function $u(t-a)$, also find the Laplace transform of $t^2u(t-3)$. (6)
- Q5. a) Using Laplace transform solve the following differential equation. (6.5)
- $$y'' + 2y' + 2y = 5 \sin t, \quad \text{where } y(0) = y'(0) = 0.$$
- b) Evaluate, using Laplace transform (6)
- i) $\int_0^\infty t e^{-3t} \sin t dt.$
- ii) $\int_0^\infty \frac{\sin t}{t} dt.$



QP031

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Unit-III

- Q6. a) Find the image of $|z| = 1$ under the transformation $w = \frac{i-z}{i+z}$, onto the w plane. (6.5)
- b) Show that the function $z|z|$ is not analytic anywhere. (6)
- Q7. a) Expand

$$f(z) = \frac{1}{(z-1)(z-2)}, 1 < |z| < 2. \quad (6.5)$$
- b) Using contour integration in complex plane evaluate (6)
- $$\int_0^\pi \frac{d\theta}{3+2\cos\theta}$$

Unit-IV

- Q8. a) Evaluate $\iint (x^2+y^2)dx dy$ throughout the area enclosed by the curves $y=4x$, $x+y=3$, $y=0$ and $y=2$. (6.5)
- b) Use Green's theorem to evaluate. (6)
- $$\int_C (x^2 + xy)dx + (x^2 + y^2)dy$$
- where C is the square formed by the lines $y=\pm 1$, $x=\pm 1$.
- Q9. a) Apply Stoke's theorem to calculate $\int_C 4ydx + 2zdy + 6ydz$, where C is the curve of integration $x^2+y^2+z^2=6z$ and $z = x+3$. (6.5)
- b) If a force $\vec{F} = 2x^2yi + 3xyj$ displaces in the xy -plane from $(0,0)$ to $(1,4)$ along the curve $y=4x^2$, find the work done. (6)

ETMA-102

P2/2