(Please write your Exam Roll No.) Exam Roll No. ....

END TERM EXAMINATION

SECOND SEMESTER [B.TECH] MAY-JUNE 2016

Paper Code: ETMA-102 **Subject: Applied Mathematics-II (Batch 2013 onwards)**

Time: 3 Hours **Maximum Marks: 75**

Note: Attempt any five questions including Q.no.1 which is compulsory. Select one question from each Unit.

Q1 (a) Change the order of integration in the following integral and evaluate: $\int_0^4 \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx$. **(4)**

(b) Prove that the function *sinh* z is analytic and find its derivative. **(4)**

(c) The $\frac{2\pi}{\omega}$ – periodic function f(t) is given by $f(t) = a\sin\omega t,\quad 0 < t < \frac{\pi}{\omega}$

$= 0 \quad , \quad \frac{\pi}{\omega} < t < \frac{2\pi}{\omega}$

Find its Laplace transform. **(4)**

(d) If $z = \sqrt{x^2+y^2}$ and $x^3 + y^3 + 3axy = 5a^2$, find the value of $\frac{dz}{dx}$, when x = y = a. **(4)**

(e) Expand $\tan^{-1}\frac{y}{x}$ in the neighborhood of (1, 1) using Taylor's Theorem. **(4)**

(f) If $u^3 + v^3 = x + y,\ u^2 + v^2 = x^3 + y^3$, show that $\frac{\partial(u,v)}{\partial(x,y)} = \frac{y^2-x^2}{24\,v\,(u-v)}$. **(5)**

Unit-I

Q2 (a) If $u = f(r,s,t)$, where $r = \frac{x}{y}, s = \frac{y}{z}, t = \frac{z}{x}$. Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. **(6)**

(b) If the sides and angles of a triangle ABC vary in such a way that its circum radius remains constant, prove that $\frac{da}{\cos A} + \frac{db}{\cos B} + \frac{dc}{\cos c} = 0$. **(6.5)**

Q3 (a) Find the maximum and minimum distances from the origin to the curve $5x^2 + 6xy + 5y^2 - 8 = 0$. **(6)**

(b) Solve the following differential equation $(x^2 - y^2 - z^2)p + 2xy\,q = 2xz$. **(6.5)**

Unit-II

Q4 (a) If $L\,(t\sin\omega t) = \frac{2\omega s}{(s^2+\omega^2)^2}$, *Evaluate*: $L\,(\omega t\cos\omega t + \sin\omega t)$. **(4)**

(b) Evaluate: $L\left[\int_0^t e^t \sin t\, dt\right]$. **(4)**

(c) A function $f(t)$ is given by $f(t) = \begin{Bmatrix} t^2\ for & 0 < t < 1 \\ 4t\ for & t > 1 \end{Bmatrix}$.

Express it in terms of unit step function and then find its Laplace transform. **(4.5)**

Q5 (a) Applying convolution, find the inverse Laplace transform of the function: $\frac{1}{(s+1)(s+9)^2}$. **(6)**

(b) Solve $\frac{dy}{dx} + y = \cos 2t,\ y(0) = 1$, using Laplace transformation. **(6.5)**

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Unit-III

Q6 (a) Obtain the Taylor series expansion of $f(z) = \frac{1}{z^2+(1+2i)z+2i}$ about $z = 0$. Also find its radius of convergence. **(6)**

(b) Evaluate: $\oint_c \frac{e^z dz}{(z-1)(z+3)^2}$ where c: $|z| = 10$. **(6.5)**

Q7 (a) Show that the transformation $\omega = z + \frac{a^2-b^2}{4z}$ transforms the circle of radius $\frac{a+b}{2}$, centre at the origin, in the z-plane into an ellipse of semi axes a and b in the ω –plane. **(6.5)**

(b) Evaluate the integral $I = \int_0^{2\pi} \frac{d\theta}{2+\sin\theta}$ using the residue calculus. **(6)**

Unit-IV

Q8 (a) If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $|\vec{r}| = r$. Show that div $r^n.\vec{r} = (n+3)r^n$. **(6)**

(b) Evaluate $\oint_c \vec{F}.\vec{dr}$ by Stoke's Theorem. Where $\vec{F} = y^2\hat{\imath} + x^2\hat{\jmath} - (x+z)\hat{k}$ and C is the boundary of the triangle with vertices at (0, 0, 0), (1, 0, 0) and (1, 1, 0). **(6.5)**

Q9 (a) Evaluate the following integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \quad dzdy\, dx$. **(6)**

(b) Apply divergence theorem to find $\iint_s \vec{F}.\hat{n}$ ds, where $\vec{F} = (2x+3z)\hat{\imath} - (xz+y)\hat{\jmath} + (y^2+2z)\hat{k}$ and S is the surface of the sphere having centre at (3, -1, 2) and radius 3. **(6.5)**

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