Roll No. .....

24022

B. Tech 3rd Semester (CS & IT) Examination – December, 2017

MATHEMATICS-III

Paper : Math-201-F

Time : Three Hours] *[Maximum Marks : 100*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Question No. 1 is ***compulsory***. Attempt total *five* questions, selecting *one* question from each Section. All questions carry equal marks.

1. (a) If $f(x) = \left(\frac{x-2}{2}\right)^2$, $0 < x < 2\pi$, find a_n.

$2.5 \times 8 = 20$

(b) Find the Fourier Cosine transform of $f(x) = e^{-ax}$.

(c) State Convolution Theorem for Fourier Transform.

(d) State Residue Theorem.

(e) Prove that :

$$\tan\left(i\log\frac{a-ib}{a+ib}\right)=\frac{2ab}{a^2-b^2}$$

(f) Evaluate $\oint_c \frac{e^{-z}}{z+1}dz$, where c is the circle $|z|=\frac{1}{2}$.

(g) If A and B are two events such that $P(A)=\frac{1}{4}$, $P(B)=\frac{1}{3}$ and $P(A\cup B)=\frac{1}{2}$. Show that A and B are independent events.

(h) If X is a normal variable with mean 30 & standard deviation 5; find the probabilities that :

(a) $20 \le X \le 40$.

(b) $X \ge 45$.

SECTION – A

2. (a) Find the Fourier series of the function :

$$f(x)=x\sin x,\ -\pi \le x \le \pi$$

Also deduce that :

$$\frac{1}{1.3}-\frac{1}{3.5}-\frac{1}{5.7}+\ldots\ldots=\frac{\pi-2}{4}$$

(b) Expand $f(x)$ as a Fourier series if :

$$\begin{aligned} f(x) &= \pi x \quad , \ 0<x<1 \\ &= \pi(2-x) \quad , \ 1<x<2 \end{aligned}$$

3. (a) Find the Fourier transform of :

$$f(x)=\begin{cases} 1-x^2 & \text{if } |x|<1 \\ 0 & \text{if } |x|>1 \end{cases}$$

Hence evaluate :

$$\int_0^\infty \frac{x\cos x-\sin x}{x^3}\cos\frac{x}{2}dx$$

(b) Find the Fourier sine transform of :

$$\frac{1}{x\left(x^2+a^2\right)}$$

SECTION – B

4. (a) If $\tan(\theta + i\phi)\tan\alpha + i\sec\alpha$, show that :

(i) $e^{2\phi} = \pm\cot\alpha/2$

(ii) $2\theta = (n + 1/2)\pi + \alpha$

(b) Determine the analytic function whose real part is :

$$e^{2x}(x\cos 2y - y\sin 2y)$$

5. (a) Define line integral of f(z). Prove that :

$$\int_C \frac{dz}{z} = -\pi i \text{ or } \pi i,$$

According as c is the semi-circular arc $|z| = 1$ above or below of the real axis.

(b) Using Cauchy's Integral Formula, Evaluate :

$$\oint \frac{\sin^6 z}{(z-\pi/3)^3}dz$$

around the circle $|z| = 1$.

SECTION – C

6. (a) Expand $e^{2z}/(z-1)^3$ about the singularity $z = 1$ in Laurant's series.

(b) Evaluate :

$$\int_0^{\pi} \frac{1}{a+b\cos\theta}d\theta,$$

where $a > b$.

7. (a) The contents of Urn I, II and III are as follow 1 white, 2 black and 3 red balls, 2 white, 1 black and 1 red balls and 4 white, 5 black and 3 red balls.

One Urn is chosen at random and two balls drawn. They happen to be white and red. What is the probability that they come from I, II or III ?

(b) Fit a normal curve to the following distributions :

x :	2	4	6	8	10
f :	1	4	6	4	1

SECTION – D

8. A survey of 320 families with 5 children each revelled the following distribution :

No. of Boys :	5	4	3	2	1	0
No. of Girls :	0	1	2	3	4	5
No. of Families :	14	56	110	88	40	12

Is this result consistent with the hypothesis that male and female births are equally probable ?

9. Using Simplex Method solve the following LPP :

Maximize : $z = 2x_1 + 5x_2$

Subject to :

$$x_1 + 4x_2 \leq 24$$

$$3x_1 + x_2 \leq 21$$

$$x_1 + x_2 \leq 9$$

$$x_1, x_2 \geq 0$$