

(b) Find the Laplace Transform of the function of period a defined by:

$$f(t) = 1, \text{ when } 0 < t < \frac{a}{2} \\ = -1, \text{ when } \frac{a}{2} < t < a$$

SECTION - D

8. (a) Form the partial differential equation by eliminating the arbitrary function from

$$z = x^n f\left(\frac{y}{x}\right).$$

(b) Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$, given that

$\frac{\partial z}{\partial y} = -2 \sin y$, when $x = 0$ and $z = 0$, when y is an odd multiple of $\frac{\pi}{2}$.

(c) Solve:

$$z(xp - yq) = y^2 - x^2$$

9. The points of trisection of a string are pulled aside through the same distance on opposite sides of the position of equilibrium and the string is released from rest. Derive an expression for the displacement of the string at subsequent time and show that the mid point of the string always remains at rest.

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Roll No.

24018

B. Tech. 2nd Semester (Common for all Branches) Examination – May, 2017

MATHEMATICS-II

Paper : Math-102-F

Time : Three Hours]

[Maximum Marks : 100

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt five questions in total, selecting at least one question from each Section. Question No. 1 is compulsory.

1. (a) Define physical interpretation of divergence.

(b) State divergence theorem of Gauss.

(c) Solve:

$$x dy - y dx = (x^2 + y^2) dx.$$

(d) Find the P. I. of $(D - 2)^2 y = e^{2x}$.

(e) Find the Laplace transform of:

$$\sin h \frac{t}{2} \sin \frac{\sqrt{3}}{2} t.$$

(f) Find the Laplace transform of $\frac{\sin 2t}{t}$.

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(g) Solve:

$$p^3 - q^3 = 0$$

(h) Solve the equation $3 \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0$, by method of separation of variables.

SECTION - A

2. (a) Find the constants a and b so that the surface $ax^2 - byz = (a+2)x$ is orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1, -1, 2)$.

(b) Show that the vector field \vec{A} , where $\vec{A} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$ is irrotational and find its scalar potential.

3. Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2) dx + (4y - 6xy) dy$, where C is the boundary of the region defined by $y = \sqrt{x}$, $y = x^2$.

SECTION - B

4. (a) State and prove the necessary and sufficient condition for the differential equation

$$Mdx + Ndy = 0 \text{ to be exact}$$

(b) Find the orthogonal trajectories of the family of coaxial circles $x^2 + y^2 + 2gx + c = 0$, g being the parameter.

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5. (a) Solve:

$$\frac{d^2 y}{dx^2} + a^2 y = \sec ax$$

(b) An electric circuit consists of an inductance of 0.1 henry, a resistance of 20 ohms and a condenser of capacitance 25 micro-farads. Find the charge q and the current i at any time t , given that at $t = 0$, $q = 0.05$ Coulomb $i = \frac{dq}{dt} = 0$, when $t = 0$.

SECTION - C

6. (a) Find the Laplace transform of $f(t)$ defined as $f(t) = |t-1| + |t+1|$, $t \geq 0$.

(b) Find the inverse Laplace Transform of:

$$\tan^{-1} \frac{2}{s}$$

(c) Find the inverse Laplace Transform of:

$$\frac{s^5}{(s^2+1)(s^2+4)}$$

by using Convolution theorem.

7. (a) Solve the integral equation

$$\int_0^t \frac{y(u)}{\sqrt{t-u}} du = \sqrt{t}$$

by Laplace Transform method.

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