7. (a) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$

and

$$x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} = 2\cos 3u \sin u.$$

(b) A rectangular box open at the top is to have a volume of 32 c.c. Find the dimensions of the box requiring least material for its construction.

## Unit–IV

8. (a) Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$.

(b) Find, by double integration, the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the y-axis.

9. (a) Find the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

(b) Prove that

$$\beta(m, n) = \frac{\Gamma m \, \Gamma n}{\Gamma m + n}$$, hence evaluate $\Gamma 1/2$.





**B.Tech. 1st Semester F-Scheme Examination,**

**December–2016**

**MATHEMATICS–I**

**Paper–MATH–101–F**

*Time allowed : 3 hours ]* *[ Maximum marks : 100*

*Note : **Question No. 1 is compulsory.** Attempt total five questions with selecting **one** question from each unit. All questions carry equal marks.*

1. (a) Discuss the behavior of series

$$\frac{\sqrt{2}-1}{3^3-1} + \frac{\sqrt{3}-1}{4^3-1} + \frac{\sqrt{4}-1}{5^3-1} + \frac{\sqrt{5}-1}{6^3-1} + \text{-------}$$

(b) Prove that

$$\log(1+x) = x - \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} \text{ ------}$$

(c) Find asymptotes parallel to co-ordinate axis of the curve

$$x^2y^2 - x^2y - xy^2 + x + y + 1 = 0$$

(d) Define rank of matrix. Find rank of matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & 0 & 5 & 7 \end{bmatrix}$$

(e) Using Cayley-Hamilton theorem, find $A^6$ if

$$\begin{bmatrix} 2 & 1 \\ 5 & -2 \end{bmatrix}$$

(f) Write the relationship between Cartesian coords and cylindrical polar coords. Also write relationship between Cartesian coords and Spherical polar coords.

(g) Define Beta and Gamma function.

(h) State Leibnitz theorem for the nth derivative of the product of two functions.

## Unit–I

2. (a) Examine the convergence of the series whose nth term is $\sqrt{n^4+1}-\sqrt{n^4-1}$.

(b) Discuss the convergence of the series

$$\frac{2}{1}x+\frac{9}{8}x^2+\frac{64}{81}x^3+ \ldots\ldots\ldots\ldots\ldots \text{to } \infty$$

3. (a) Test for absolute/conditionally convergence of the series $2-\frac{4}{2!}+\frac{8}{3!}-\frac{16}{4!}+\frac{32}{5!}\ldots\ldots\ldots\ldots$

(b) For what values of x are the following series convergent : $\frac{1}{1-x}+\frac{1}{2(1-x)^2}+\frac{1}{3(1-x)^3}+\ldots.\infty$

## Unit–II

4. (a) Find non-singular matrices P and Q such that PAQ is in the normal form for the matrix

$$A=\begin{bmatrix}0 & 2 & 3 & 4\\ 2 & 3 & 5 & 4\\ 4 & 8 & 13 & 12\end{bmatrix}$$





(b) Find the value of $\lambda$ such that the following equations have unique solution :

$\lambda x+2y-2z-1=0, \quad 4x+2\lambda y-z-2=0,$
$6x+6y+\lambda z-3=0$

and use matrix method to solve these equations when $\lambda=2$

5. (a) If $\lambda$ is an eigen value of a non-singular matrix A, show that

(i) $\lambda-1$ is an eigen value of $A-1$

(ii) $\frac{|A|}{\lambda}$ is an eigen value of adj. A

(b) Show that the matrix

$$A=\begin{bmatrix}1 & 2 & 0\\ 2 & -1 & 0\\ 0 & 0 & -1\end{bmatrix}$$

satisfies its own characteristic equation and hence or otherwise find $A^{-1}$.

## Unit–III

6. (a) If $\log y=\tan^{-1}x$, show that $(1+x^2)y_{n+2}+\{2(n+1)x-1\}y_{n+1}+n(n+1)y_n=0$ and hence find $y_3$, $y_4$ and $y_5$ at $x=0$.

(b) Show that the radius of curvature $\rho$ at P on an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is given by $\rho=\frac{CD^3}{ab}$ where CD is the semi-diameter conjugate to CP.

(c) Expand $e^x \log(1+y)$ in powers of x and y upto terms of third degree.

 [P.T.O.