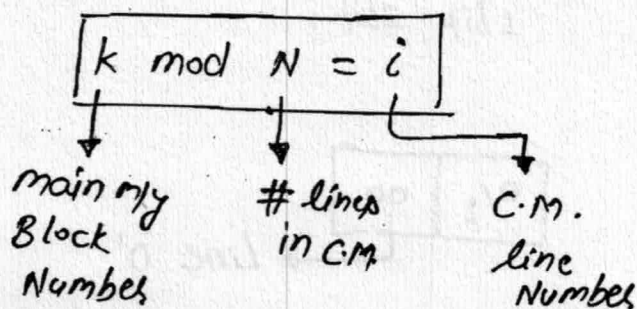


⇒ "Direct mapping" :-

In this technique $\text{mod}()$ function is used to map the data that is :

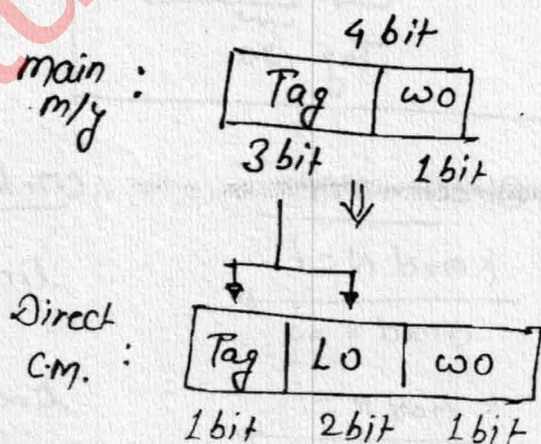


0	$\bmod 4 = 0$
1	$\bmod 4 = 1$
2	$\bmod 4 = 2$
3	$\bmod 4 = 3$
4	$\bmod 4 = 0$
5	$\bmod 4 = 1$
6	$\bmod 4 = 2$
7	$\bmod 4 = 3$

Direct C.M.

0	0	m	2m...
1	1	(m+1)	(2m+1) ..
⋮	⋮	⋮	⋮
(m-1)	(m-1)	(2m-1)	(3m-1) ..

* Above function shows the relationship between main m/y block and cache m/y line therefore Address Binding is:-



main
m/y

Tag

3bit

Direct C-M

Tag	Lo
-----	----

1bit 2bit

B0 [000]
B4 [100]

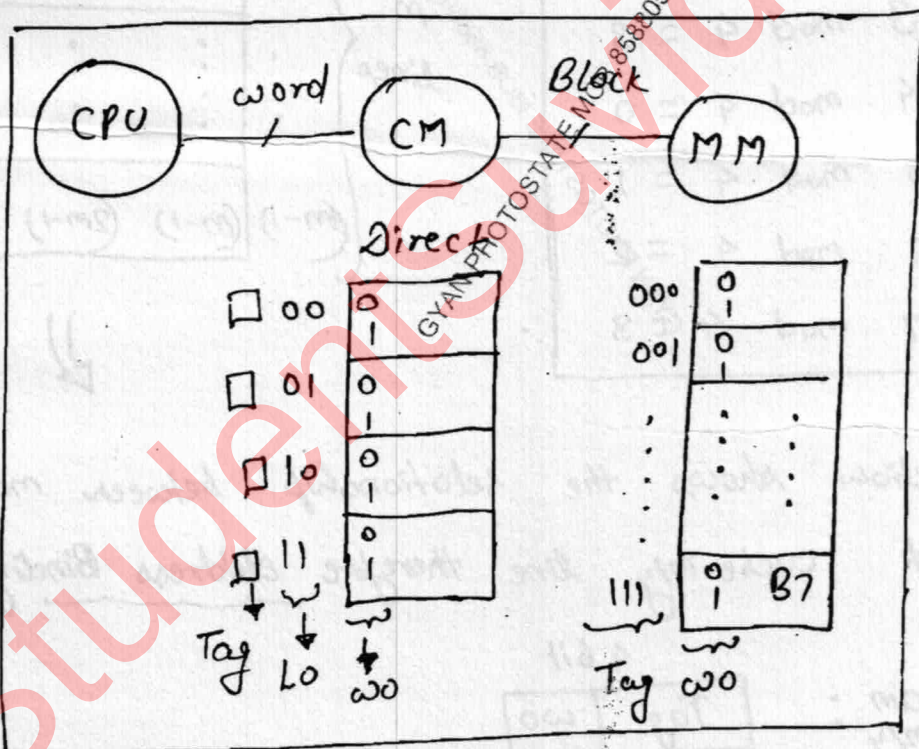
0/1	00
-----	----

Line '0'

B1 [001]
B5 [101]

0/1	01
-----	----

Line '1'



mm Block

B0'
[000]
B7
[111]
B4
[100]

Direct - MAP

$k \bmod N = i$
 $0 \bmod 4 = 0$
 $k \bmod N = i$
 $2 \bmod 4 = 2$
 $k \bmod N = i$

cm-Line

line '0'
line '3'
line '0'

Date
21/08/2017

Accessing Sequence :-

machine Instⁿ: `mov r0, [1001]`



CPU generated the mly request

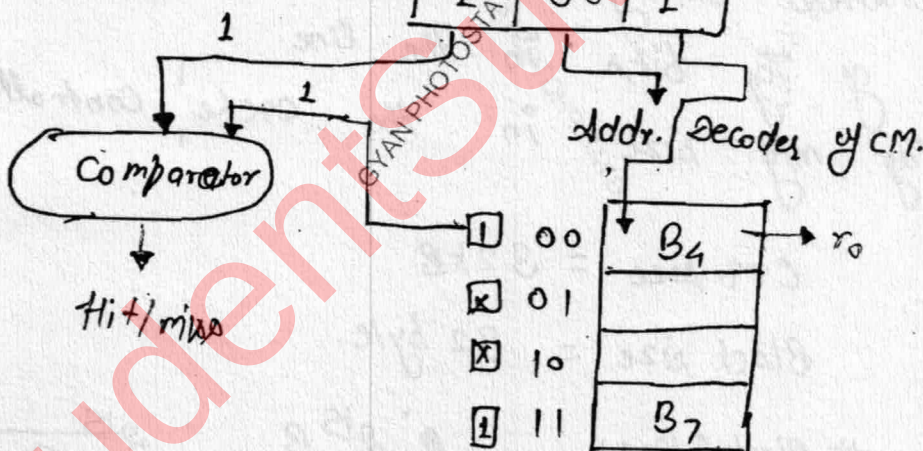
1001 | $\overline{RD}$

↓
Direct C.M.

↓
Address format

Tag	LO	WO
16bit	2bit	1bit

1 | 00 | 1



During the mapping process some of the Tag bits are connected to a address logic of cache memory and rest of the bits are stored in the cache controller as Tag. so Tag mly size:

$$\text{Tag mly size} = \# \text{ lines in CM} * \# \text{ tag bits in line}$$

In the Direct Cache design, Explicit Replacement policies (FIFO, LRU) are not required to replace the data because each main memory block is having fixed location in the cache memory.

Que:- Consider 32 kB direct mapped cache organized into a 32 byte Block. cache m/y data is a subset of 2^{32} Byte m/y space. In the cache controller each tag comprising of 1 update bit. Calculate the following:-

- a) # of lines in cache
- b) # of Blocks in m.m.
- c) Address Binding
- 4) # of Tag bits in the line
- 5.) Tag m/y size in the cache controller.

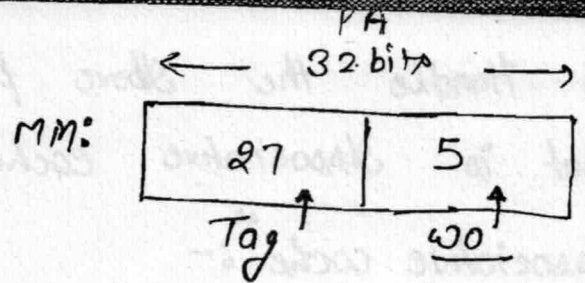
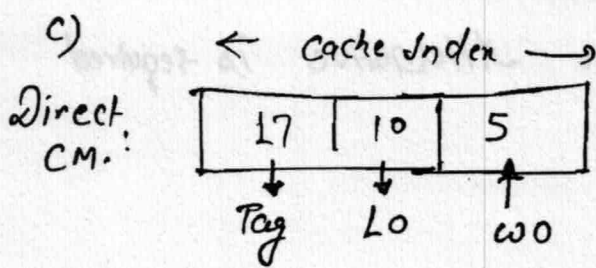
Solⁿ:-

$$C.M. \text{ size} = 32 \text{ kB}$$

$$\text{Block size} = 32 \text{ byte.}$$

$$\begin{aligned} \# \text{ Block (lines) in cm.} &= \frac{2^{15} \text{ B}}{32 \text{ B}} = \frac{2^{15}}{2^5} = 2^{10} \\ &= \frac{2^{15}}{2^5} = 2^{10} \\ &= 1 \text{ k} \end{aligned}$$

$$\begin{aligned} \# \text{ Blocks in m.m.} &= \frac{2^{32} \text{ B}}{32 \text{ B}} = \frac{2^{32}}{2^5} = 2^{27} \\ &= 128 \text{ M Blocks} \end{aligned}$$



d) 17

e)

$$\begin{aligned} \text{Tag my size} &= \# \text{ lines in cache} * \# \text{ tag bits in the line} \\ &= 2K * (17 \text{ bit} + 1 \text{ update bit}) \\ &= 18K \text{ bits} \end{aligned}$$

* Note :-

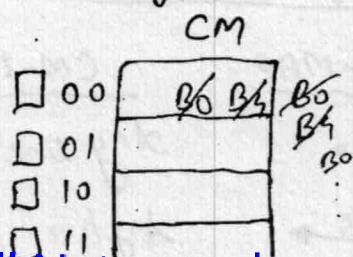
:- Limitation in the Direct mapping is Each line is capable to hold only one Block (Tag) at a time therefore # of 'Conflict misses' will be increased

:- when the CPU frequently refers the multiple Blocks which are mapped into a same cache line then cache Blocks are continuously swapping so Hit ratio falling down this phenomenon is called as "Thrashing" that is described below:

CM Contain 4 Lines (Empty)

MM Block reference: B0, B4, B0, B4, B0, B4...

Execution:



compulsory miss

$$\begin{aligned} B0 - M: & 0 \bmod 4 = 0 \\ B4 - M: & 4 \bmod 4 = 0 \rightarrow \text{conflict} \\ B0 - M: & 0 \bmod 4 = 0 \\ B4 - M: & 4 \bmod 4 = 0 \end{aligned}$$

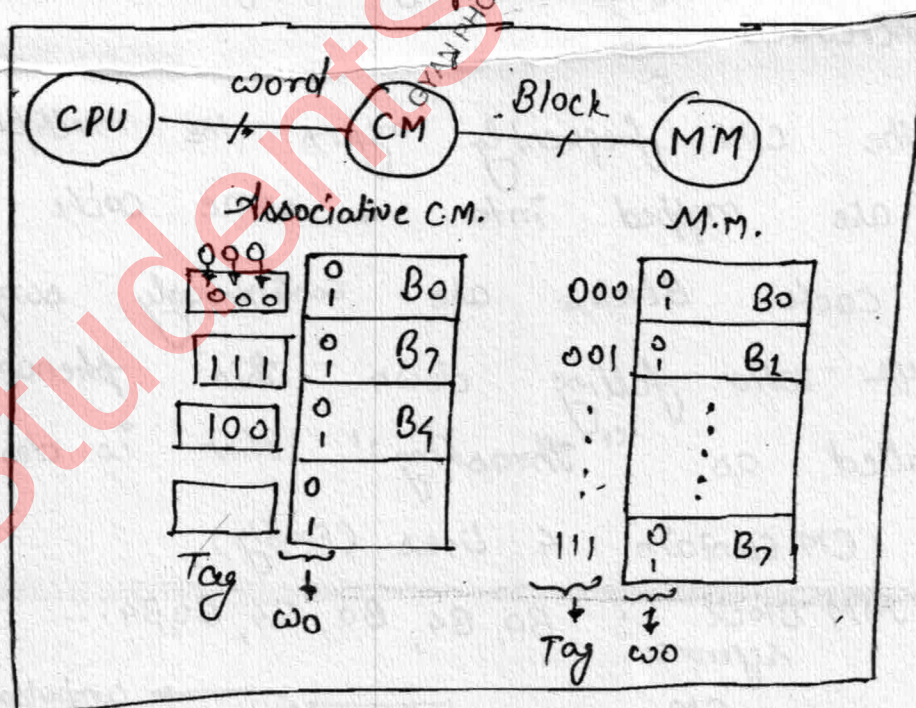
To Handle the above problem Alternative is required that is Associative cache.

"Associative cache" :-

Associative cache is designed without Address called as content addressable memory.

In this cache design any main-memory Block can be mapped into any cache m/y line in a sequence so that all the lines are effectively used. Replacement will be started only when the cache is full.

During the mapping process, complete Physical Tag is stored in the cache controller so Tag-size is increased in the cache line.



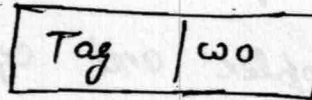
B₄
[100]

sequence →

Any line in sequence.

Address Binding :-

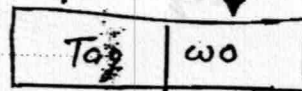
MM :



3bit

1bit

Associative
CM :



3bit

1bit

$$\text{Tag mly size} = \# \text{ lines in CM} * \# \text{ Tag bits in line}$$

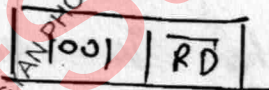
$$= 4 * 3 \text{ bits}$$

$$= 12 \text{ bits}$$

Accessing sequence -

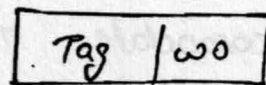
machine Instⁿ: `mov r0, [100]`

CPU generates the mly request



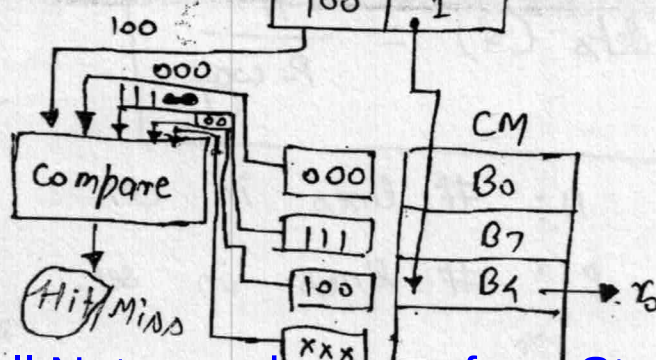
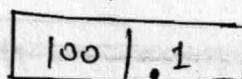
Associative CM.

Address Format



3bit

1bit



* Limitation in the Associative cache is all the Tags in the Cache Controller is compared with a CPU generated Tag in parallel so implementation of a parallel search comparison circuit become very complex and expensive. Search Time is added to a storage access time. Hence alternative is required name as: 'Set Associative Cache.'

* In the Associative cache design explicit replacement techniques are used to replace the data when the cache is full (FIFO, LRU).

* # "Set Associative Cache"

Set Associative Cache is used to compromise the Disadvantages in the Direct and associative cache designs.

In this organization lines are grouped into a sets to ~~aca~~ accomodate the more than one Block (Tag) in the set at a time.

$$\# \text{ sets } (S) = \frac{N}{P\text{-way}}$$

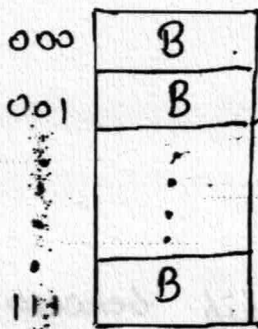
N: # lines in CM.

P: # lines in set.

eg: Consider 2-way set associate cache of 8B with 2B block:

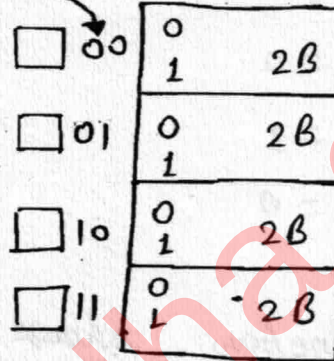
Before orgⁿ

8B $\rightarrow$ CM.



After orgⁿ into lines

- # lines (N) = $\frac{8}{2} = 4$
- $4 * 2B \Rightarrow 8B$

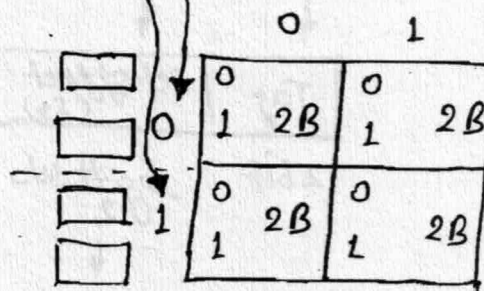


After organized into a sets

$$\# \text{ sets } (S) = \frac{N}{P\text{-way}}$$

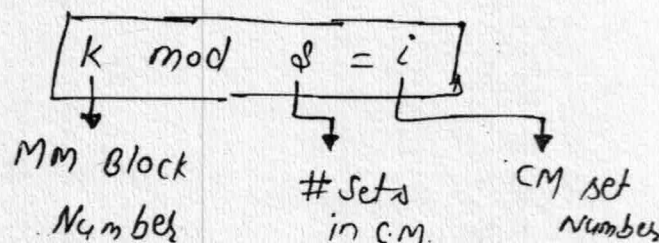
$$= \frac{4}{2} \Rightarrow 2$$

$$2 * 2 * 2B \Rightarrow 8B$$



$\left\{ \begin{array}{l} S: \text{rows} \\ P: \text{cols} \end{array} \right\}$

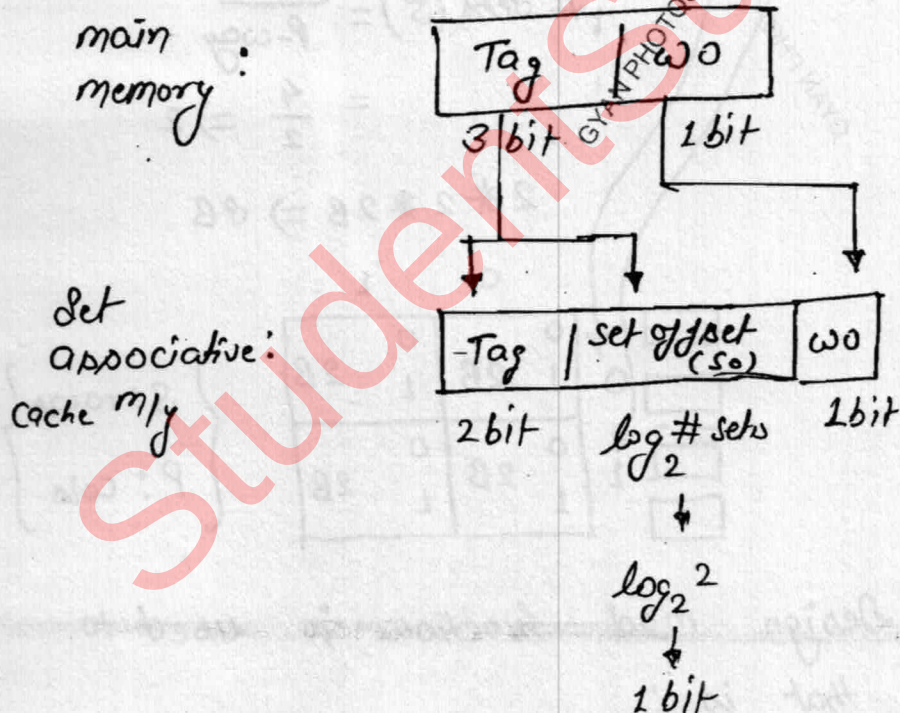
* In this cache design mod function is used to map the data that is:



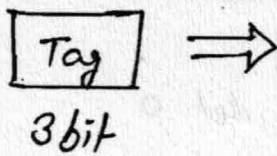
$$\left\{ \begin{array}{l} 0 \bmod 2 = 0 \\ 1 \bmod 2 = 1 \\ 2 \bmod 2 = 0 \\ 3 \bmod 2 = 1 \\ 4 \bmod 2 = 0 \\ 5 \bmod 2 = 1 \\ 6 \bmod 2 = 0 \\ 7 \bmod 2 = 1 \\ 8 \bmod 2 = 0 \end{array} \right.$$

mapping function shows the relationship between the Block and set no; Address Binding is:

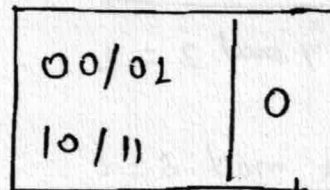
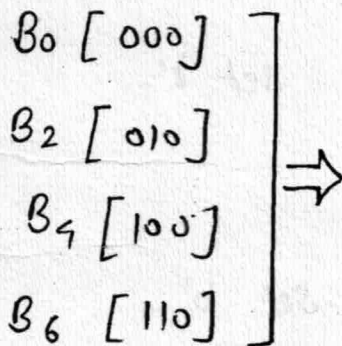
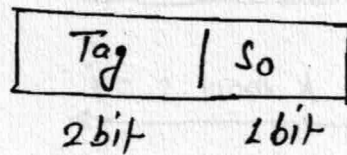
Address Binding is :-



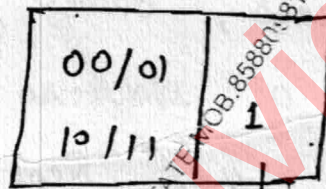
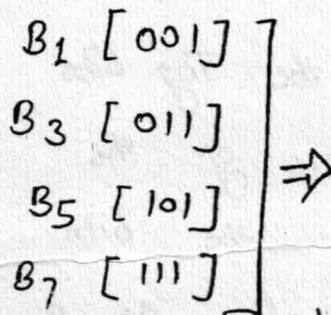
MM



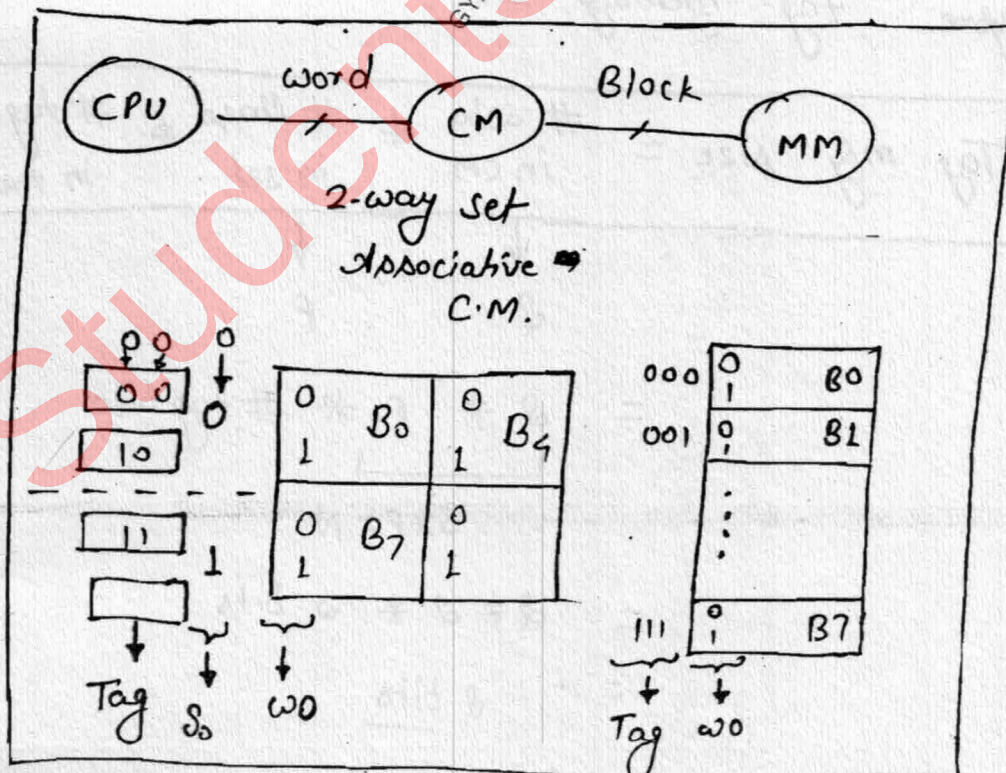
Set Associative m/y



Set '0'



Set '1'



MM Block

Set Associative
~~cache~~ ~~map~~

C.M. Set

B₀
[000]

$$\begin{array}{l} k \bmod s = i \\ \hline 0 \bmod 2 = 0 \end{array}$$

Set '0'

B₇
[111]

$$\begin{array}{l} k \bmod s = i \\ \hline 7 \bmod 2 = 1 \end{array}$$

Set '1'

B₄
[100]

$$\begin{array}{l} k \bmod s = i \\ \hline 4 \bmod 2 = 0 \end{array}$$

Set '0'

In the mapping Process, some of the Tag bits are connected to a address logic of the Cache - memory and the Rest of the bits are stored in the cache controller as a Tag therefore tag memory size;

Tag m/y size =	# sets in CM.	*	# lines in set	*	# tag bits in the line
----------------	------------------	---	-------------------	---	---------------------------

↓

↓

$$= 2 * 2 * \text{\# tag bits}$$

$$S * P = N$$

$$= 2 * 2 * 2 \text{ bits}$$

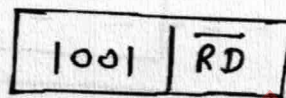
$$= 8 \text{ bits}$$

* Set Associative cache is designed with a explicit replacement policies, used to replace the data when the set is full.

Accessing Sequence :-

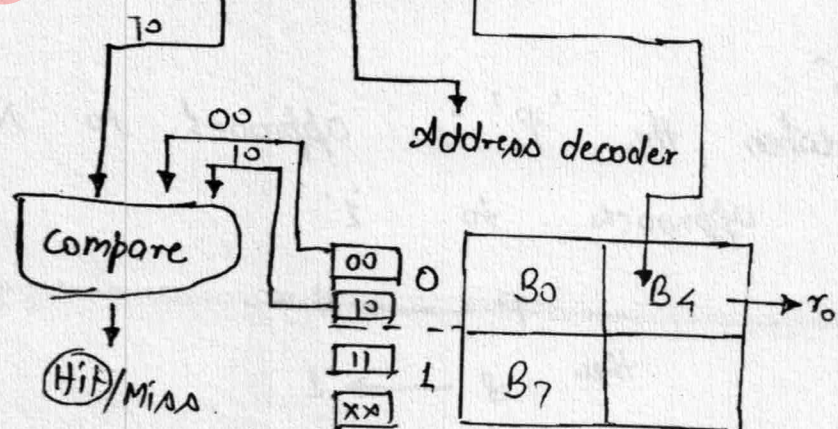
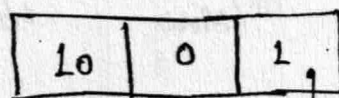
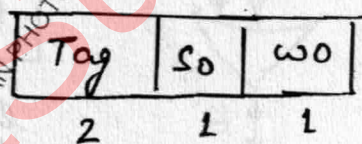
machine Instr: `Mov r0, [100]`

↓
CPU generates the m/y Request



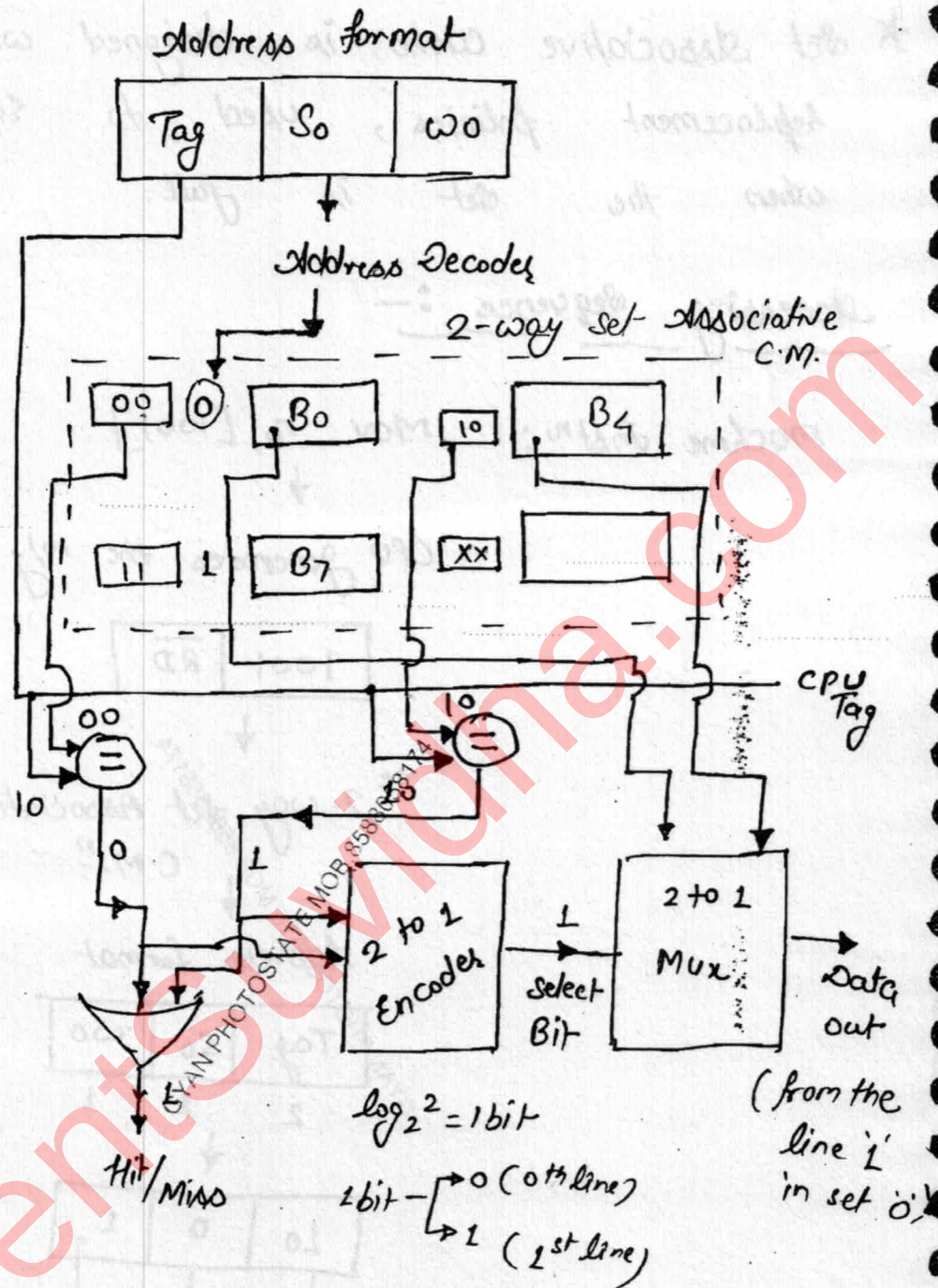
↓
"2-way Set Associative C.M."

Address format



Comparison :-

Ckt



Note :-

when the 'P' is approach to N then ϕ is approach to 1

$$P \rightarrow N$$

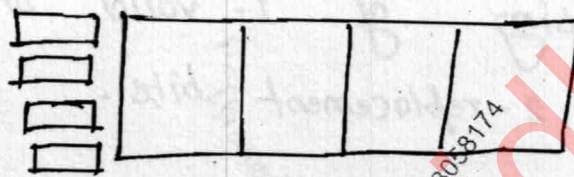
$$\text{then } \phi \rightarrow 1$$

then, cache m/y behaves as a fully associative.

eg: 4 way set associative
Cache of 8B with 2B Block

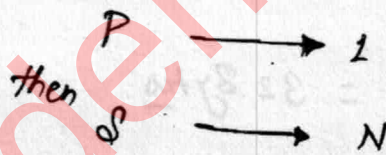
$$* \text{ \# lines } (N) = \frac{8}{2} = 4$$

$$* \text{ \# sets } (S) = \frac{N}{P\text{-way}} = \frac{4}{4} = 1$$



fully associative

2.) when P is approach to 1 then S is approach to N , then cache behaves as a Direct.



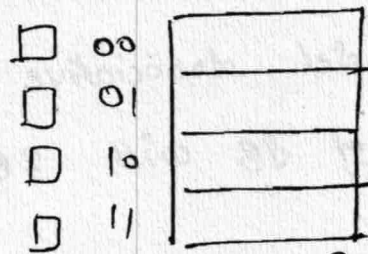
then set Assoc. C.M. $\rightarrow$ Direct

eg:

1-way set associative cache
of 8B with 2B Block.

$$* \text{ \# lines } (N) = \frac{8}{2} = 4$$

$$\text{ \# sets } (S) = \frac{N}{P\text{-way}} = \frac{4}{1} = 4$$



Direct C.M.

Ques: Consider a 16-way set-associative cache of 64kB organized into 9 32 Byte Block. CPU generates 32 bit address to access the data. In the Cache Controller Each Tag comprising of 1- valid bit, 1 modified bit and 2-replacement bits.

a) Show the address Binding.

b) # tag bits in the line

c) Tag m/y size.

$$P = 16$$

$$C.M. \text{ size} = 64kB = 16 \text{ bit}$$

$$m/m \text{ size} = 2^{32}B$$

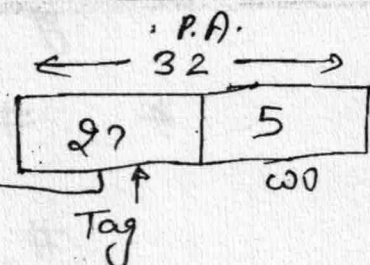
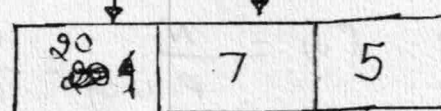
$$\text{Block size} = 32 \text{ Bytes}$$

$$\# \text{ lines } (N) = \frac{2^{16}B}{2^5B} = 2^{11} = 2K$$

$$\# \text{ sets } (S) = \frac{2^{11}}{2^4} = 2^7$$

a) Address Binding :-

Set
Ass. :-
m/y



b) # Tag bits = 20
in line

c). Tag mly size = $S * P * \# \text{ tag bits}$
 $= 2^7 * 16 * (20 \text{ bit} + 1 \text{ valid bit} + 1 \text{ mod bit} + 2 \text{ replaced bit})$
 $= 2^7 * 2^4 * 24 \text{ bits}$
 $= 48 \text{ k bits}$

"Replacement Algorithms" :-

:- Replacement algorithms are used in the cache design, To Replace the data when the cache is full they are :

- 1) Random
- 2) FIFO
- 3) LRU

In FIFO, replace the Block (cache block) which is having the longest time-stamp.

In LRU, replace the cache block which is present in the cache longest time without reference.

Que:- Consider 4-Block cache (Initially empty) with the following main mly Block references 4, 5, 7, 12, 4, 5, 13, 4, 5, 7. Identify the Hit ratio using :

- ~~AB~~ FIFO
- LRU
- Direct mapped cache
- 2-way set associative with LRU.

a)

4	5	7	12	4	5	13	4	5	7
4	4	4	4	4	4	13	13	13	13
	5	5	5	5	5	5	4	4	4
		7	7	7	7	7	7	5	5
			12	12	12	12	12	12	7
F	F	F	F			F	F	F	F

$$\text{Hit ratio} = \frac{2}{10} = 20\% = 0.2$$

b)

4	5	7	12	4	5	13	4	5	7
4	4	4	4	4	4	4	4	4	4
	5	5	5	5	5	5	5	5	5
		7	7	7	7	13	13	13	13
			12	12	12	12	12	12	7
F	F	F	F			F			F

$$\text{Hit ratio} = \frac{6}{10} = 60\% = 0.6$$

c) Direct mapped :-

4 5
0 1

CM.

0	4 4 12
1	5 5 13
2	
3	7 ✓

$$k \bmod N = i$$

4	C	C	C	C	✓
	4	5	7	12	4
	M	M	M	M	M
	C		✓		
	13	4	5	7	
	M	H	M	H	

$$\frac{3}{10} = 0.3$$

5 compulsory miss

2 conflict miss

d). 2-way set associative (with LRU)

	0	1
0	4	12
1	13	7

4	5	7	12	4	5	13
M	M	M	M	H	H	M

4	5	7
H	H	H

$$k \bmod s = i$$

$$\downarrow$$

$$\bmod 2$$

$$4 \text{ Hit} = \frac{4}{10} = 0.4$$

	0	1
0	4	12
1	5	13

$$\left. \begin{array}{l} 4 \bmod 2 = 0 \\ 5 \bmod 2 = 1 \\ 7 \bmod 2 = 1 \\ 12 \bmod 2 = 0 \\ 13 \bmod 2 = 1 \end{array} \right\}$$

$$\frac{4}{10} = 0.4$$

4 - M
5 - M
7 - M

12 - M

4 - H
5 - Hit
13 - m

4 - Hit

5 Hit, 7 miss
(7 mod 2 = 1)