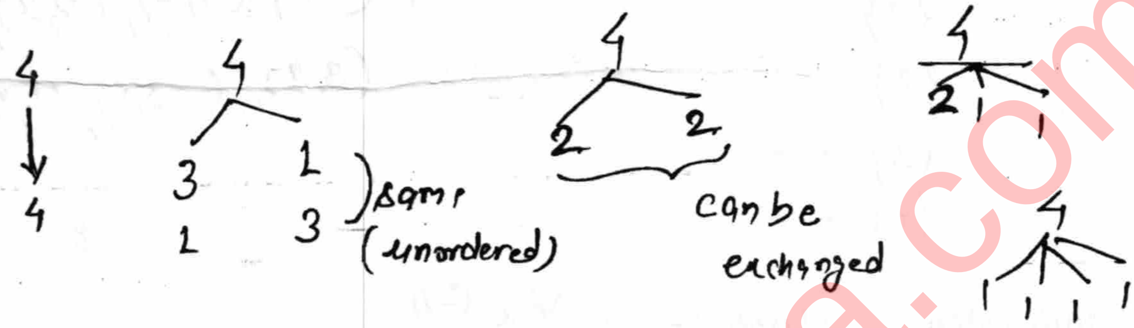
Theorem :- Every equivalence Rel^n correspond to unique partition.

so,

:- # of unodered partition = # equivalence Rel^n

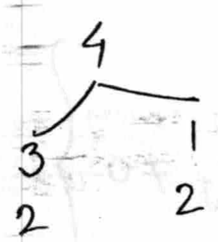


$\frac{4}{4!} = 1$ $\frac{4!}{3!\,1!} + \frac{4!}{2!\,2!\,2!}$

$\frac{4!}{2!\,1!\,1!\,2!}$ $\frac{4!}{(1!)^4\,4!} = 1$

~~1er~~ add all = $\underline{15}$

:- exactly 2 Block partition :-



$\Rightarrow \frac{4!}{3!\,1!} + \frac{4!}{2!\,2!\,2!}$

$\Rightarrow 7$

Theorem 2. Correspond to every partition π of A there is a unique equivalence Rel R such that $A/R = \pi$

eg $A = \{1, 2, 3, 4\}$

$\pi = \{ [1, 2, 3], [4] \}$
$\qquad\quad A_1 \qquad\quad A_2$

$A/R = \pi$

R = equivalence Relⁿ

$R = \{ A_1 \times A_1 \cup A_2 \times A_2 \}$

$= \{ \{1, 2, 3\} \times \{1, 2, 3\} \cup \{4\} \times \{4\} \}$

$= \{$

Que: π = 4 Block, 3 of them → 3 element
1 Block → 4 element

~~# eg~~ size of equivalence Relⁿ = ?

$|R| = (3 \times 3) \times 3 + 4 \times 4$

$= 27 + 16 = \boxed{43}$

:– set Other method for partition (If R having meaning)

$\omega = \{$ cat, bat, ball, catch, doll $\}$

$R_1 = \{ (x, y) \mid x \,\&\, y$ start with 'same' letter$\}$ on ω
equivalence Relⁿ

$\omega / R_1 = \{ \{$cat, catch$\}, \{$bat, ball$\}, \{$doll$\} \}$

$R_2 = \{ (x, y) \mid |x| = |y| \}$ on ω

$\omega/R_2 = \{ \{cat, bat\}, \{ball, doll\}, \{catch\} \}$

Note :- ~~Quotient set~~ :- "Index of R" = size of Quotient set

= # distinct equivalence classes

~~Index~~

Theorems :-

1.) R is equivalence Rel^n $\Leftrightarrow$ R^{-1} is equivalence Rel^n

2.) R is partial order Rel^n $\Leftrightarrow$ R^{-1} is also partial order Rel^n

(Becoz Reflexive, Transitive, Antisymmetric) are having $R^{-1} = R$

3) R is an equivalence Rel^n and S is a equiv. Rel^n on a Common set A

A

R is ER   S is ER   then

$\Rightarrow$ $R \cap S$ is also E.R. on A

If R is E.R. & S is E.R on A

$\Rightarrow$ $R \cup S$ may or may not E.R on A

:- In $R \cup S$ $\Rightarrow$ Transitive Property violates so its may or may not E.R.

eg:- R & S are symmetric on A

I $R \cup S$ — Both

II $R \cap S$ —

** Note :-

Reflexive & symmetric closed under :-

R, S — R^{-1}, $R \cup S$, $R \cap S$

(Transitive) T — R^{-1}, $R \cap S$, ~~$R \cup S$~~

* v.imp E.R. — R^{-1}, $R \cap S$, ~~$R \cup S$~~

** Note :-

eg:- $(R \cup S)^{\infty} \Rightarrow (R \cup S)^{+}$ (smallest E.R containing R & S) (LUB) lowest upper Bound

R — E.R., S — E.R.

$R \cap S$ — Largest E.R $\subset$ R & S (~~largest~~ E.R.) subset of R & S

~~Smallest equivalence Reln~~ (GLB)

R — S

:- $R^0 = I_A$ (identity)

$R^1 = R$

$R^2 = R \circ R$

$R^3 = R \circ R^2 = R^2 \circ R$

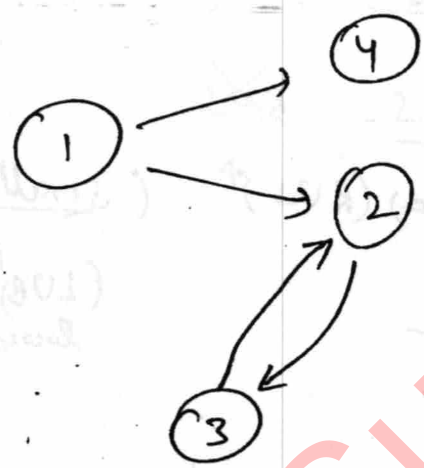
$R^m \circ R^n = R^n \circ R^m$

$R^{-2} = R^{-1} \circ R^{-1} = \{(3,1), (2,2), (3,3)\}$

$R^{-1} = \{(2,1), (3,2), (2,3), (4,1)\}$

Powers, R^2, R^3 by digraph

$R = \{(1,2), (2,3), (3,2), (1,4)\}$



$R^2 = \{(1,3), (2,2), (3,3)\}$

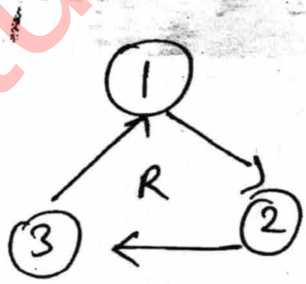
$R^3 = \{(1,2), (2,3), (3,2)\}$

Note :-

$(x,y) \in R^{\infty}$ Iff Direct Path from $x \to y$

$(x,y) \in R^{n}$ Iff Direct Path from $x \to y$ of length 'n'

eg :-



$R^{29} = ?$

$R^{29} = \{(1,3), (3,2), (2,1)\}$

$29 \bmod 3 = 2 \Rightarrow R^2 = \{(1,3), (3,2), (2,1)\}$

Que:-

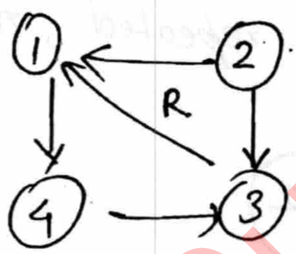
$$M_{R^3} = \begin{array}{cc} & j \\ i & \begin{bmatrix} \quad & \quad \end{bmatrix} \end{array}$$

i, j th element of M_{R^3} is 4
then fill :-

$\exists$ 4 paths of length 3 for i to j in digraph

$M_{R^3} = M_R \times M_R \times M_R$

Que:-



$M_{R^3} = ?$

check by options → matrix for length 3 path.

⇒ Reduction of R to B := $(R \cap B \times B)$

$B \subseteq A$ $\qquad$ $A = \{1, 2, 3, 4\}$

$B = \{1, 2, 3\}$ $\qquad$ $R = \{(1,1), (2,3), (3,2), (1,4)\}$

Remove 4 element set)

$R = \{(1,1), (2,3), (3,2)\}$

Functions :-

:- function is a unique valued relation.

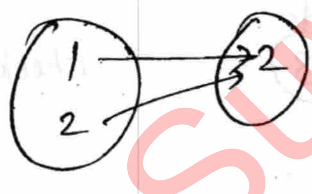
like (x, y) ordered pair x I/P, y O/P

:- "Every input should have unique O/P."

$$R = \{(1,2), (2,3), (3,1)\}$$

Every function is relation but Reverse is not true.

:- If 2nd element (O/P) repeated, no harm



eg:- ✓ $\{(x,y) \mid y = 3x+1\}$ on $Z \times Z$ funct^n

✓ $\{(x,y) \mid y = x^2\}$ on Real No. funct^n

✓ $\{(x,y) \mid y = \log x\}$ on R funct^n

× $\{(x,y) \mid y = \sqrt{x}\}$ on R If sign + or − given then funct^n

× $\{(x,y) \mid y = \sin^{-1}(x)\}$ on R Not funct^n

$$y = \sin^{-1}(0) \quad y = \pm n\pi$$

$$n = 0, 1, 2, \ldots$$

:- If it's a Principle value of $\sin^{-1}(x)$, then its a function.

eg: $y = 0$ x is even
$y = 1$ x is odd } funct^n

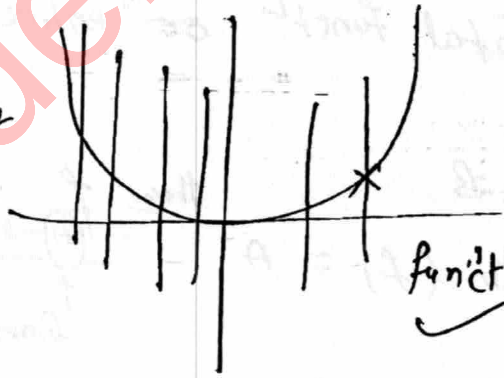
$y = 0$ or 1 x is even
$y = 2$ x is odd } ✗ not funct^n.

eg: $y = |x|$ is funct^n (many to one).

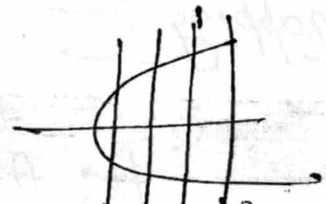
<u>graph</u>:-

:- for checking funct^n, Draw vertical lines,
If line intersect the graph only at one point then it is funct^n.

$y = x^2$



funct^n



Not funct^n
✗
$y = \sqrt{x}$

$y = \sin x$

$y = \sin^{-1} x$
✗

1. Def funct^n
2. Domain, Range
3. Types of funct^n
4. ~~Counting of funct^n~~
5. f^{-1}, fog.

Types of funct^n :-

1. mapping (Total funct^n or entire funct^n)
* 2. 1-1 funct^n (INJective funct^n)
3. ONTO funct^n (Surjective funct^n)
4. 1-1 & ONTO funct^n (Bijective funct^n) or 1-1 correspondence funct^n

mapping By default

5. Permutation funct^n

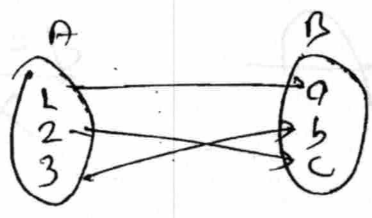
:- mapping :- (Total funct^n or entire funct^n)

$f : A$ to B iff Domain $(f) = A$

then $f_{A \to B}$ (Total funct^n representation)

Domain

$f_{A \to B}$ iff Domain $(f) = A$



mapping-? $f(x) = \log x$ on R ✗ on $\underline{R^+}$

$f(x) = \sin x$ on R

$f(x) = e^x$ on R

$f(x) = 3x+1$ on R

Onto :- (Surjective) (check (find) Range) :- $f_{A \to B}$

iff $\boxed{Range(f) = B}$

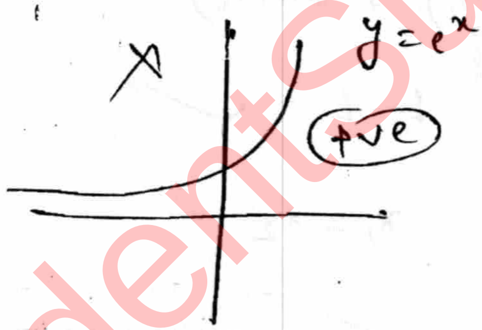
✗ $f(y) = \sin x$ on R

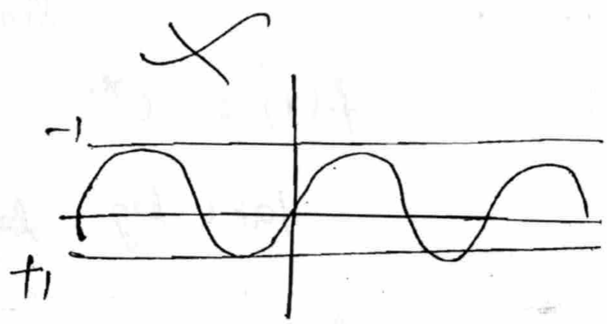
$x = \sin^{-1} y$

$\sin x \to -1$ to $+1$

so Not onto.

✗ $f(x) = \underline{e^x}$ on R



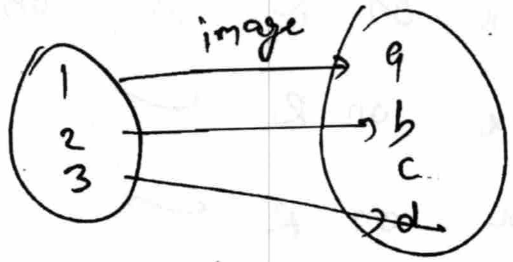


∴ "If funcⁿ is not onto then it will be into"

Into :- funcⁿ iff $Range(f) \subset B$

(Proper subset of B)

i.e. atleast one element in B lonely.



into

1-1 (Injective) :-
function

$$\boxed{f \text{ is } 1\text{-}1 \text{ iff } f(x_1) = f(x_2) \Rightarrow x_1 = x_2}$$

contrapositive

1-1 :- function $\boxed{x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)}$

$f(x) = \sin x$ Not 1-1

$\sin(x_1) \neq \sin(x_2)$

$f(x) = e^x$ 1-1 ✓

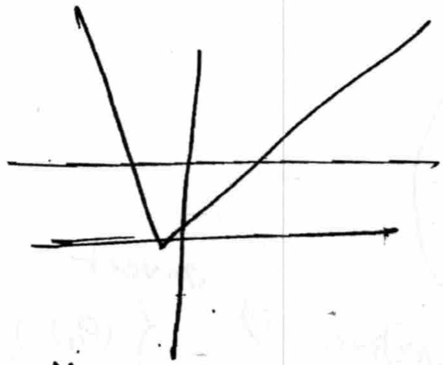
take log ~~log~~ $x_1 = x_2$

$f(x) = 3x + 1$ ✓

so its Bijective (injective + surjective)

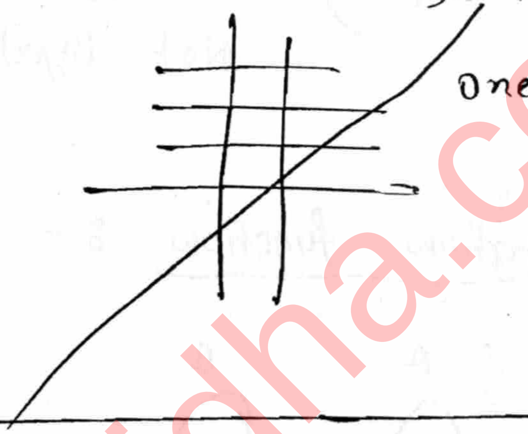
:- To check the 1-1 function on graph, Draw Horizontal Lines. If Horizontal lines cuts more than one point then not one to one.

$y = |x|$ ✗



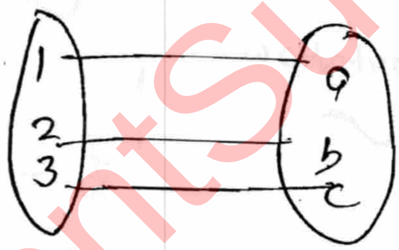
Not one to one

$y = 3x + 1$



Bijective :- funct^n

$\boxed{\text{Bijective } A \to B \text{ iff } |A| = |B|}$

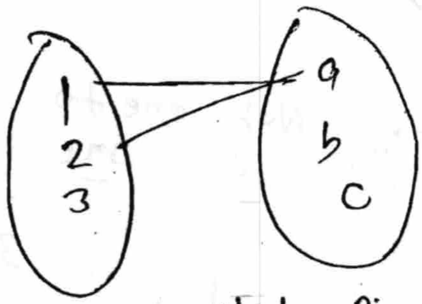


⇒ Between two infinite, Bijection is possible if Both countably infinite or countably finite.

⇒ Bijection funct^n is possible if Both finite set have same # of elements

⇒ finite to infinite ✗
Infinite to finite ✗
Infinite to infinite ✓

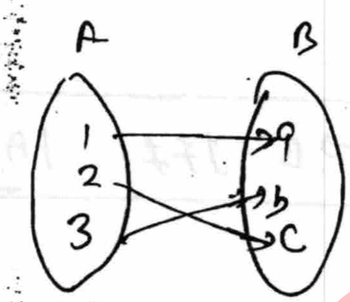
:- f^{-1} Exist Iff f is Bijective



Not Bijective $\Rightarrow$ Invert $\{(a,1),(a,2)\}$

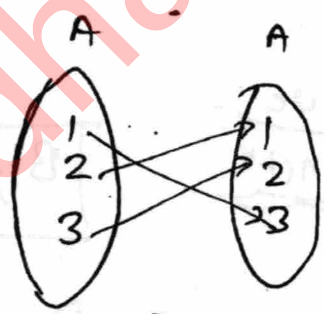
Not invertable.

:- Permutation function :-



Not Permutation

Bijection ✓



Permutation funct^n

$$f_1 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$$

$$f_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

$$f_3 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

Permutation funct^n

for $\boxed{n = n!}$

Counting of funct^{w} :-

1) funct^{n} ? $|A| = m \longrightarrow |B| = n \longrightarrow$ $\underline{n^m}$

2) $\cancel{f}$? Partial funct^{w} $|A| = m \longrightarrow |B| = n \longrightarrow$ $\underline{(n+1)^m}$

3) ? fn $|A| = m \longrightarrow |B| = n$ are 1-1 ? $\underline{{}^nP_m}$

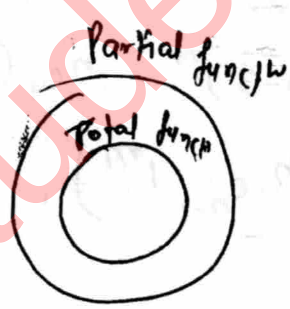
4) " are many-1 ? $\underline{n^m - {}^nP_m}$

5) " are onto ?

6.) INto ?

7) $|A| = n$ $|B| = n$ Bijective ? $\underline{{}^nP_n = n!}$

8) $|A| = n$ Permutation ? $\underline{n!}$



Truly total funct^{n} = Partial funct^{n} − Total funct^{n}

$= \underline{(n+1)^m - n^m}$

Onto funct^{n} :-

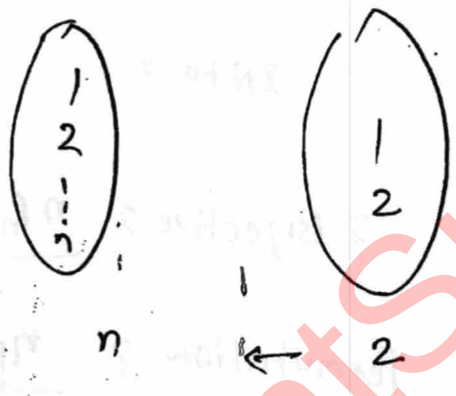
$\boxed{n^m - \text{into funct}^n}$

INto funct^{n} :-

$$n(① \cup ② \cup ③ \cup \dots)$$

$$\Rightarrow \left({}^nC_1 \times (n-1)^m - {}^nC_2 (n-2)^m + {}^nC_3 (n-3)^m \right.$$

$$\left. - {}^nC_4 (n-4)^m + \dots {}^nC_0 (0)^m \right)$$



so

ONTO :- $n^m - \left({}^nC_1 \times (n-1)^m - {}^nC_2 (n-2)^m + {}^nC_3 (n-3)^m - {}^nC_4 (n-4)^m - \dots - {}^nC_{n-1} 1^m \right).$

eg: $2^n - (2)$

$\Rightarrow 2^n - 2$

eg $|A| = 10$ $|B| = 4$ $f_{A \to B}$

$$4^{10} - \left({}^4C_1 \times 3^{10} - {}^4C_2 \times 2^{10} + {}^4C_3 \times 1^{10} \right)$$

Date
31/08/2017

$\star \rightarrow$ f is invertible iff f is Bijection
(1-1 & onto)

$\star \rightarrow$ f^{-1} is Bijective Iff f is Bijection

eg: $f(x) = \frac{1}{x-3}$

$f^{-1}(x) = ?$

find $f^{-1}(y)$, then replace y with x

let

$\Rightarrow f(x) = y, \quad y = \frac{1}{x-3}$

$x - y = 1/y$

$\boxed{x = \frac{1}{y} + 3}$

$\Rightarrow x = f^{-1}(y)$

so $\boxed{f^{-1}(y) = \frac{1}{y} + 3}$

$f^{-1}(x) = \frac{1}{x} + 3$

eg:- $f(x) = e^x$

$f(x) = y = e^x$

$\log y = x$

$x = \log y$

$x = f^{-1}(y) = \log_e y \Rightarrow \boxed{f^{-1}(x) = \log_e x}$

(2) $f(x) = 3x+1$

$y = 3x+1$

$\frac{y-1}{3} = x$

$x = f^{-1}(y) = \frac{y-1}{3}$

$$\boxed{f^{-1}(x) = \frac{x-1}{3}}$$

<u>other method :-</u>

$f(x) = 3x+1$

$f^{-1}(x) = ?$

Put any value :-

$f(2) = 3\cdot 2+1$

$= 7$

$f(2) = 7$

$f^{-1}(7) = 2$

a) $\frac{x+2}{3} \Rightarrow \frac{7+1}{3} = x$

b) $3x-1 \Rightarrow 21-1 = 20$

c) $\frac{1}{3x-1} \Rightarrow \frac{1}{21-1} = \frac{1}{20}$

d) $\frac{x-1}{3} \Rightarrow \frac{7-1}{3} = 2$

Que :- $f(x,y) = (x+y, x-y)$

$f^{-1}(x,y) = ?$

$x=3, y=2$

$f(x,y) = (5, 1)$

$f^{-1}(x,y) = (3,2)$

a) $(x-y, x+y)$

b) $(2x+1, 2y+1)$

c) $\left(\frac{x-y}{2}, \frac{x+y}{2}\right)$

d) $\left(\frac{x+y}{2}, \frac{x-y}{2}\right)$

a) $5-1=4$, 6

b) 11, 3

c) 2, 3

d) 3, 2

Let

$$f(x,y) = (z_1, z_2)$$
$$(x,y) = f^{-1}(z_1, z_2)$$

$z_1 = x+y$
$z_2 = x-y$
$z_1 + z_2 = x$

$$x = \left(\frac{z_1+z_2}{2}\right) \quad y = \frac{z_1 - z_2}{2}$$

Put $z_1 = x$
$z_2 = y$

$$\left(\frac{x+y}{2}, \frac{x-y}{2}\right)$$

Composition :-

$$f\circ g(x) = f(g(x))$$
$$g\circ f(x) = g(f(x))$$

$$f(x) = 3x+1$$
$$g(x) = \sin(x^2)$$

$$f\circ g(x) = f(g(x)) = f(\sin x^2) = 3\sin x^2 + 1$$

$$g\circ f(x) = g(f(x)) = g(3x+1) = \sin(3x+1)^2$$

$$\boxed{f\circ g \neq g\circ f}$$

⇒ composition of function & Relation is not commutative but associative.

$f_{A\to B}$

$$\boxed{\begin{array}{l} f\circ f^{-1} = I_B \\ f^{-1}\circ f = I_A \end{array}} = \{(a,a), (b,b), (c,c)\}$$

$$f_{(1,2,3)} \longrightarrow (a,b,c)$$

$$f \circ f^{-1} = I_B = \{(a,a),(b,b),(c,c)\}$$

$$f^{-1} \circ f = I_A = \{(1,1),(2,2),(3,3)\}$$

– for any Bijective funct^{n} [Bijection $A \times A$]

$$\boxed{f \circ f^{-1} = f^{-1} \circ f = I_A}$$

:– If $\boxed{f \circ g = g \circ f = I \Leftrightarrow f = g^{-1} \;\&\; g = f^{-1}}$

eg:– $f \circ f^{-1} = I$

$f(x) = 3x+1$

$f^{-1}(x) = \frac{x-1}{3}$

$f \circ f^{-1}(x) = f(f^{-1}(x)) = f\left(\frac{x-1}{3}\right) = 3\frac{(x-1)}{3} + 1$

$= x$

→ If $f \circ g$ exist, $g \circ f$ may or may not exist

★ → f is 1-1 g is 1-1 → gof is 1-1

★ → f is onto g is onto → gof is onto

★ → f is Bijective g is Bijective → gof is also Bijective.

one way only

gof is 1-1 → ~~f is 1-~~

★ gof is 1-1 → 'g' need not to be 1-1, but 'f' has to be 1-1

★ gof is onto → 'g' has to be onto, f need not to be onto

Group Theory :-

1. Binary operation : $*$

$(S, *)$: Algebric Structure

- Groupoid
- Semigroup
- monoid
- Group
- Abelian

- Closure → Groupoid
- Associative → Semigroup
- Identity → Monoid
- Inverse → Group
- commutative → Abelian

:- Binary operation : function of 2 variables

eg :- which one is Binary op^n :

$\{z, *\}$ Parent

a) $a*b = a+b$ ✓

b) $a*b = \sqrt{ab}$ ✗

c) $a*b = a$ ✓

d) $a*b = \sin^{-1}(a,b)$ ✗

Binary op^n — $*(a,b)$, o/p should be unique.

:- $a*b = a-b$ ✓

are not closed under $(N, *)$

$2-3 = -1$ is not Natural No.

eg :- $(Z, *)$

I. $a * b = a + b$ → abelian

II. $a * b = a \times b$ — monoid

III. $a * b = a^b$ —

IV. $a * b = a$ — semi-group

V. $a * b = \max\{a, b\}$ or min → semigroup

VI. $a * b = a + b - ab$ — monoid

VII. — abelian

*	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

upper triangle & lower should be same (commutative)

	a	b	c
a	a	a	b
b	a	b	c
c	b	c	c

b is e (identity)

	Closure	Associative	Identity	Inverse	Commutative
I	✓	✓	✓	✓	✓
II	✓	✓	✓	✗ $a^{-1} = 1/a$	✓
III	✗	✗	✗	✗	✗
IV	✓	✓	✗	✗	✗
V	✓	✓	✗	✗	✓
VI	✓	✓	✓ $e = 0$	✗	✓
VII	✓	✓ (given)	✓ $e = a$	✓ $a^{-1} = a$, $b^{-1} = d$, $c^{-1} = c$ have to be	✓

:- closure → $\forall a,b \in S \quad a*b \in S$

:- Associative – $a*(b*c) = (a*b)*c$

:- identity – $\exists e \; \forall a \in S \quad a*e = a = e*a$

:- inverse – $\exists a \in S \; \exists a^{-1} \in S \quad a*a^{-1} = e = a^{-1}*a$

:- commutative – $\forall a,b \in S \quad a*b = b*a$

eg:- $a*(b*c) = (a*b)*c$

$a*b^c \neq a^b*c$

a

⇒ identity:

$a^e = a = e^a$

eg:-

$\boxed{\cancel{a}+e - ae = \cancel{a}} = e+a-ea$

$e(1-a) = 0$

$e=0$

inverse:

$\forall a \; \exists a^{-1} \quad a*a^{-1} = e = a^{-1}*a$

$a+a^{-1} = 0 = a^{-1}+a$

$a^{-1} = -a$