Arguments :-

P_1 [If you work hard, you will pass Exam.
Premise 1 Premise 2

P_2 [you are working hard therefore you will pass exam.] ϕ
Premise 2 conclusion

Argument always ends with 'therefore'

after therefore :- only one sentence that is conclusion

P_1, P_2 therefore ϕ

:- Falacy :- Invalid Argument

validity of Argument :-

$(P_1, P_2, P_3 \ldots P_n) \vdash \phi$ is valid Iff

$(P_1 \wedge P_2 \wedge P_3 \ldots \wedge P_n \Rightarrow \phi) = 1$ (Tautology)

i.e. conjunction of premises implies conclusion must be Tautology.

:- Correctness of conclusion doesn't make sure that Argument is invalid.

Ex:-
P_1: If you work hard, you will fail
P_2: you are working hard therefore
ϕ: you will fail. (valid Argument)

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Que: [If you $\underbrace{\text{work hard}}_{p}$, you will $\underbrace{\text{pass exam}}_{q}$

you are $\underbrace{\text{working hard}}_{p}$ therefore you will $\underbrace{\text{pass exam}}_{q}$.

$P_1 : \; p \Rightarrow q$

$P_2 : \; \dfrac{p}{q}$

$\cancel{p \Rightarrow q}$ $(p \Rightarrow q, p) \vdash q$

$[(p \Rightarrow q) \wedge p] \Rightarrow q$

$[(p' + q) \cdot p] \Rightarrow q$

$\overline{pq} + q$

$p' + q' + q$

$p' + 1$

$\underline{1}$ (Tautology)

Ex:- If you $\underbrace{\text{work hard}}_{p}$ & $\underbrace{\text{have talent}}_{q}$ then you will $\underbrace{\text{become Musician}}_{r}$

you $\underbrace{\text{working hard}}_{p}$

you $\underbrace{\text{dont have talent}}_{q'}$

$\therefore$ I am not going to be musician

Not valid

may or may not

$P_1 : \; (p \wedge q \Rightarrow r)$

$P_2 : \; p$

$P_3 : \; \dfrac{q'}{r'}$

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Que: I If it rains, the match will not be played.
II. The match was played.

q: ~~It~~ ~~is~~ was not raining.

Rules of Inference :- [always valid]

1. Conjunction
2. Addition
3. Modus ponens
4. Modus tollens
5. Disjunctive Syllogism
6. Hypothetical Syllogism
7. ~~Conjunctinctive~~ Constructive dilemma
8. Distructive dilemma

① Conjunction:-

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

$$\begin{array}{ll} R & \text{(Reflexive)} \\ S & \text{(symmetric)} \\ T & \text{(transitive)} \\ \hline \therefore R \wedge S \wedge T & \Rightarrow \text{eqivalence} \end{array}$$

② Addition:-

$[p] \Rightarrow p \vee q$

$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array}$$

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③ Modus ponens :- (Rule of detetchment)

$$[(p \Rightarrow q) \wedge p] \Rightarrow q$$

$$[p \wedge (p \Rightarrow q)] \Rightarrow q$$

④ Modus tollens :- (Rule of contrapositive)

$$[(p \Rightarrow q) \wedge \sim q] \Rightarrow \sim p$$

HF = P (House on fire), Smoke = S

Ex: $(HF \Rightarrow S) \wedge \sim S \Rightarrow \sim HF$

⑤ Disjunctive syllogism :-

$$[(p \vee q) \wedge \sim p] \Rightarrow q$$

$$[(p \vee q) \wedge \sim q] \Rightarrow p$$

⑥ Hypothetical syllogism :-

$$[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$$

(Transitive Rule of implication).

Ex :-

Even $\Rightarrow \div 2$

$\div 2 \Rightarrow 2x,\ x \in Z$

Even $\Rightarrow 2x,\ x \in Z$

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7) Constructive dilemma :-

P_1: $p \Rightarrow q$

P_2: $r \Rightarrow s$

P_3: $p \vee r$

$\therefore$ $q \vee s$

$$(p \Rightarrow q) \wedge (r \Rightarrow s) \wedge (p \vee r) \Rightarrow (s \vee q)$$

$H_1F \Rightarrow SH_1$

$H_2F \Rightarrow SH_2$

$\sim SH_1 \vee \sim SH_2$

$\therefore \sim H_1F \vee \sim H_2F$

8) Distructive Dilemma :-

$$\{(p \Rightarrow q) \wedge (r \Rightarrow s) \wedge (\sim p \vee \sim r)\} \Rightarrow \{\sim s \vee \sim q\}$$

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Predicate Logic :-

US (universal specification)

UG (universal generalization)

ES (Existance specification)

EG (Existence generalization)

US :-

:- $\forall_x (P x) \Rightarrow P(x_1)$

UG $\Rightarrow$ US (universal specification)

UG :-

:- $P(x_1) \wedge P(x_2) \wedge P(x_3) \Rightarrow \forall_x (P(x))$

(universal generalization)

ES :-

$\exists_x (P(x)) \Rightarrow P(x_1)$

EG :- $P(x_1) \Rightarrow \exists_x P(x)$

$\boxed{P(x_1) \Rightarrow \forall_x P(x)}$ X Not valid

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Predicate Logic :-

Given : $P(x)$, $P(x,y)$, $P(x,y,z)$ check T or F

Ex:-

$P(x)$: x is Even

$P(x)$: $x^2 = -1$

$\exists x \; P(x)$

Not true

No real No.

default domain: real Numbers

O/w domain will be given

If $\forall x \; P(x)$ is true, then $\exists x \; P(x)$ also be true.

Ex:-

1) $P(x,y)$: $x+y = y+x$

I. $\forall x \; P(x,y)$ ✓

II. $\exists x \; P(x,y)$ ✓

2) $P(x,y)$: $x+y = 10$

I. $\forall x \; P(x,y)$ ✗

II. $\exists x \; P(x,y)$ ✓

Ex:- $P(x)$: x is even

I. $\forall x \; P(x)$ ✗ } → strongest start from here

II. $\exists x \; P(x)$ ✓

III. $\forall x \sim P(x)$ ✗

IV. $\exists x \sim P(x)$ ✓

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$$\neg(\forall x\, P(x)) = \exists x \sim P(x)$$

for 2 variables:-

Positive integer

$P(x,y)$: $x+y=10$ $\{domain = Z^+\}$

order of checking

I $\forall x \forall y\ P(x,y)$ ✗ (1)

II $\forall x \exists y\ P(x,y)$ (3)

III $\exists y \forall x\ P(x,y)$ (2) $\Rightarrow \boxed{y = 10 - x}$

IV $\exists x \exists y\ P(x,y)$

V $\forall y \exists x\ P(x,y)$

VI $\exists x \forall y\ P(x,y)$

$P(x,y) = x + y^2 = 10$

I $\forall x \forall y\ P(x,y)$ ✗

II $\forall x \exists y\ P(x,y)$ ✗

III $\exists y \forall x\ P(x,y)$ ✗

IV $\exists x \exists y\ P(x,y)$ ✓

V $\forall y \exists x\ P(x,y)$ ✓

VI $\exists x \forall y\ P(x,y)$ ✗

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Note:- $\left.\begin{array}{l}\exists_x\ P(x)\\ \forall_x\ (P(x))\end{array}\right\} \rightarrow$ with quantifiers $P(x)$ is become Proposition

Properties of Predicates:-

I. $\forall x\ P(x) \Rightarrow \exists_x\ P(x)$
$\times \Leftarrow$

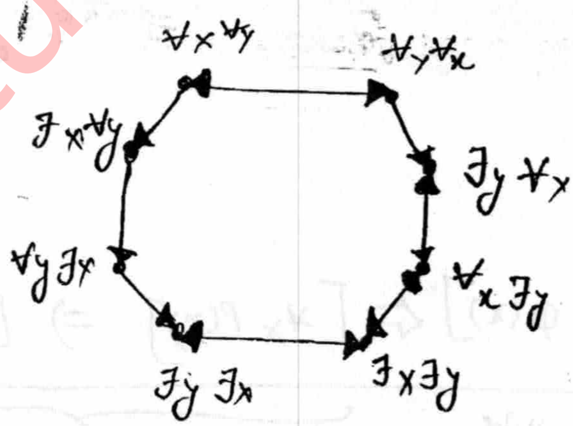
II. $\exists x\ \forall y\ P(x,y) \Rightarrow \forall y\ \exists_x\ P(x,y)$
$\times \Leftarrow$

III. $\forall x\ \forall y\ P(x,y) \Leftrightarrow \forall y\ \forall x\ P(x,y)$ $\Big\{$ $\Leftrightarrow\ =\ \equiv$ Biconditional equal to equivalent.

$P \Leftrightarrow q \vee r$ is T
then
$\underline{P \equiv q \vee r}$.

Ex:- $\underline{a \Leftrightarrow b}$ is T
then
$a + b' + c = ?$ is T.
$b + b' + c$
$1 + c = 1$ ✓ $\underline{T}$.

IV. $\exists_x\ \exists_y\ P(x,y) \Leftrightarrow \exists_x\ \exists_y\ (P(x,y))$.



$\nexists$

$\underline{\exists_x}\ (\forall y)\ P(x,y) \Rightarrow$
$\underline{\exists_x\ (\forall y)\ (P(x..}$

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⇒ Note :-

$$\forall x\,[P(x) \wedge \phi(x)] \Leftrightarrow (\forall x\, P(x)) \wedge [\forall x\, \phi(x)]$$

$$\exists x\,[P(x) \vee \phi(x)] \Leftrightarrow (\exists x\, P(x)) \vee [\exists x\, \phi(x)]$$

$$\forall x\,[P(x) \vee \phi(x)] \underset{\Leftarrow}{\overset{\Rightarrow \times}{}} [\forall x\, P(x)] \vee [\forall x\, \phi(x)]$$

$$\exists x\,[P(x) \wedge \phi(x)] \underset{\times \Leftarrow}{\Rightarrow} [\exists x\, P(x)] \wedge [\exists x\, \phi(x)]$$

Dated 23/08/2017

$$\boxed{\forall x\,[P(x) \Rightarrow \phi(x)] \Rightarrow [[\forall x\, P(x)] \Rightarrow [\forall x\, \phi(x)]]}$$

this one way true only -

$P_1 \Rightarrow \phi_1$

$P_2 \Rightarrow \phi_2$

$\vdots$

$P_n \Rightarrow \phi_n$

(P_1, P_2, P_3, P_4)

$\Downarrow$

ϕ_1, ϕ_2, ϕ_3

so this true

so we can't make

$\forall x\, \phi(x)$ false

$$\forall x\,[P(x) \Rightarrow \phi(x)] \Leftarrow [\forall x\, P(x)] \Rightarrow [\forall x\, \phi(x)] \quad \times \text{ false}$$

$P_1 \Rightarrow \phi_1$ can't be true for P_1 only

$P_2 \Rightarrow \phi_2$

Let take this true

$P_1, P_2, P_3 \ldots \Rightarrow \phi_1, \phi_2, \phi_3$

all have to be true

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$$\exists x \underbrace{(P(x) \rightarrow \phi(x))}_{x \text{ common}} \Rightarrow [\underbrace{[\exists x P(x)]}_{\text{let this true}} \Rightarrow \exists_x \phi(x)]$$

$P_5 \rightarrow \phi_5$

$\downarrow$

P_6 so this false

$$\exists x (P(x) \rightarrow \phi(x)) \Leftarrow [(\exists x P(x)] \rightarrow \exists x \phi(x)]$$

$\downarrow$

$P_6 \rightarrow \phi_5$

$P_1 \rightarrow \phi_1$

$\vee P_2 \rightarrow \phi_2$

$\vee P_3 \rightarrow \phi_3$

$P_6 - \phi_6$

$P_5 - \phi_5$

Implication is true for

$\cancel{\phi}$ $P \rightarrow \phi$

f or t

so If $P_6 \rightarrow \phi_5$ True

so

P_6 is true

or

If P_6 is false

then

$P_6 \rightarrow \phi_5$

is true

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$\forall$	$\wedge$	$\Rightarrow$	2	2
$\exists$	$\vee$	$\Leftarrow$	2	2
$\forall$	$\vee$	$\Rightarrow$	1	2
$\exists$	$\wedge$	$\Rightarrow$	1	2
$\forall$	$\Rightarrow$	$\Rightarrow$	1	2
$\exists$	$\Rightarrow$	$\Rightarrow$	1	2
			8	Possibilities
			True	

Properties :-

$$\forall x \, (P(x) \wedge \varphi) \Leftrightarrow [\forall x \, P(x)] \wedge \varphi$$

$$\forall x \, (P(x) \vee \varphi) \Leftrightarrow [\forall x \, P(x)] \vee \varphi$$

$$\exists x \, [P(x) \wedge \varphi] \Leftrightarrow [\exists x P(x)] \wedge \varphi$$

$$\exists x \, [P(x) \vee \varphi] \Leftrightarrow [\exists x \, P(x)] \vee \varphi$$

$$\forall x \, [P x \vee \varphi] \Rightarrow \exists x$$

$$\exists x \, [P(x) \wedge \varphi)]$$

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Predicate Translation :-

) :- English → Symbolic Logic

$\forall$ \$ $\exists$

:- Every student in the class have studied calculus.

given :-

Let $S(x)$ = x is a student (No need) → If domain given

$C(x)$ = x has studies calculus.

:- Domain is given :-

:- (Consider all the student in the class)

:- $\forall x\, C(x)$

Every

$\forall$:- "All" ≡ Every

"Plural" = Every

Humming Birds eat Honey → all Humming Birds

or

Students = every.

$\exists$:- some,

Singular

At least one

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If domain not given :- → $\forall x \; (\Rightarrow)$ / $\exists x \; (\wedge)$ { If domain not given }

$$\forall x \; (S(x) \Rightarrow C(x))$$

$\forall x$ → represent Everything in universe

$\Rightarrow$ → for $\forall(x)$ '$\Rightarrow$' come to reduce the domain to the 'class'

with

$$\exists x \; (S(x) \wedge C(x))$$

$\exists x$ → Some (there exist) Student

→ '$\Rightarrow$' can't used

$\wedge$ → with $\exists x$ '$\wedge$' is used as some students from ~~universe~~ class read calculus

Example :- (Reduced domain)

Store Room contain only Red Bicycle :- $B(x)$

all objects in store Room : $R(x)$

$$\forall x \; (R(x) \wedge B(x))$$

All objects are Bicycle and they are red

$$\forall x \; (R(x) \Rightarrow B(x))$$

all Red objects are Bycle (allow green car)

$$\forall x \; (B(x) \Rightarrow R(x))$$

Every Bicycle are red (allow red car)

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* Adjective $\equiv$ and ($\wedge$)

there ~~are~~ is a Red Bouncing ball in Room

Adj Adj Adj

$$\exists x \, (R(x) \wedge Bo(x) \wedge B(x))$$

Bouncing

eg:-

1) There is a boy who is tall

and adj

same as: There is a tall boy

2) There is ball which is red

and adj

same as: There is a red ball

$$\forall x \, (F(x) \wedge C(x)) \Rightarrow D(x)$$

fast car Dangerous

adj

Every fast car on express way is dangerous

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:- $\exists x$ not distributive over '$\wedge$' op^n:

$$\exists x\,(B(x) \wedge C(x)) \quad \times$$

for:- there exist a bicycle and car in the room

$$\exists x\, B(x) \wedge \exists x\, C(x)$$

NO, NOT :-

$$:- \forall x\,[(F(x) \wedge C(x)) \Rightarrow D(x)]$$

a) $\forall x\,[\neg D(x) \Rightarrow (\neg F(x) \vee \neg C(x))]$

b) $\forall x\,(\neg F(x) \vee \neg C(x) \vee D(x)]$

as: $P \Rightarrow q \equiv \sim q \Rightarrow \sim p \equiv \neg P \vee q$

No / NoT :-

:- 1) No Hard working Person is poor

:- 2) Not all Hard working Persons are Poor

$H(x)$ = Hard working

$P(x)$ = Poor (x is Poor)

1) ≡ Every Hardworking Person is rich

$$\forall x\,(H(x) \Rightarrow (\neg P(x)))$$

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1) Not a Hardworking Person is poor. Put Not & a

Not a Hardworking Person is poor

$$\neg\left(\exists x\left(H(x) \wedge P(x)\right)\right]$$

(or) $\forall x \left[\neg H(x) \vee \neg P(x)\right]$

$\forall x \left[H(x) \Rightarrow (\neg P(x))\right]$

Note:-

$P \Rightarrow q = \neg P \vee q$

$P \vee q = (\neg P) \Rightarrow q$

or

$P' \rightarrow q$

or

$q' \rightarrow P$ $\Rightarrow \neg(\neg q) \vee P$

$q \vee P \Rightarrow \equiv P \vee q$

2. Not all Hardworking Persons are Poor.

$\equiv$ Some Hardworking Persons are Rich

$$\exists x\left(H(x) \wedge \neg P(x)\right]$$

or

$$\neg\left(\forall x\left(H(x) \Rightarrow P(x)\right]\right]$$

(or) $\exists x\left(H(x) \wedge \neg P(x)\right]$

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:- 'AND' sometimes meaning 'OR'

:- All Diamonds & Pearls are Precious

$\forall [D(x) \wedge P(x)] \Rightarrow Pr(x)$ ✗

$\forall [D(x) \vee P(x)] \Rightarrow Pr(x)$ ✓

:- Lions & Tigers attacks when they are Hungry or threatened.

Plural — all lions

If $\Leftarrow$

$\forall x \, (L(x) \vee T(x)) \Rightarrow [H(x) \vee Th(x) \Rightarrow A(x)]$

$\forall x \, (L(x) \vee T(x)) \Rightarrow [H(x) \vee Th(x) \Rightarrow A(x)]$

:- 'OR' Never becomes 'AND'

Note :-

~~$P \Rightarrow (q \Rightarrow r)$ ✓ $\equiv (q \Rightarrow P) \Rightarrow r \equiv ($~~

★★ $P \Rightarrow (q \Rightarrow r) \equiv ((P \wedge q) \Rightarrow r) \equiv q \Rightarrow (P \Rightarrow r)$

$\forall x \, (H(x) \vee Th(x)) \Rightarrow ((L(x) \vee T(x)) \Rightarrow A(x)) \simeq$

$\forall x \, ([(L(x) \vee T(x)) \wedge (H(x) \vee Th(x))] \Rightarrow A x) \simeq$

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eg:- Purple mushrooms are poisonous :-

I. $\forall x\,(P(x) \wedge M(x)) \Rightarrow Po(x)$

II $\forall x\; P(x) \Rightarrow (M(x) \Rightarrow Po(x))$

III. $\forall\; M(x) \Rightarrow (P(x) \Rightarrow Po(x))$

I $\forall x \forall y \rightarrow f(x,y)$

II $\forall x \exists y \rightarrow f(x,y)$

III $\exists y\; \forall x \rightarrow f(x,y)$

IV $\exists x \exists y \rightarrow f(x,y)$

V. $\forall y \exists x$

VI $\exists x \forall y$

eg:- $F(x,y)$: x & y are friends

(domain: all students in c/g).

1) $\boxed{\forall x \forall y\; F(x,y)}$ In clg all are friendly with each other

2) $\forall x \exists y\; F(x,y)$ → Every body in the clg have a friend (or) atleast one friend

3) $\exists y\; \forall x\; F(x,y)$ ⇒ there is a person who is friendly with all

⇓ same

$\forall y \exists x\; F(x,y)$

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eg:- Some Boys are taller than all the girls in the class

$B(x)$;

$G(x)$;

$T(x,y)$; x is taller than y

I. Both domain are given:

$x \rightarrow$ all Boys in class

$y \rightarrow$ All girls in class

$$\exists x \forall y \, T(x,y)$$

IV No domain given:-

$$\exists x \left(B(x) \wedge \forall y \left(G(y) \Rightarrow T(x,y)\right)\right)$$

$\equiv \exists_x \cancel{\forall_y} (B(x) \wedge (G(y) \Rightarrow T(x,y)))$ reduce domain.

II $x \rightarrow$ all boys in class

$B(x)$ Not req.

$$\cancel{\exists x}(\quad \exists x \left(\forall y \left(G(y) \Rightarrow T(x,y)\right)\right)$$

III $y \rightarrow$ all girls in class.

$G(y) \rightarrow$ Not req.

$$\exists x \left(B(x) \wedge \forall y \, T(x,y)\right)$$

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Eg:- domain - "Table" $A(x)$: x is an apple.

i) There is an apple on the Table

$\geq 1 \ni \exists_x A(x)$

* ii) There is ~~an~~ exactly one apple on the table

$= 1$

$:- \exists!_x A(x)$

(or)

$$\boxed{\exists_x A(x) \wedge \forall_y (A(y) \Rightarrow y = x)}$$

* iii) There is atleast 2 apples on the table

i.e. ≥ 2 apples

$\exists_x A(x) \wedge \exists_y A(y) \wedge x \neq y$

iv) There is exactly two apples on the table

$= 2$

$$\exists_x \exists_y \Big(A(x) \wedge A(y) \wedge x \neq y \wedge \forall z \big(A(z) \Rightarrow z = x \text{ or}(\vee)\ z = y\big)\Big)$$

(or)

$$\exists_x A(x) \wedge \exists_y A(y) \wedge x \neq y \wedge \forall z \big(A(z) \Rightarrow z = x \text{ or } z = y \big)$$

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v) There are ~~to~~ atmost two apples on the table i.e. $\underline{\leq 2 \text{ apples}}$.

~~$\forall_x \neg A(x) \vee (\forall_x$~~

$$\underbrace{\left(\forall_x \neg A(x)\right)}_{0 \text{ apples}} \vee \underbrace{\left(\exists_x \left(A(x) \wedge \forall_y (A(y) \Rightarrow y = x)\right)\right)}_{1 \text{ apple}} \vee \underbrace{\sim}_{2 \text{ apples}}$$

$$\underbrace{\exists_x \exists_y \left(A(x) \wedge A(y) \wedge x \neq y \wedge \forall_z (A(z) \Rightarrow z = x \text{ or } z = y)\right)}_{2 \text{ apples}}$$

(or)

$$\equiv \underbrace{\forall_x \cdot \forall_y \forall_z \left(A(x) \wedge A(y) \wedge A(z) \Rightarrow x = y \text{ or } x = z\right)}_{\text{atmost } 2 \text{ apples}}$$

Consistent :-

:- Consistent :- $(P_1, P_2, P_n \ldots)$ is consistent iff

$\underline{(P_1 \wedge P_2 \wedge P_3 \ldots)} \; \underline{\neq 0} \rightarrow$ satisfiable $<^{T}_{CT}$

eg:-

$= 1$

$= p' + q$]

Not have to be zero

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eg: P_1: T ↑ V H ↑ → OP

P_2: H ↑

P_3: ¬ OP

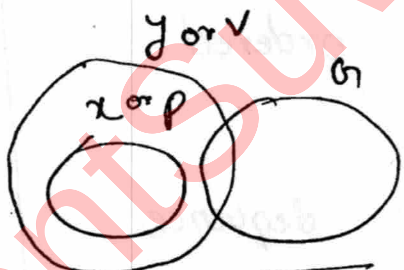
((P ↑ q).

1) eg:- P_1: Every Paint is varnish (x)

P_2: Some varnishes are green (y) or V

∴ ~~Every~~ Some Paint is ~~varnish~~ green X (G)

Venn diagram:-



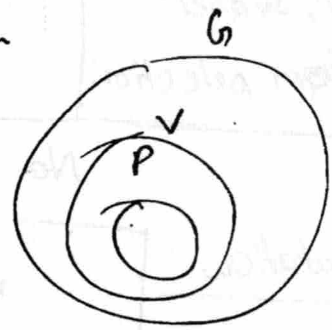
so ∴ Every Paint is not green

2)

Every Paint is varnish

Every varnish is green

∴ ~~Every~~ Some Pain is green



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