

Theory of Computation

1. Introduction

2. Regular Language & Finite Automata

50%

3. Context Free Language & Push Down Automata

5. ~~Finite~~ REC & RE languages & Turing Machine

Recursive Recursive Enumerable

Context Sensitive Language (belongs to Turing Machine)

Introduction -
- Languages
- Grammar
- Machine

Languages - bunch of string

Grammar - Way to represent language, rules so that valid string can be generated.



$L(G_1)$

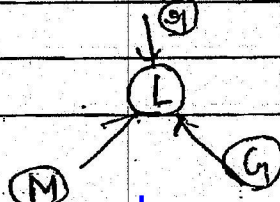
↳ language which follow G_1

$L(M)$ - Machine corresponding to language L , it is unique

Machine - set of command which accept only valid string

$L(M) = \text{accept string}$
if $\{ \text{string} \in L \}$

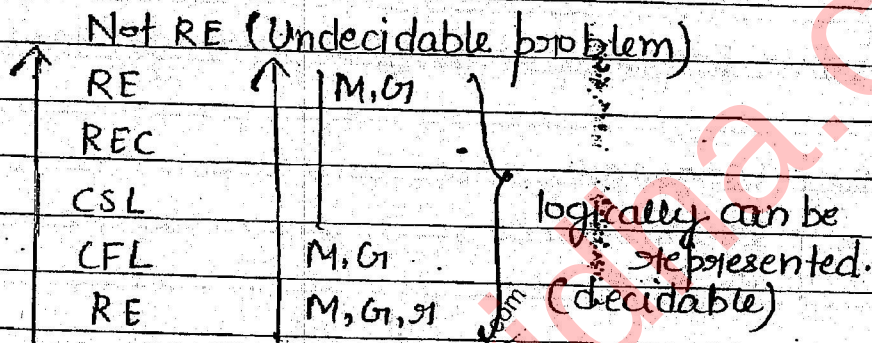
Regular Expression (r.e.) - formal description or way to represent language



EXCELLENT

- i.e. can not generated ^{be} every language.
- It can only generate regular language (simplest one)
- i.e. is basically a formulae representation of all rules.

• There are some language which can ^{not} be Represented by Machine & Grammar, these are "not RE" which are those i.e. No logic can be written



"Not All Jobs can be Automated"

Positive Aspect of TOC - can be done
 (-)ve Aspect of TOC - cannot be done.

¶ If A problem cannot be solved if they ~~are~~ cannot be represented by a particular language.

Recognizing a language - solving the problem

$L = \{s_1, s_2, s_3, s_4\}$

if $L = s_2$ (as input) it is recognizable

$L = s_5$ (not a recognizable)

(True but cannot be proved.

Language - 1 Alphabet (Σ)

2. string

3. Concatenation ($L_1 L_2$)

4. Reversal L^R

5. Null String

6. Length of String

7. prefix

8. Suffix

9. Substring

10. Power of string (w^n)

11. Σ^* , Σ^+

w-string

12. Language, $L \subseteq \Sigma^*$

Σ^* - Universal Language on Alphabet Σ .

13. Representation of languages (Informal or formal)

• You cannot write formal language for "NOT RE"

If two language represent the same logic, known as equivalent language.

Desirable problem - if any algorithm exist to solve that particular problem

two formal language are interconvertible

$$M_1 = M_2 \text{ if } L(M_1) = L(M_2)$$

14. Operations on languages

• Union • Intersection • Set Difference • Complement

• Symmetric difference.

15. L^R (Reversal)

16. $L_1 L_2$ - Concatenation of two Language

17. $L^2, \dots, L^n$ ($n \geq 0$) - power of language

(Here n can never be negative as there can never

inverse of language possible)

18. L^* , L^+

② ~~can~~ (ϵ does not belong to Alphabet)

1. ALPHABET - non empty finite set of symbols

↑ you can not break them down

$$\Sigma = \{a, b\}$$

$$\Sigma = \{0, 1\} \text{ - Binary alphabet}$$

$$\Sigma = \{0\} \text{ - Unary Alphabet}$$

$$\Sigma = \{ \} \text{ X (not an alphabet)}$$

$$\Sigma = \{a, b, c, 0, 1, \dots\} \text{ X (it is not finite)}$$

$$\Sigma = \{123, 345, \dots\} \text{ X (you can use them not)}$$

$$\Sigma = \{01, 0, 1\} \rightarrow \text{Not valid as you are dividing the symbol}$$

$$\Sigma = \{0, 1, 2\} \text{ - ternary alphabet}$$

technically allowed?

2. String - (Σ^*) combined sequence of ϵ or more symbol taken from alphabet set (Σ)

finite no of

② ϵ - Epsilon - reserved for null string is a valid string

$$\Sigma = \{\epsilon\} \rightarrow \text{not a valid alphabet as it is a valid string not a symbol}$$

(ϵ - can be replaced by 1)

$$\Sigma = \{a, b\}$$

$$L = \{a, b, ab, ba\} \text{ Valid}$$

3. Null String - length = ϵ

4. length of string - no of symbols in a string

$$|aba| = 3$$

$$|\epsilon| = 0$$

$$|a| = 1$$

• Every symbol is a string of length 1 = True but ~~not~~ every string is a symbol.

• Order of alphabet does not matter but in string, order matter

$$\{a, b\} = \{b, a\}$$

$$ab \neq ba$$

Concatenation - If two strings u, v then concatenation is $u \cdot v$ or uv

$$u = aab$$

$$uv = aabba$$

$$v = ba$$

$$\text{length of given } |uv| = |u| + |v|$$

$$1. \quad uv \neq vu \quad (\text{not commutative})$$

$$2. \quad |uv| = |u| + |v|$$

$$3. \quad u(vw) = (uv)w \quad - \text{associative}$$

$$4. \quad u \cdot \epsilon = u$$

5. ϵ - identity element for concatenation as after concatenation no change occurs.

Reversal - string - u^R It is also known as Transpose operation.

$$\text{If } u = aab$$

$$u^R = baa$$

$\neq$ means may or may not)

$$1. \quad u \neq u^R$$

$$2. \quad |u| = |u^R|$$

$$3. \quad (uv)^R = v^R u^R \quad (\text{Reversal law})$$

$$4. \quad (u^R)^R = u$$

$$5. \quad u = u^R \quad \text{iff } u \text{ is palindrome}$$

$$\text{Ex - } u = abba$$

5. Every string on unary alphabet are palindrome.

Every palindrome must be in form ww^R (Even palindrome)
 wxw^R (odd palindrome)

Language
 $\left\{ \begin{array}{l} L_o - \text{odd palindrome} \\ L_e - \text{even palindrome} \\ L_p - \text{Palindrome} \end{array} \right\}$

$$(ww^R)^R = (w^R)^R (w)^R$$

$$= ww^R$$

$$(wxw^R)^R = (wx)^R w$$

$$= (w^R)^R (x^R) w$$

$$= wxw^R$$

as x is a symbol

$$\therefore x \in \Sigma$$

$$w \in \Sigma^*$$

Σ^* is collection of every possible string on Σ .

String cannot be infinite but Σ^* can be infinite.

~~7-Proof~~ Theorem - Min. cardinality/size of alphabet = 1

Ques - On Alphabet Σ , how many string of length n is possible?

as for Σ

$$\Sigma = \{a, b\}$$

$$n = 3$$

$$\{a^3, a^2b, aba, abb, baa, bba, bab, b^3\}$$

$$8 (2^3)$$

Ques - On Alphabet Σ , how many string of length upto or at most length n ?

$$= \sum_{i=0}^n (\Sigma^i) = \frac{|\Sigma|^{n+1} - 1}{|\Sigma| - 1}$$

In $\Sigma = 2$ = $2^0, 2^1, 2^2, 2^3$

$\Sigma = 3$ = $1 +$

Ques - on a Alphabet Σ , how many even palindrome of length n , odd palindrome?

$\Sigma = \{a, b\}$ $n=2$ even - $ab \quad ba \quad aa \quad bb$

(3) if n odd: all odd

n even

$abba \quad abab \quad aaaa \quad bbbb \quad abab \quad aabb \quad bbaa \quad baba$
 $baab$

w

$n=3 = w w^R$

fixed last $aab \mid baa$

$\frac{n}{2}$ combination $|\Sigma|^{\frac{n}{2}}$

for even palindrome - $|\Sigma|^{\frac{n}{2}}$
for odd

for even = $|\Sigma|^{\frac{n}{2}}$
for odd = $|\Sigma|^{\frac{n}{2}}$

$aab \mid baa$

$\frac{n}{2} \times \Sigma$

$= |\Sigma|^{\frac{n}{2}} \times |\Sigma|$
odd
palindrome

for w for

odd palindrome - $|\Sigma|^{\frac{n+1}{2}}$

Ex $n=11$ length $n=11$, Alphabet = binary, No of even palin.
 $2^6 = 64$

for even palindrome upto length n , if n is even

$$|\Sigma|^0 + |\Sigma|^2 + |\Sigma|^4 + \dots + |\Sigma|^{\frac{n}{2}}$$

$$= \frac{|\Sigma|^{\frac{n}{2}+1} - 1}{|\Sigma| - 1}$$

$$\text{if } n \text{ is odd} = [|\Sigma|^0 + |\Sigma|^1 + |\Sigma|^2 + \dots + |\Sigma|^{\frac{n-1}{2}}] \times |\Sigma|$$

EXCELLENT

finding no of even palindrome upto

$$= \left[\frac{|\Sigma|^{n+1} - 1}{|\Sigma| - 1} \right] \times |\Sigma|$$

- You are initially finding the no of even palindromes & then again finding odd no of palindromes by multiplying by $|\Sigma|$

$$\begin{array}{c} \nwarrow \nearrow \\ W \times W^R \\ \uparrow \\ |\Sigma| \end{array} \rightarrow \text{no of even palindromes}$$

for even palindromes of length at most n

$$\text{for } n=1 \rightarrow |\Sigma| \quad \text{for } n=8 \rightarrow 2$$

$$\text{for } 0 \rightarrow |\Sigma|^{0/2}$$

$$2 \rightarrow |\Sigma|^{1/2}$$

$$4 \rightarrow |\Sigma|^{2/2}$$

$$8 \rightarrow |\Sigma|^{3/2}$$

$$\therefore |\Sigma|^{0/2} + |\Sigma|^{1/2} + |\Sigma|^{2/2} + |\Sigma|^{3/2}$$

$\therefore$ After Generalization

$$|\Sigma|^{0/2} + |\Sigma|^{1/2} + |\Sigma|^{2/2} + \dots + |\Sigma|^{n/2}$$

$$= \left[\frac{|\Sigma|^{n/2+1} - 1}{|\Sigma| - 1} \right]$$

for odd let $n=9$

$$n=0 \rightarrow |\Sigma|^{0/2} \times |\Sigma|$$

$$n=2 \rightarrow |\Sigma|^{1/2} \times |\Sigma|$$

$$n=4 \rightarrow |\Sigma|^{2/2} \times |\Sigma|$$

$$n=8 \rightarrow |\Sigma|^{4/2} \times |\Sigma|$$

$$= \left[\frac{|\Sigma|^{n+1/2} - 1}{|\Sigma| - 1} \right] \times |\Sigma|$$

PREFIX - it is set of front part of string

$$\text{Prefix of } w = \{ u \mid w = uv \}$$

$$\text{if } w = 00, \text{ prefix of } w = \{ \epsilon, 0, 00 \}$$

Null is a prefix of string always & string itself a prefix.

Suffix Back part

$$\text{Suffix of } w = \{ v \mid w = uv \}$$

$$w = 001, \text{ suffix} = \{ \epsilon, 1, 01, 001 \}$$

Suffix can be null & the string itself.

$$\text{Ques - } \text{Prefix}(w) \cap \text{Suffix}(w) = \{ \epsilon, w \}$$

~~False~~ False
as

$$w = 000$$

$$\text{prefix}(w) = \{ \epsilon, 0, 00, 000 \}$$

$$\text{suffix}(w) = \{ \epsilon, 0, 00, 000 \} \neq \{ \epsilon, w \}$$

$$\{ \epsilon, w \} \subseteq \text{prefix}(w) \cap \text{suffix}(w)$$

yes it is

Ques If a string of length n , $\text{prefix}(w) = n+1$

$$\{ \text{no of elem in } \text{prefix}(w) = n+1 \}$$

$$\text{Ques - } \{ \text{string of length } n, \text{ suffix}(w) = n+1 \}$$

cardinality

- Prefix & Suffixes are always unique.

Proper Prefix - if $w = '00'$

$\text{prefix}(w) \neq \{0, \epsilon\} \Rightarrow$ proper prefix
i.e. w is excluded.

every prefix other than string itself

proper suffix - same

$\therefore \{ \text{Max length} = n \}$
 $\downarrow$ cardinality

substring - A substring s of w ,

$$s = \{z \mid uzv\}$$

- set of 0 or more consecutive symbol of given w .
- Prefix & suffix both are substring.
- Null is always a substring.

$w = abcd$, $s = \{ \epsilon, a, b, c, d, ab, bc, cd, abcd \}$

but $s \neq \{abd\}$ as not consecutive

- substring may or may not be unique.

$\{ \text{prefix}(w) \cup \text{suffix}(w) \} = \text{subset of substring}$

ques - a string of length n with no repeated symbol.
No. of repeated sub substring of length n

$$n=0 = 1 \rightarrow 1$$

$$n=1 = \epsilon, a \rightarrow 2$$

$$ab \quad n=2 = \epsilon, ab, ba, aa, bb \rightarrow 5$$

$\therefore \{ \text{No. of substring of length } n = 5 \}$

No of ^{sub} string possible = $\frac{n(n+1)}{2} + 1$

No of subwords without $\epsilon = \frac{n(n+1)}{2}$

Ques

No of substring of string of length n , with repetition of symbol in it?

AXIOMATIZABLE

AXIOMATIZABLE

$n = 13$

A - 3 times

I = 2 time

no of diff word = $13 - 2 - 1$
= 10

$\therefore$ 1 length $\rightarrow 10$

2 length $\Rightarrow$ check for repetition

AX	AT	AB
MA	ZA	
IO	IZ	
IX	TI	

$\rightarrow$ only different element

(no repetition around)

$\therefore$ Now for 12 bcoz

No repeated words

12 possible combination for 2 length

$$\Rightarrow \frac{12 \times (12+1)}{2}$$

$$= 78$$

$$78 + 10 = 88$$

If AX

AX then we will subtract 1 $12 - 1 = 11$

We do not check for Repetition we were not checking for 3 length

for 3 length bcoz when no repetition for as no repiten
2 there is no chance for repetition of in 2 as well
string of length 3.

EXCELLENT

Powers of string - it is basically concatenation.

w^{-1} as $w \cdot w^{-2}$ nothing such can happen.

$w^0 = \epsilon$ (it means it is independent of what actually string is)

$$w^1 = w$$

$$w^2 = w \cdot w$$

$$w^3 = w \cdot w^2$$

$$= w \cdot w \cdot w$$

$$w^m \cdot w^n = w^{m+n}$$

$$(w^m)^n = w^{mn}$$

$$[w = 01 \quad w^2 = 0101]$$

w^{-1} is not possible
as for $w^0 = w \cdot w^{-1}$

$$\uparrow$$

$$\epsilon = w \cdot w^{-1}$$

nothing is possible which
after concatenation
result into
 ϵ (null
string)

$$((01)^2(100)^3)^R$$

$$= (0101100100100)^R$$

$$= 0010010011010$$

$$= ((001)^3(10)^2)$$

$$\left[(u^m v^n)^R = (v^R)^n (u^R)^m \right]$$

II. Σ^* , Σ^+

Σ^* - Every possible string on given alphabet.

• also known as star closure / Kleen's closure / iteration.

$$\Sigma = \{0\}$$

$$\Sigma^* = \{\epsilon, 0, 00, 000, 0^9, \dots\}$$

$$\Sigma = \{0, 1\}$$

$$\Sigma^* = \{\epsilon, 0, 1, 00, 01, 10, 11, 000, \dots\}$$

$$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \dots$$

• Σ^* is always infinite. It can never be finite.

 Σ^+ - positive closure

$$\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \dots$$

$$\Sigma = \{0\}$$

$$\Sigma^+ = \{0, 00, 000, \dots\}$$

$$\boxed{\Sigma^+ = \Sigma^* - \epsilon}$$

it does not contain null string

$$(\Sigma^+ \cup \epsilon = \Sigma^*)$$

Ques - $(\Sigma^+, \cdot) \rightarrow$ it has identity element (no)
as it does not contain ϵ .

$(\Sigma^*, \cdot)$ - it contains identity element (yes)

~~Closed - Groupoid~~

~~CL + Assoc~~

if $u \in \Sigma^*$, $v \in \Sigma^*$ then $u.v \in \Sigma^*$

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(Closed - Groupoid)

Closed + Associative - semigroup

Closed + Associative + Identity $\rightarrow$ Sem Monoid

Closed + Associative + Identity + Inverse $\rightarrow$ Group

Closed + Associative + Identity + Inverse + commutative -
- Abelian Group

$\Sigma^+ \rightarrow$ semigroup

$\Sigma^* \rightarrow$ Monoid

• Inverse of a string does not exist.

$(\mathbb{Z}, +)$ - Abelian Group

13. Representation of Languages

12. $L \subseteq \Sigma^*$ $\Rightarrow$ bunch of strings on Σ of specific meaning for Grammar to them.

Ex-

$\Sigma = \{0\}$

$\Sigma^* = \{\epsilon, 0, 00, 000, \dots\}$

$L_1 = \{\epsilon\}$ - Null language - length = 0 (cardinality)

$L_2 = \{\epsilon\} \rightarrow$ Language having null string

$\downarrow$ length No of elemt = 1

$|L_1| = 0$

$|L_2| = 1$

$L_3 = \{0, 00\}$

$L_4 = \{00, 0000, 0^5\}$

$\rightarrow$ Finite

A Language can be finite or infinite

$L = \{0^{2n} \mid n \geq 0\}$

∞ infinite

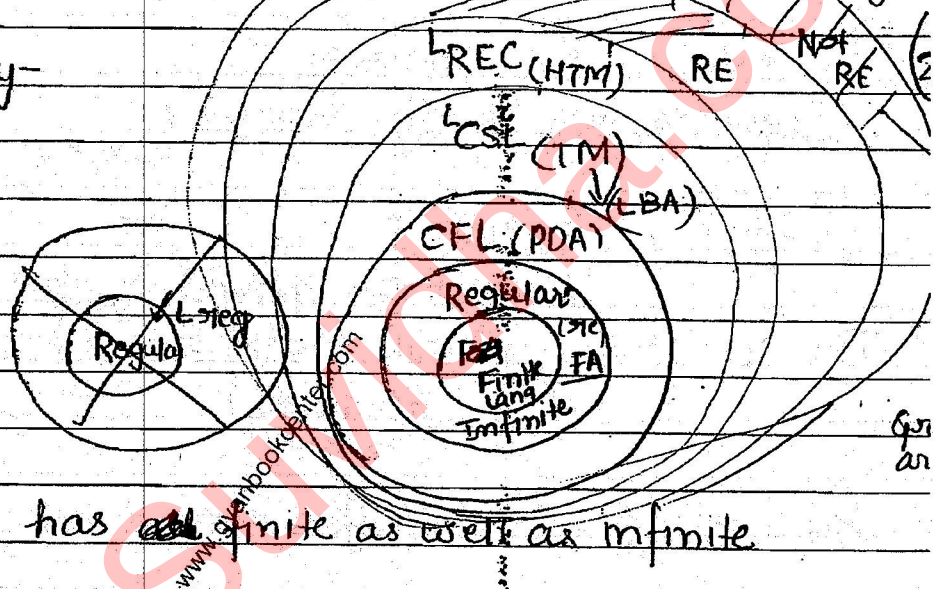
EXCELLENT

$\{0^{2^n}\}$ $0^+ \cdot^*$

When some restrictions are putted on a language Σ^* , then it forms a language.

Regular Language - A language which can be represented using s.e.s known as (Regular language)

Chomsky Hierarchy -



Regular language has ~~all~~ finite as well as infinite automata.

Regular Expression $*, +, \cdot, (), 0, 1$

(it does not allow intersection & complement)

- Regular Expression is one which for which FA can be defined.
- FA has only finite memory i.e. limited Memory.
- Each R-L is a finite automata
- Each finite automata is RL but each RL is not FA.
- Each CFL is not RL but each RL is CFL.

$L = \{0^n 1^n \mid n \geq 0\}$ CFL but not RL as it requires memory for remembering n no. of Zero.

EXCELLENT