SPUR GEAR

LECTURE - 5
13/06/2017

The aim of this topic is to determine module of gear.

(1) Addendum = 1m

(2) Dedendum = 1.157 m

3) face width b = 10 m

4) clearance = 0.157 m

(5) $P_c = \pi m$

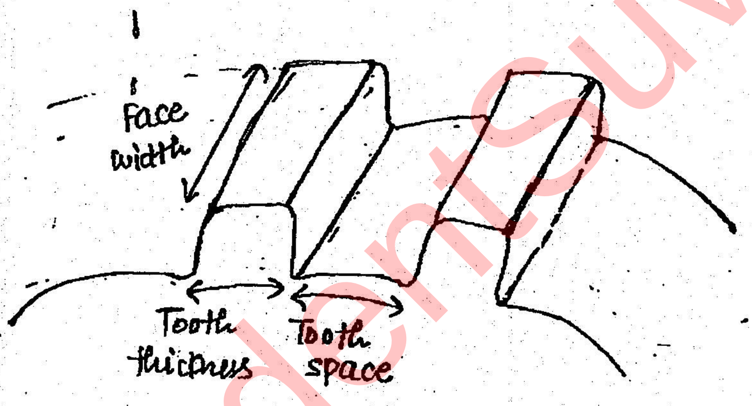
(6) $P_d = \frac{1}{m}$

(7) $P_c \cdot P_d = \pi$

(8) $m = \frac{D}{Z}$

(9) T = Torque.

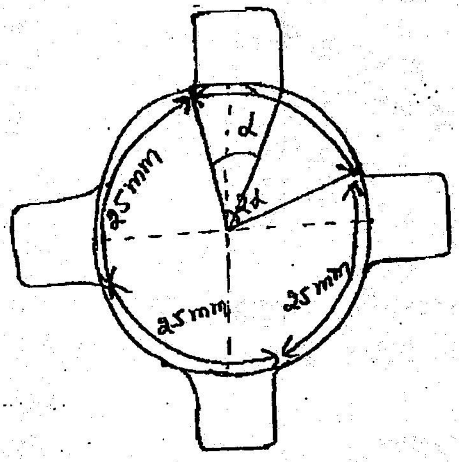
where :- m → module
Z → No. of teeth.



$\boxed{\text{Tooth space} - \text{Tooth thickness} = \text{Backlash}}$

Tooth space ≃ Tooth thickness

eg.



$\pi D = 100mm$

$Z = 4$

$P_c = \pi m = \frac{D}{Z}\pi$

$P_c = \underline{25 mm}$

Tooth space + Tooth thickness = P_c

$$\boxed{\text{Tooth thickness} = \text{Tooth space} = \frac{P_c}{2}}$$

$$2\alpha = \frac{360}{Z}$$

$$\boxed{\alpha = \frac{360}{2Z}}$$

angle caused by Tooth thickness/Tooth space on center

⇒ when two gears are mating together their circular pitch must be equal.

$P_{c_1} = P_{c_2}$

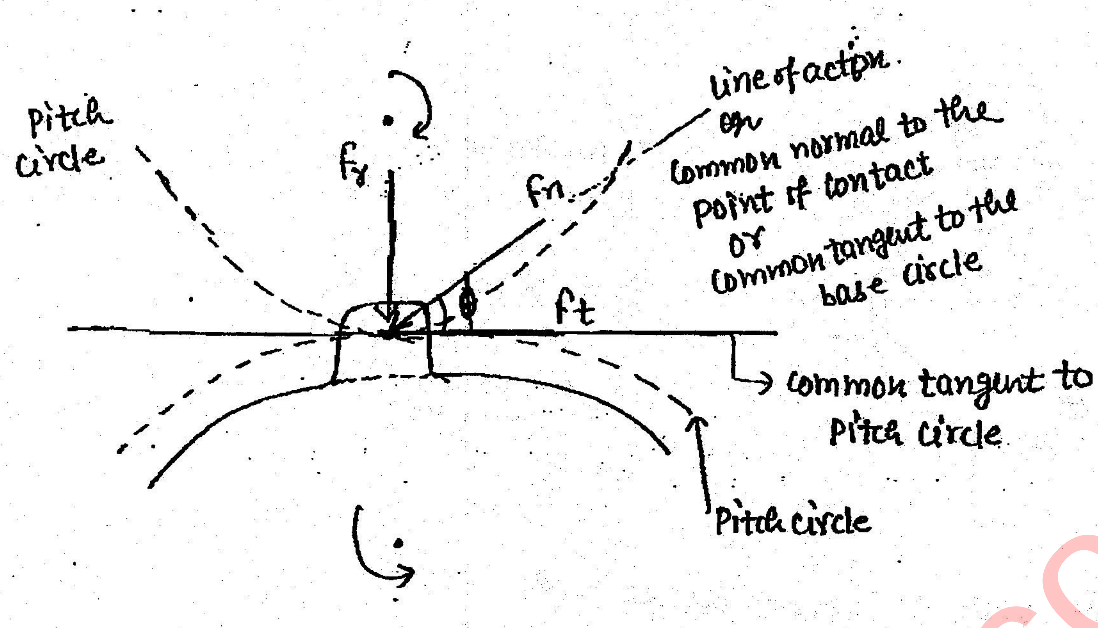
$\pi m_1 = \pi m_2$

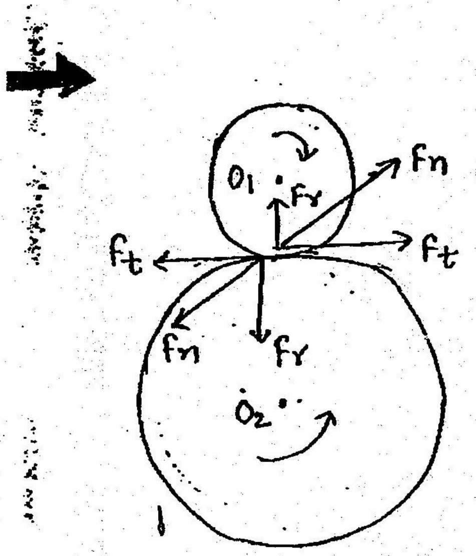
$\boxed{m_1 = m_2}$

Force analysis used for Gear tooth :-

$$\boxed{\begin{aligned} F_t &= F_n \cos\phi \\ F_n &= \frac{F_t}{\cos\phi} \end{aligned}}$$

$$\boxed{\begin{aligned} F_r &= F_n \sin\phi \\ F_r &= F_t \tan\phi \end{aligned}}$$





Pinion Z_1

$m_1 = m_2 = m$

Gear Z_2

Pinion:- $P_1 = \frac{2\pi N_1 T_1}{60}$

$T_1 =$ known

$T_1 = F_t \times \frac{D_1}{2}$

$F_t = \frac{2T_1}{D_1}$, $\left[\begin{array}{l} m = \frac{D_1}{Z_1} \\ D_1 = mZ_1 \end{array}\right]$

$F_t = \frac{2T_1}{mZ_1}$

$F_t =$ known

$F_n = \frac{F_t}{\cos\phi}$

$F_n =$ known

$F_r = F_t \tan\phi$

$F_r =$ known

Gear :-

$$T_2 = F_t \times \frac{D_2}{2} = F_t \times \frac{mZ_2}{2}$$

T_2 = known

Gear Ratio :-

$$\boxed{G.R = \frac{Z_2}{Z_1} = \frac{D_2}{D_1} = \frac{N_1}{N_2}}$$ always

Case 1 :- when $\eta_m = 100\%$

* If nothing mentioned in question then $\eta_m = 100\%$.

$$Power_2 = Power_1$$

$$P_2 = P_1$$

$$\frac{2\pi N_2 T_2}{60} = \frac{2\pi N_1 T_1}{60}$$

$$\boxed{\frac{N_1}{N_2} = \frac{T_2}{T_1}}$$

$\therefore$ $$\boxed{G.R = \frac{Z_2}{Z_1} = \frac{D_2}{D_1} = \frac{N_1}{N_2} = \frac{T_2}{T_1}}$$ when $\eta_{mechanical} = 100\%$.

Case 2 :- when $\eta_m \neq 100\%$

η_m = given

$$P_2 = \eta_m \cdot P_1$$

$$\frac{2\pi N_2 T_2}{60} = \eta_m \cdot \frac{2\pi N_1 T_1}{60}$$

$$\frac{N_1}{N_2} = \frac{T_2}{T_1 \eta_m}$$

$\therefore$ $$\boxed{G.R = \frac{Z_2}{Z_1} = \frac{D_2}{D_1} = \frac{N_1}{N_2} = \frac{T_2}{\eta_m T_1}}$$

$$\boxed{\begin{array}{l}\text{Resultant force on pinion} = F_n \\ \text{Resultant force on gear} = F_n\end{array}}$$

ues .13

$N_1 = 1440$ rpm

$R.R = 10:1$

$N_2 = 144$ rpm

$P_2 = \eta_m P_1$

$$\frac{2\pi N_2 T_2}{60} = \eta_m \times \frac{2\pi N_1 T_1}{60}$$

$$\frac{2\pi(144)56.36}{60} = \eta_m \times 1\times10^3$$

$$\boxed{\eta_m = 85\%}$$

ues .29

Power = 3kw

$\omega = 200$ rad/sec

$P = T\omega$

$3\times10^3 = T\times 200$

$T = 15$ Nm

$$F_t = \frac{2T}{D} = \frac{2\times15}{105}$$

$F_t = 600$ N

$$F_n = \frac{F_t}{\cos\phi} = \frac{600}{\cos 20}$$

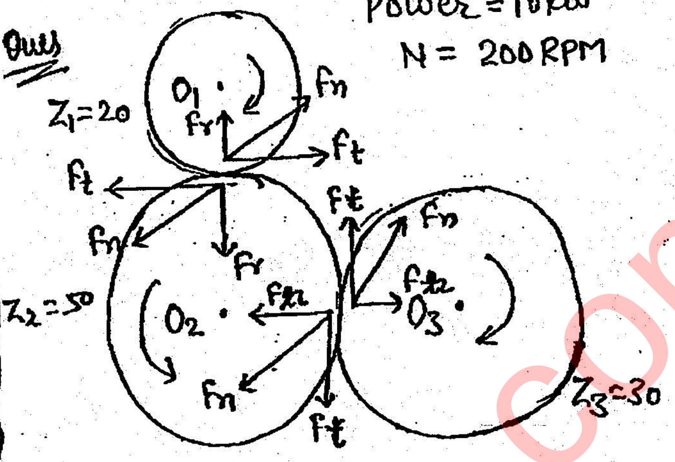
$$\boxed{F_n = 638.5\text{ N}}$$

Ques

m = 5mm

Power = 10kw

N = 200 RPM



Find out the resultant force on gear 1, gear 2 and gear 3. Also find torque of gear 1, 2, 3.

Sol^n: Second gear is called idler gear. (एक से Power लेके दूसरे मे transmit कर देता है)(O_1 से लेके O_3 मे)

$\therefore$ Net power of O_2 gear = 0

$\therefore$ $\boxed{T_2 = 0}$

Gear 1:- $P = \frac{2\pi N_1 T_1}{60}$

$$10\times10^3 = \frac{2\pi(200)T_1}{60}$$

$$\boxed{T_1 = 477.46\text{ N-m}}$$

$$F_t = \frac{2T_1}{D_1} = \frac{2\times477.46}{5\times20\times10^{-3}}$$

$F_t = 9549.2$ N

$$F_n = \frac{F_t}{\cos\phi} = 10162.1\text{ N}$$

$$F_r = F_t \tan\phi$$

$$\underline{F_r = 3475.6\,N}$$

<u>Gear 2 :-</u>

$$T_2 = 0$$

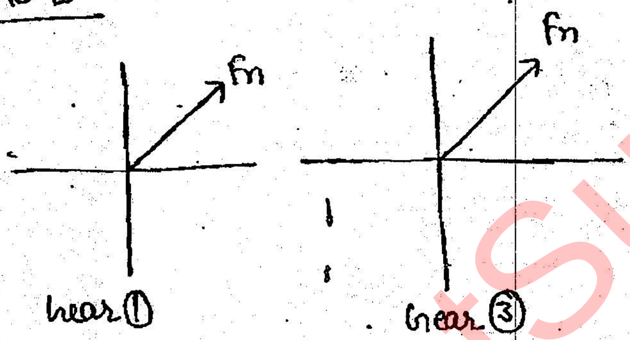
<u>Gear 3 :-</u>

$$T_3 = F_t \times \frac{D_3}{2}$$

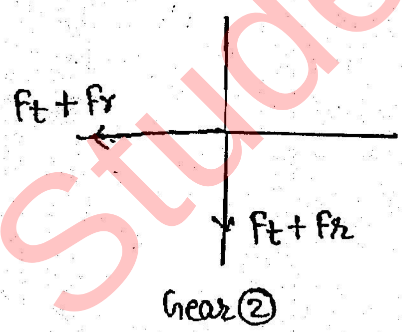
$$= 9549.2 \times \frac{5 \times 10^{-3} \times 30}{2}$$

$$\boxed{T_3 = 719.19\ Nm}$$

<u>F.B.D</u>



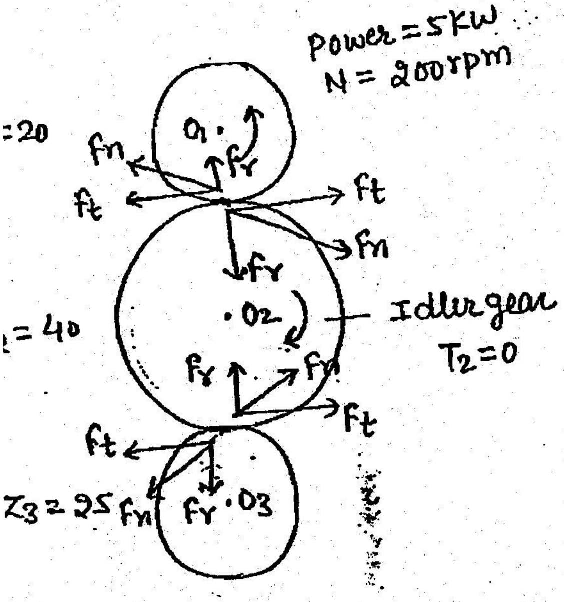
$$\boxed{R_1 = R_3 = 10162.1N}$$



$$R_2 = \sqrt{2}\,(F_t + F_r)$$

$$\boxed{R_2 = 18419.8\ N}$$

$m=5, \phi=20^\circ$



$$\text{Power} = \frac{2\pi N_1 T_1}{60}$$

$$5\times10^3 = \frac{2\pi \times 2000 \times T_1}{60}$$

$$\boxed{T_1 = 238.73\ N\cdot m}$$

$$F_t = \frac{2T_1}{D_1} = \frac{2T_1}{mZ_1}$$

$$F_t = \frac{2\times238.73}{5\times20\times10^{-3}} = 4774.6\ N$$

$$F_n = \frac{F_t}{\cos\phi}$$

$$\boxed{F_n = 5081.07\ N}$$

$$F_r = F_t \tan\phi$$

$$= 4774.6 \tan 20^\circ$$

$$\boxed{F_r = 1737.8\ N}$$

Gear 2:

$$\boxed{T_2 = 0}$$

Gear-3

$$T_3 = F_t \times \frac{D_3}{2}$$

$$= 4774.6 \times \frac{5\times25\times10^{-3}}{2}$$

$$\boxed{T_3 = 298.41\ Nm}$$

$$\boxed{R_1 = R_3 = F_n = 5081.07\ N}$$

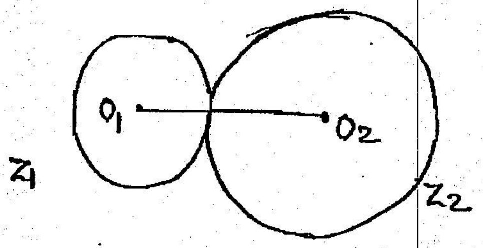
Gear 2

$$R_2 = 2F_t$$

$$= 2\times 4774.6$$

$$\boxed{R_2 = 9549.6\ N}$$

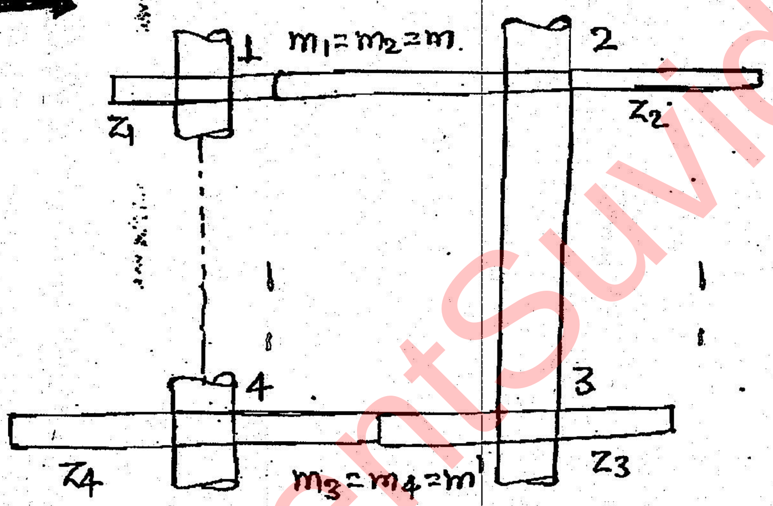
Concept on Centre Distance :-



$$CD = O_1O_2 = r_1 + r_2$$

$$CD = \frac{mZ_1}{2} + \frac{mZ_2}{2}$$

$$\boxed{CD = \frac{m(Z_1+Z_2)}{2}}$$



$m_1 = m_2$ (Because meshing Gear)

$m_3 = m_4$ (meshing Gear)

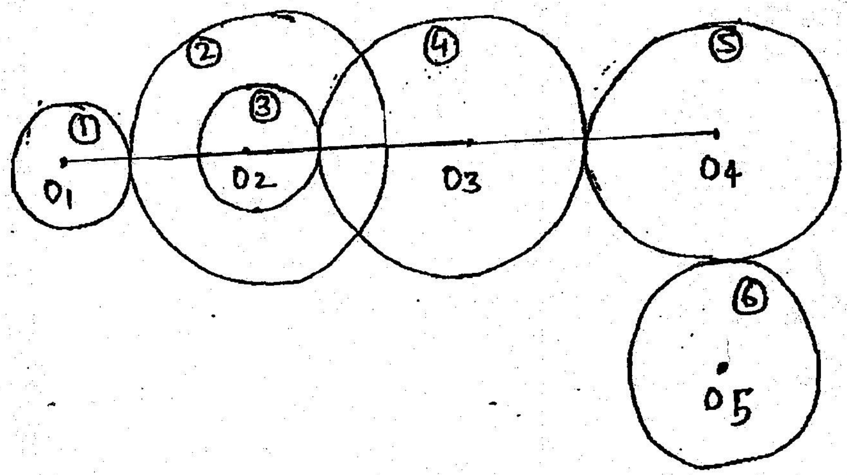
$$r_1 + r_2 = r_3 + r_4$$

$$\boxed{\frac{m(Z_1+Z_2)}{2} = \frac{m(Z_3+Z_4)}{2}}$$

and

$\boxed{\omega_2 = \omega_3}$ (Because Gear 2 and Gear 3 are on same shaft)

⇒



⇒ $m_1 = m_2$

⇒ $\omega_2 = \omega_3$

⇒ $m_3 = m_4 = m_5$

⇒ $O_1O_4 = r_1 + r_2 + r_3 + 2r_4 + r_5$

⇒ $O_4O_5 = r_5 + r_6$

∴ ⇒ $O_1O_5 = \sqrt{(O_1O_4)^2 + (O_4O_5)^2}$

Ques 5.18

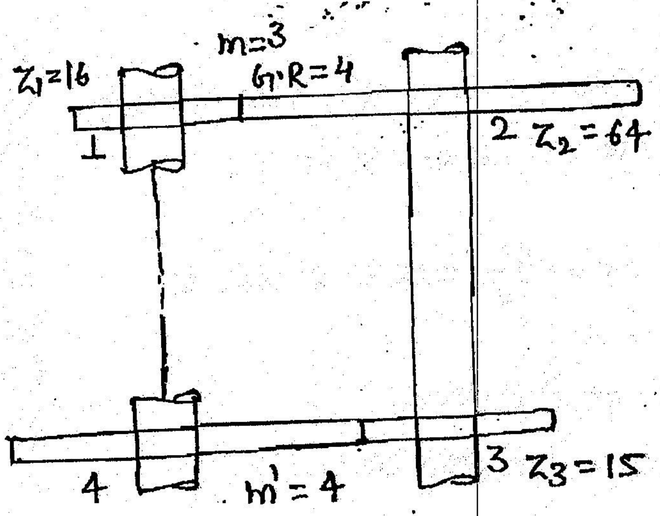
$$\text{Arc of Contact} = \frac{\text{Path of Contact}}{\cos\phi}$$

$$\text{Contact Ratio} = \frac{\text{Arc of contact}}{P_c}$$

$$= \frac{19}{\cos 20 \times \pi \times 5}$$

$$\boxed{C.R = 1.28}$$

Ques 5.12



$$CD = \frac{m(z_1+z_2)}{2} = \frac{m'(z_3+z_4)}{2}$$

$$\frac{3(16+64)}{2} = \frac{4(15+z_4)}{2}$$

$$\boxed{z_4=45}$$

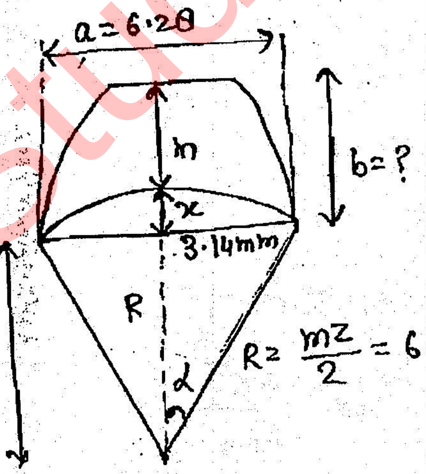
and $\boxed{CD = 120\,mm}$

Ques 5.22

$$\text{Tooth thickness} = \frac{P_c}{2}$$

$$a = \frac{\pi \times 4}{2}$$

$$\boxed{a = 6.28\,mm}$$



$$R = \frac{mZ}{2} = 64\,mm$$

$$b = m + x \quad \text{——— (1)}$$

$$x = R - y$$

and $y = \sqrt{R^2 - (3.14)^2}$

$$= \sqrt{64^2 - (3.14)^2}$$

$$y = 63.92\,mm$$

$\therefore$ $x = 64 - 63.92$

$$x = 0.08\,mm$$

$\therefore$ $\boxed{b = 4.08\,mm}$

OR $\alpha = \frac{360}{\pi \times 2 \times 2} = 2.8$

$$y = R\cos\alpha$$

then $x = R - y$

NOTE :-

There is maximum chance of interference in case of Rack & pinion arrangement because the addendum circle radius of Rack is infinity.

Hence always design any gear by assuming rack & pinion to avoide interference.

$$\boxed{T_{min} = \frac{2a_r}{\sin^2\phi}}$$

$a_r = 1 \rightarrow$ Full depth

$a_r = 0.8 \rightarrow$ For stub tooth

* Full depth and $\phi = 20^\circ \Rightarrow T_{min} = 18$ teeth

Stub tooth and $\phi = 20^\circ \Rightarrow T_{min} = 14$ teeth

Full depth and $\phi = 14.5^\circ \Rightarrow T_{min} = 32$ teeth

Stub tooth and $\phi = 14.5^\circ \Rightarrow T_{min} = 26$ teeth

DESIGN OF SPUR GEAR

ere
B.M → max^m
↓
itical
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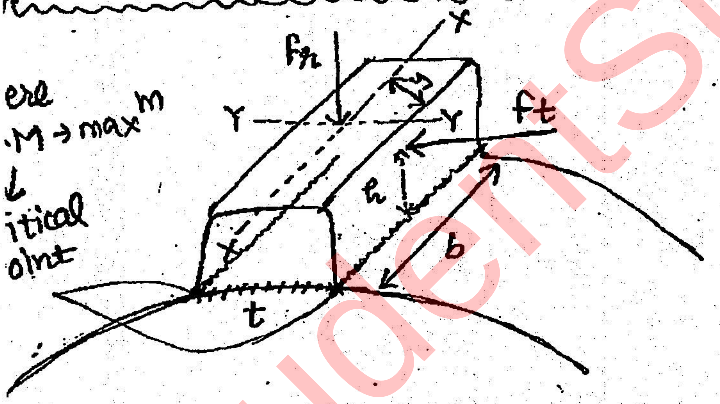
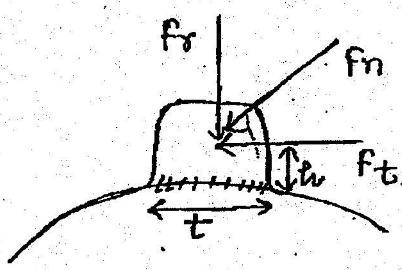




or Gear Tooth :-

$F_r =$ Axial compressive load $\Rightarrow$ compressive stress

$F_t = T.S.L$ — Constant shear force $\Rightarrow$ Direct shear stress

— variable Bending Moment $(F_t \cdot x)$

⇓

Bending stress (Normal stress)

$$\tau_{dir.} = \frac{FA\bar{y}}{I_{NA} \cdot b}$$

Conclusion :-

(1) Due to axial compressive force gear tooth is subjected to compressive stress.

(2) Due to constant shear force F_t gear tooth is ~~this~~ subjected to direct shear stresses (effect of T.S.L)

(3) Due to variable Bending Moment $(F_t \cdot x)$ gear tooth is subjected to variable bending stresses.

(4) While designing a gear tooth the effect of direct shear stress and compressive stress will be neglected only bending stress will be taken into consideration.

$$(\sigma_b)_{max} = \frac{M_{max} \cdot y_{max}}{I_{NA}}$$

$$M_{max} = F_t \times h$$

$\Downarrow$

$M_{x\text{-}x}$

$$I_{NA} = I_{x\text{-}x} = \frac{bt^3}{12}$$

and $y_{max} = \frac{t}{2}$

$$(\sigma_b)_{max} = \frac{(F_t h)(t/2)}{bt^3/12}$$

$$\boxed{(\sigma_b)_{max} = \frac{6 F_t h}{bt^2}}$$

Safe Condition :-

$$(\sigma_b)_{max} \le \sigma_{per}$$

$$\frac{6 F_t h}{bt^2} \le \sigma_{per}$$

$$F_t \le \frac{bt^2}{6h} \sigma_{per}$$

$$(F_t)_{max} = \frac{bt^2}{6h} \sigma_{per}$$

$$(F_t)_{max} = mb \cdot \left(\frac{t^2}{6hm}\right) \sigma_{per}$$

$$\left(\frac{t^2}{6hm}\right) = Y \rightarrow$$ Lewi's form factor OR Tooth geometry factor

$$(F_t)_{max} = mbY (\sigma_{per})_b$$

and $(\sigma_{per})_{bending} = (\sigma_b)$

$$\therefore \boxed{(F_t)_{max} = mbY [\sigma_b]}$$

Beam strength of gear tooth

OR Safe condition

$$\boxed{F_{actual} \leq (F_t)_{max}}$$

BEAM STRENGTH :-

It is defined as the maximum tangential load that a gear tooth can wear without any bending failure.

Lewi's Form Factor (Y) :-

$\boxed{Y = \pi y}$ where $y \rightarrow$ Tooth form factor

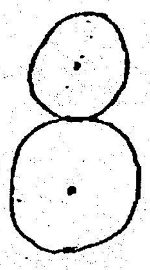
$y = \left(0.154 - \frac{0.912}{z}\right)$ for full depth and $\phi = 20^\circ$

$$\boxed{Y = \pi\left[0.154 - \frac{0.912}{z}\right]}$$ for full depth and $\phi = 20^\circ$

Hence lewi's form factor depends on no. of teeth, pressure angle and geometry of the tooth profile. (Basically module)

$\therefore$ $Z_p < Z_G$

and $Y_p < Y_G$



$(F_t)_{max} = 45$

$(F_t)_{max} = 60$

$$\boxed{(F_t)_{max} = m\,b\,Y\,[\sigma_b]}$$

onclusion :-

1) weaker gear is a gear which has minimum value of beam strength and always design for weaker gear.

2) when pinion and gear both are made of same material then $(\sigma_b)_p = (\sigma_b)_G$

$m_1 = m_2$

$b_1 = b_2$

$$Y_P < Y_G$$

$$\therefore \ [(F_t)_{max}]_P < [(F_t)_{max}]_G$$

Hence pinion is weaker so desigen for pinion.

(3) when pinion and gear both are made of different materëal

then $(\sigma_b)_P \neq (\sigma_b)_G$

then compare the product of $Y[\sigma_b]$ and design for the gear which has minimum value of this product.

Actual load :-

$$\text{Power} = \frac{2\pi NT}{60}$$

T = known

$$F_t = \frac{2T}{D} = \frac{2T}{mZ}$$

F_t = known $\Rightarrow$ Static load

But Actual load = Dynamic load

F_d = actual load or Dynamic load

$$\boxed{F_d > F_t}$$

$$\boxed{F_d = F_t \cdot S \cdot C_v}$$

Here $S \rightarrow$ service factor

$C_v \rightarrow$ velocity factor

- $C_v = \frac{3+V}{3}$ when $V \leq 10\ m/s$
- $C_v = \frac{6+V}{6}$ when $V > 10\ m/s$

* when the value of S & C_v greater than 1 $\rightarrow$ multiplied

* when the value of S & C_v less than 1 $\rightarrow$ Devided.

(Because $F_D > F_t$)

Safe Condition :-

$$F_{act} \leq (F_t)_{max}$$

$$\boxed{(F_t \times C_v \times \delta) \leq b m Y [\sigma_b]}$$

$\frac{us}{.20}$ Power = 3 kw

$n = 20$ rps

$$P = \frac{2\pi NT}{60}$$

$$P = \frac{2\pi NT}{60} = 2\pi n T$$

$$3 \times 10^3 = 2\pi (20) \times T$$

$$T = 23.87 \text{ N-m}$$

$$F_t = \frac{2T}{mZ} = \frac{2 \times 23.87}{3 \times 16 \times 10^{-3}}$$

$$F_t = 994.71 \text{ N}$$

$$F_t \times C_v \times \delta = b m Y [\sigma_b]$$

$$994.71 \times 1.5 \times 1 = 36 \times 3 \times 0.3 [\sigma_b]$$

$$\boxed{\sigma_b = 46.05 \text{ MPa}}$$

Assumptions Made in Lewi's Equation :-

1. The effect of direct shear and compressive ~~shear~~ stress are neglected.
2. Gear tooth assume as a cantilever beam.
3. Gear tooth assume prismatic throughout.
4. Contact Ratio assume as 1.

5. The effect of stress concentration factor are neglected.

Reason for Dynamic Load :-

1. Deflection of the tooth under load.
2. Inaccuracy of the tooth profile.
3. Presence of stress concentration factor.
4. Presence of Jerk and impact.
5. Inertia of the Rotating part

WEAR STRENGTH OF THE GEAR TOOTH :-

It is defined as the maximum value of the load that a gear tooth can bear without any wear.

$$\boxed{F_w = D_p \cdot b \cdot Q \cdot K}$$

wear strength

Bear strength is always check for pinion because pinion is subjected to more wear.

$D_p \rightarrow$ Pitch dia. of Pinion

$b \rightarrow$ Face width

$Q \rightarrow$ Ratio factor

$$\boxed{Q = \frac{2G}{G \pm 1}}$$

(+) ⟶ External Gearing
(−) ⟶ Internal Gearing

$G \rightarrow$ Gear ratio

$K \rightarrow$ material combination factor

$$\boxed{K = \frac{(\sigma_{es})^2 \sin\phi \left[\frac{1}{E_P} + \frac{1}{E_G}\right]}{1.4}}$$

$\sigma_{es} \rightarrow$ surface endurance limit

$\phi \rightarrow$ pressure angle

$E_p, E_G \rightarrow$ young's modulas of pinion and Gear

Safe condition :-

$F_{act} \leq F_W$

$F_d \leq F_W$

$$\boxed{(F_t \times C_v \times f_s) \leq D_p \cdot b \cdot Q \cdot K}$$

Eg: $(F_t)_{max} = 60KN$

$F_W = 50KN$

$F_{act} \leq 50KN$

Types of WEAR :-

1) Abrasive wear :- (mostly occur in open gear)

This failure occurs due to presence of Foreign material due to lubrication or dust deposit on the mating tool.

2) Scoring wear :-

This failure occurs due to metal to metal contact between two gears due to fail of lubrication.

3) Corrosive wear :-

This failure occurs due to chemical reaction b/w lubrication lubricant) and mating gear.

4) Pitting wear :-

This failure occurs due to repeative stresses under cyclic loading.