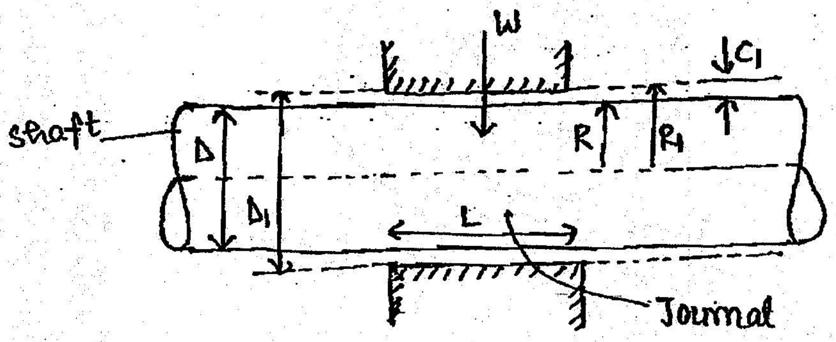
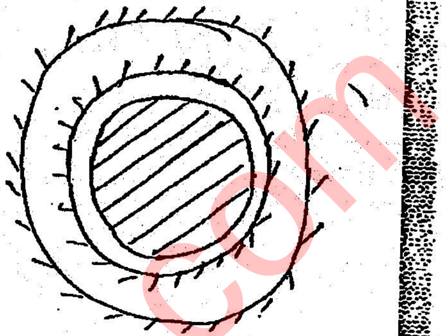
Imp.

JOURNAL BEARING

Journal Bearing is a sliding contact radial load bearing generally operating with hydrodynamic lubrication.





R = Radius of shaft / Journal

R_1 = Radius of bearing

D = Dia. of shaft / Journal

D_1 = Dia. of bearing

W = radial load

L = length of Journal / Bearing

C_1 = Radial clearence

$$\boxed{C_1 = R_1 - R}$$

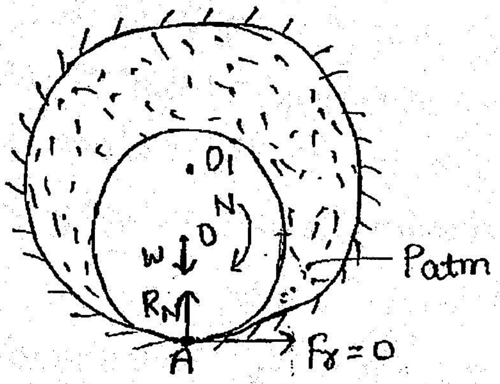
C = Diametrical clearence

$$\boxed{C = D_1 - D = 2C_1}$$

Hydrodynamic lubrication :-

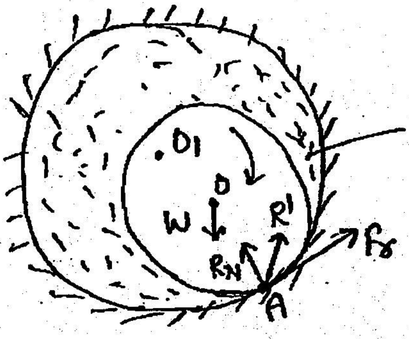
Case I :- Stationary condition

$N = 0$



$$\boxed{R_N = W}$$

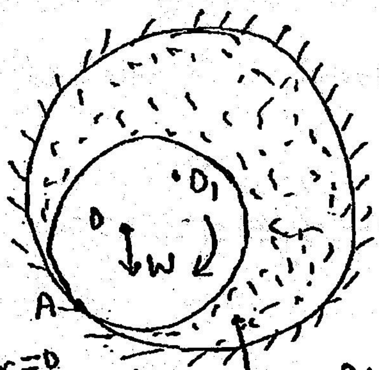
Case II :- $N_1 > N = 0$



$R' = W$

Case III $N_2 > N_1 > N = 0$

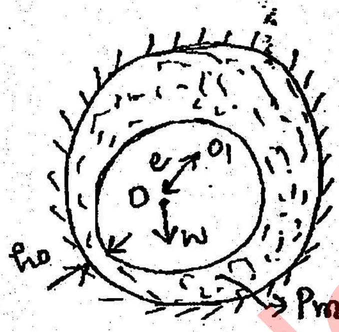
Just want to lift



Case III :- $N_{max} > N_2 > N_1 > N = 0$

Journal Bearing at Dynamic Condition

Journal Bearing at max^m speed condition



h_o = minimum film thickness

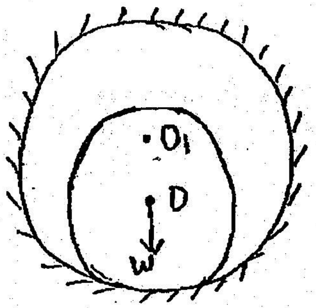
Conclusion :-

1) As the speed of shaft increases pressure of lubricant also increases and this liquid lubricant is converting into a sticky film and when this lubricant moves from wider space to narrow space pressure of the lubricant becomes maximum. This phenomena of lubricant is known as wedging action or converging-action or dynamic action.

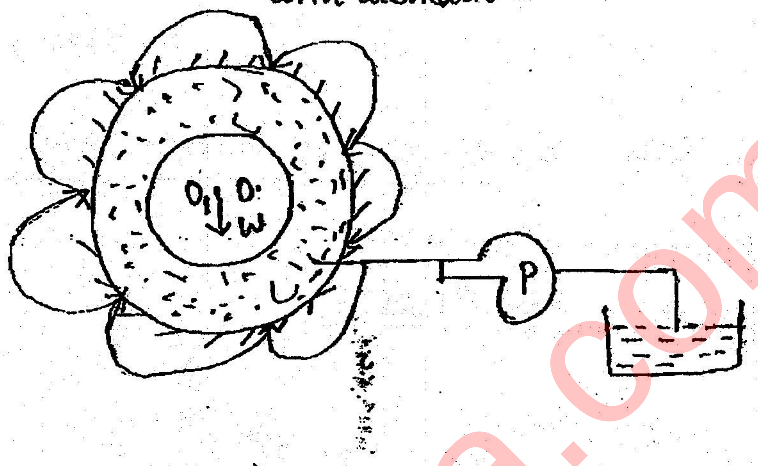
and this high pressure is responsible to lift the shaft.

Hydrostatic lubrication :-

Case I :- Stationary condition without lubrication



Case II :- Stationary condition :- with lubrication



Hydrdynamic lubrication	Hydrostatic lubrication
⇒ lubricant is supplied into the bearing at atmospheric pressure.	⇒ At higher pressure.
⇒ pressure of the lubricant increases due to wedging action/convergent action of the lubricant.	⇒ pressure of the lubricant increases by an external device like pump.
⇒ Metal to metal contact is avoided at high speed hence suitable for highly speed application.	⇒ metal to metal contact is avoided at stationary condition.
⇒ motion of the shaft is ecentric with respect to the bearing.	⇒ motion of the shaft is cocentric with respect to the bearing.
⇒ cost of the lubrication is less	⇒ cost of lubrication is more.
⇒ starting torque is high	⇒ Starting torque is less.
⇒ It is used in Application where shaft is subjected to lighter load at stationary condition.	⇒ It is used in application where shaft is subjected to higher load at stationary condition.

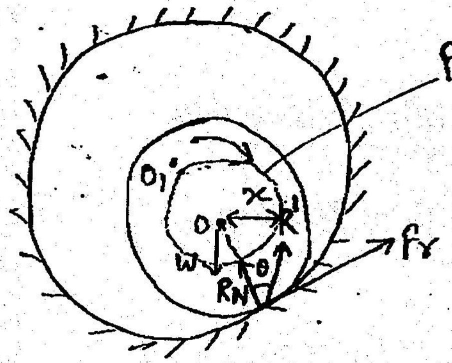
⇒ It is used in most of the industrial application like IC engine crank shaft, steam and gas turbine, electric motor etc.

⇒ It is used in heavy machinery like bowl mills in thermal power plant, vertical turbo generator, large size concrete mixtures etc.

Frictional Torque and Concept of friction circle in Journal Bearing

Case II:- [Because max^m loss occurs]

Design of Bearing for max^m loss



$R' = W$

$\therefore T_f = Wx$

$\sin\theta = \frac{x}{R}$

$x = R\sin\theta$

$T_f = WR\sin\theta$

θ = very less

$\sin\theta = \theta = \tan\theta$

$T_f = WR\tan\theta$

and $\tan\theta = \mu$

$\therefore$ $T_f = \mu WR$

Power loss $= T_f \times \omega$

$= \mu WR \cdot \omega$

Power loss $= \mu WV$

OR $V = \frac{\pi DN}{60}$

$x \rightarrow$ friction circle radius

$x = R\sin\theta = R\tan\theta$

$x = R\mu$

$\therefore$ $x = f(R, \mu)$

Conclusion -

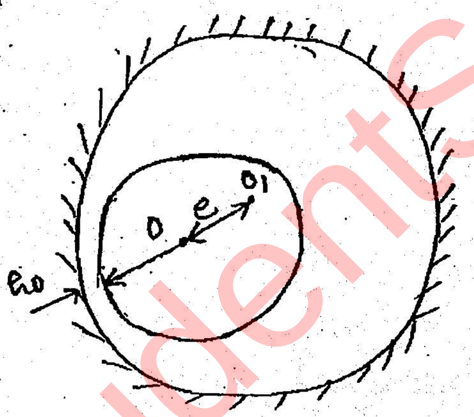
(i) when the shaft is in stationary condition (in absence of friction) reaction offerred by the bearing is inline with line of action of the load acting from the Journal.

(2) when shaft is in motion (presence of friction) the resultant force R' (Resultant of Normal reaction and friction) is deviated from the line of action of the load acting from the Journal by a distance of x which is known as friction circle radius.

(3) A circle drawn from centre of the shaft by taking radius x is known as friction circle.

Terminology Used In Journal Bearing :-

Journal Bearing at dynamic Condition



$$e + R + h_0 = R_1$$

→ **Ecentricity :-** The distance b/w centre of the shaft and centre of the bearing.

$$e + R + h_0 = R_1$$

$$e + h_0 = R_1 - R$$

$$e = C_1 - h_0$$

$$e = \frac{C}{2} - h_0$$

Ecentricity Ratio (ϵ) :- (Attitude)

It is defined as the ratio of ecentricity to radial clearence.

$$\epsilon = \frac{e}{C_1}$$

$$= \frac{\frac{C}{2} - h_o}{\frac{C}{2}}$$

$$\boxed{\epsilon = 1 - \frac{2h_o}{C}}$$

Bearing clearence :-

C = Diametrical clearence
C_1 = Radial clearence

If no mentioned in question then clearence = Diametrical clearence

Clearence ratio :- (Diametrical clearence ratio) or $\left[\frac{C}{D}\right]$ ratio

$C\uparrow \quad \left(\frac{C}{D}\right)\uparrow \quad$ power loss $\downarrow \quad W\downarrow$

$C\downarrow \quad \left(\frac{C}{D}\right)\downarrow \quad$ power loss $\uparrow \quad W\uparrow$

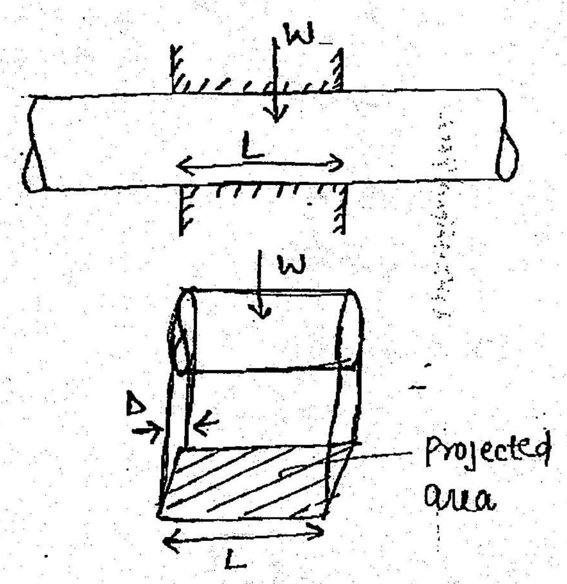
$\therefore$

$$\boxed{0.001 \le \left(\frac{C}{D}\right) \le .002}$$

Bearing Pressure :-

$$\text{Pressure} = \frac{\text{load}}{\text{Projected area}}$$

$$\boxed{P_{ind} = \frac{W}{L \cdot D}}$$



Safe condition

$$P_{ind} \le P_{per}$$

$\frac{W}{L \cdot D} \leq P_{per}$

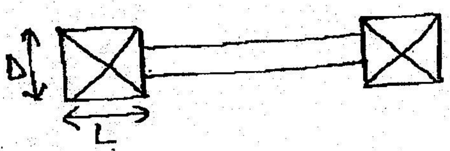
$$W_{max} = L \cdot D \cdot P_{per}$$

Strength of Journal Bearing

$D = D_1$

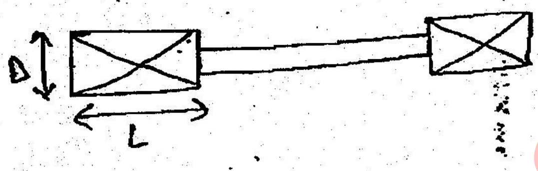
$\left[\frac{L}{D}\right]$ Ratio :- D → Bearing Diameter

$\frac{L}{D} < 1$, Short Bearing



$\frac{L}{D} = 1$, square Bearing

] Simple Support



$\frac{L}{D} > 1$, Long Bearing] fixed support

Beam ⇒ Support

shaft ⇒ Bearing

Point load	UDL	
$\delta = \frac{WL^3}{48EI}$	$\delta = \frac{5WL^4}{384EI}$	Simple Supported
$\delta = \frac{WL^3}{192EI}$	$\delta = \frac{WL^4}{384EI}$	Fixed Support

Power loss :-

$$T_f = \mu W R$$

$$\text{Power loss} = \mu W V$$

Coefficient of friction (μ) :-

$$\mu = f\left[\left(\frac{Zn}{P}\right), \left(\frac{D}{C}\right), \left(\frac{L}{D_1}\right)\right]$$

By Mcc kee's

$$\mu = 2\pi^2\left(\frac{Zn}{P}\right)\left(\frac{D}{C}\right) + K$$

$\left(\frac{Zn}{P}\right)$ → Bearing characteristics No.

Z → absolute viscocity of lubricant (Pa-s)

n → speed of shaft "rps" (revolution per sec)

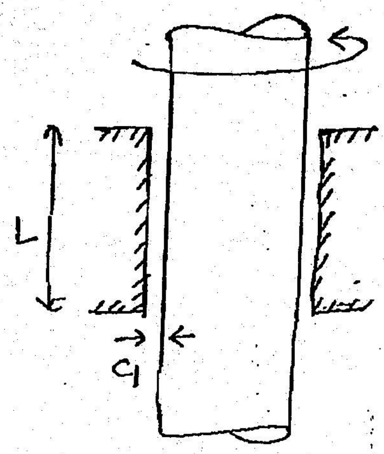
K → leakage factor

$K = 0.002$ when $.75 \le \frac{L}{D_1} < 2.8$

$K = -0.003$ when $\frac{L}{D_1} \ge 2.8$

Derivation of Mcc-kee's :-

$$\tau = \mu \frac{dv}{dz}$$ F.M



$$\tau = Z \frac{dv}{dz}$$

$$\frac{F_r}{A} = Z \frac{V}{C_1}$$

$$\frac{f_r}{2\pi RL} = \frac{z\,\pi DN}{\frac{c}{2}}$$

$$f_r = \frac{2\pi^2 zn \cdot D \cdot R \cdot L}{\frac{c}{2}}$$

and

$$T_f = f_r \times R$$

$$T_f = \frac{2\pi^2 zn D R^2 L}{\frac{c}{2}} \quad \text{--- (1)}$$

$$T_f = \mu W R \quad \text{--- (2)}$$

eq(1) = eq(2)

$$\mu W R = \frac{2\pi^2 zn D R^2 L}{\frac{c}{2}}$$

$$\boxed{\mu = 2\pi^2 \left(\frac{zn}{P}\right)\left(\frac{D}{c}\right)}$$

⇒ Sommerfield NumBER:

Sommer field number remains constant for a given bearing in different condition hence it is used to corelate working condition of Different machine operating with same bearing.

$$\boxed{S = \frac{zn}{P}\left(\frac{D}{c}\right)^2}$$ (unit less)

$$\boxed{S_1 = S_2}$$ for a given bearing.

* A Bearing is defined by its Sommerfield number.

{GATE BOX}

$$T_f = \mu W R$$

$$\text{Power loss} = \mu W V$$

$$P_{ind} = \frac{W}{L \cdot D}$$

$$\mu = 2\pi^2\left(\frac{Zn}{P}\right)\left(\frac{D}{c}\right)$$

$$S = \frac{Zn}{P}\left(\frac{D}{c}\right)^2$$

Shaft Diameter

$D = 50 - 50\times10^{-3}$
$= 49.95\,mm$

Ques A natural feed Journal Bearing of Diameter 50mm and length 50mm operating at 20 rps carries a load of 2KN. The lubricant used has a viscosity of 20 mPa-s and the radial clearence is 50 µm. The sommerfield No. for the bearing is ——.

$$S = \frac{Zn}{P}\left(\frac{D}{c}\right)^2$$

$Z = 20\times10^{-3}$ Pa-s

$n = 20$ rps

$$P = \frac{W}{LD} = \frac{2000}{(.05)(.05)}$$

and $c_1 = 50\,\mu m$

$$S = \frac{20\times10^{-3}\times 20}{\frac{2000}{(.05)^2}}\left(\frac{.05}{2\times50\times10^{-6}}\right)^2$$

$$\boxed{S = 0.125}$$

Ques A Journal Bearing has a shaft Diameter of 40mm and length of 40mm. The shaft is rotating at 20 rad/sec. The lubricant used has a viscocity of 20 mPa-sec and the radial clearence is .020 mm. The loss of the torque due to viscocity of the lubricant.

$w = 2\pi n$

$$T_f = \mu W R$$

and
$$\mu = 2\pi^2\left(\frac{Zn}{P}\right)\left(\frac{D}{c}\right)$$

$$\therefore \quad T_f = 2\pi^2\left(\frac{Zn}{\frac{W}{LD}}\right)\cdot\left(\frac{D}{c}\right)\cdot WR$$

$$T_f = 2\pi^2 (Zn)\left(\frac{D}{c}\right)(LD)\cdot\frac{D}{2}$$

$$= 2\pi^2\left(20\times10^{-3}\times\frac{20}{2\pi}\right)\left(\frac{.04}{.02\times10^{-3}\times2}\right)(.04)^2\left(\frac{.04}{2}\right)$$

$$\boxed{T_f = .04 \text{ N-m}}$$

Ques A lightly loaded full Journal Bearing has Journal diameter of 50mm, bush bore of 50.05mm and bush length of 20mm. If rotational speed of the Journal is 1200 rpm and the average Viscocity of the lubricant is 0.03 Pa-sec. The power loss in watt will be ——.

$$c = 50.05 - 50 = .05 \text{ mm}$$

$$\text{Power loss} = \mu W V$$

$$= 2\pi^2\left(\frac{Zn}{\frac{W}{LD}}\right)\left(\frac{D}{c}\right)W\cdot V$$

$$= 2\pi^2 (Zn)\left(\frac{D}{c}\right)(LD)V$$

$$= 2\pi^2 (0.03)\left(\frac{1200}{60}\right)\left(\frac{.05}{.05\times10^{-3}}\right)(.02)(.05)\left(\frac{\pi\times.05\times1200}{60}\right)$$

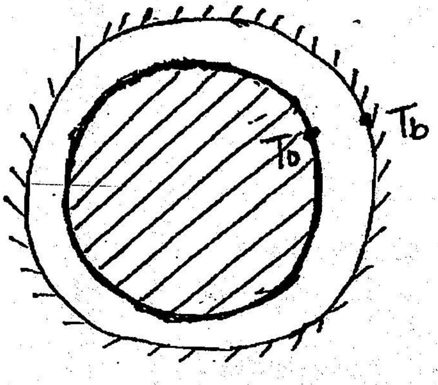
$$\boxed{\text{Power loss} = 37.2 \text{ W}}$$

Mass flow rate of coolant Required in Journal Bearing :-

T_o → maximum Temp. required that lubricant wear without failure

T_a → ambient temp.

T_b → Outer temperature (surface) of the bearing



Powerloss = Heat Generated

Heat Dessipiated :-

$H \cdot D = h A \Delta T$

$HD = h(\pi D L)(T_b - T_a)$

$HD = C_D D L (T_b - T_a)$

$C_D = (A h)$ watt/m^2-K → Heat Dissipiation coeffient.

Let us assume

$$T_b - T_a = \frac{T_o - T_a}{2}$$

$$H \cdot D = C_D D L \frac{(T_o - T_a)}{2}$$

if $\left[\begin{matrix} HG < HD \\ HG = HD \end{matrix}\right]$ No Coolant Required

If $H_G > H_D$

Hence coolant Required

$$\dot{m}_c C_{P_c} (\Delta T)_{coolant} = H_G - H_D$$

$\dot{m}_c$ = known

Ques A full Journal bearing lubricated with a lubricant having viscosity = 0.04 pa-sec. Journal length and Diameter both are equal to 60 mm and supports the shaft rotating at 2000 rpm. The max^m Temp. that lubricant can wear is 70°c and room temp. is 25°c. Assuming $\frac{c}{D}$ ratio .001 and Heat Dissipation co-efficient 400 W/m^2K.

Determine →

(i) Rate of Artificial cooling required.

(ii) mass flow rate of the coolant required if temp. difference of coolant b/w outlet and inlet is 25°c and Cp for coolant 1.8 $\frac{KJ}{kgK}$.

Powerloss = HG

$H_G = \mu W V$

$$= 2\pi^2 \left(\frac{ZN}{\frac{W}{LD}}\right)\left(\frac{D}{C}\right) \cdot W \, \frac{\pi D N}{60}$$

$$= 2\pi^2 (ZN) \left|\frac{D}{C}\right| LD \cdot \frac{\pi D N}{60}$$

$$= 2\pi^2 \left(.04 \times \frac{2000}{60}\right)(1000)(.06)^2 \frac{\pi (.06)(2000)}{60}$$

$H \cdot G = 595.32$ Watt

$$H_D = C_D \cdot DL \left(\frac{T_0 - T_a}{2}\right)$$

$$H \cdot D = 400(0.06)^2\left(\frac{70-25}{2}\right)$$

$H \cdot D = 32.4$ kW

$A \cdot C = H \cdot G - H \cdot D$

$$A \cdot C = 562.92 W$$

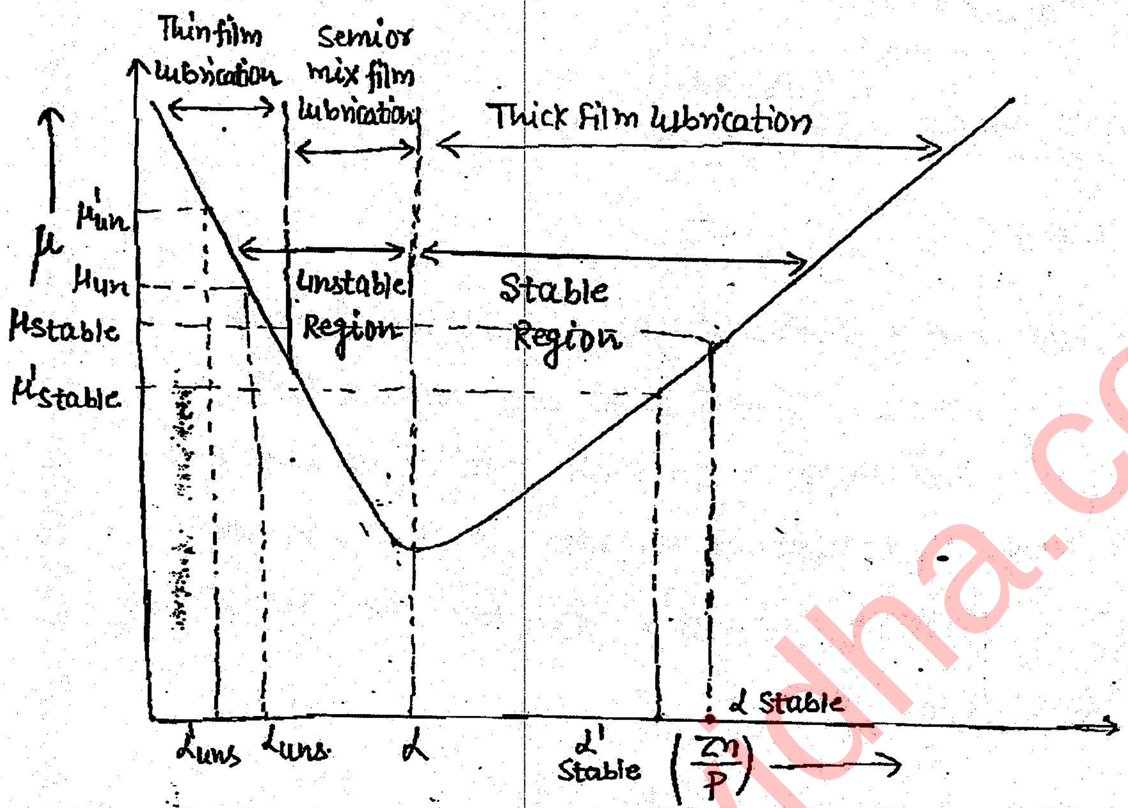
$$\dot{m}_c C_{P_c} (\Delta T)_{coolant} = A \cdot C$$

$$\dot{m}_c \times 1.8 \times 10^3 \times 25 = 562.92$$

$$\dot{m}_c = 12.5 \; g/sec$$

Significance of Bearing Characterstic Number

LECTURE-3
11/06/2017

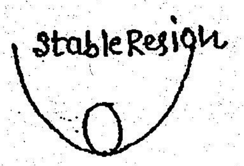


Unstable Region

Load ↑ → Power loss ↑ → Temp ↑ → Z ↓ → $\left(\frac{Zn}{P}\right)$ ↓ → μ ↑ → Power loss ↑

Stable Region

oad ↑ → Powerloss ↑ → Temp. ↑ → Z ↓ → $\left(\frac{Zn}{P}\right)$ ↓ → μ ↓ → Power Loss ↓



For stable condition

$$\alpha = \left(\frac{Zn}{P}\right)_{min}$$

OR

$$\left(\frac{Zn}{P}\right)_{min} \leq \left(\frac{Zn}{P}\right)$$

Bearing modulus

In practical case → (For Design also)

$$3\alpha \leq \left(\frac{Zn}{P}\right) \leq 15\alpha$$

For static loading →

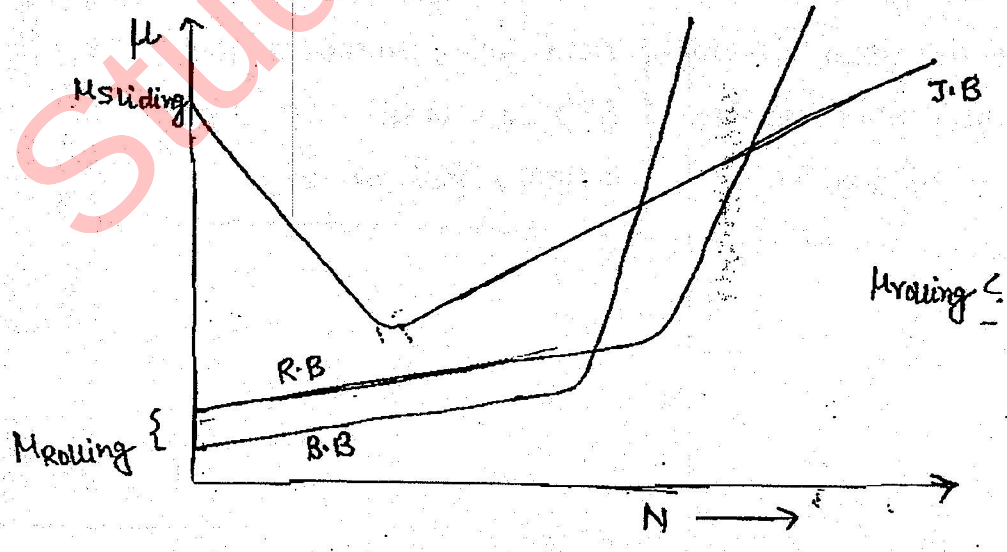
$$3\alpha \leq \left(\frac{Zn}{P}\right) \leq 5\alpha$$

For impact and fatigue loading →

$$12\alpha \leq \left(\frac{Zn}{P}\right) \leq 15\alpha$$

V.Imp

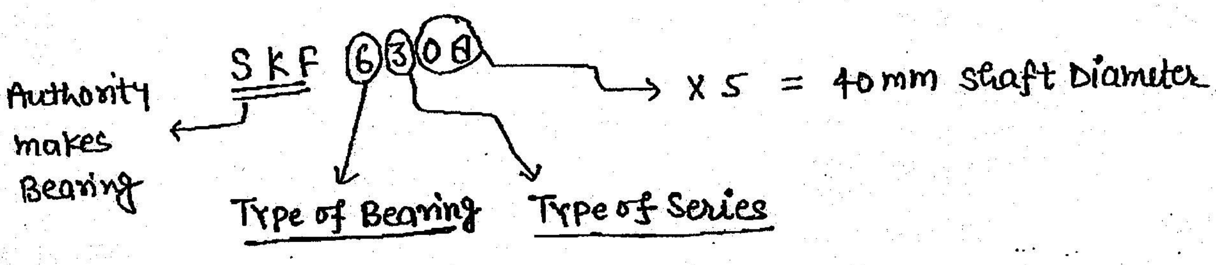
ANTI FRICTION BEARING



$\mu_{rolling} <<< \mu_{sliding}$

Parameter	Journal Bearing	Antifriction Bearing (Babbit material) (made by powder metallurgy process)
Speed	Used for high speed application.	Used for low and medium speed application.
Load	Only radial load	Radial and axial both
Cost	less	more
Machine service	Machine is continuing service	Intermittent service required.
Noise	Minimum from all Bearing.	Maximum noise.
Radial space	Less space	more space
Axial space	more space	less space
Damping capacity	more	less
Lubrication	Liquid lubricant and continuous lubrication.	Semi solid lubricant (grease) and periodic lubrication.
Type of failure	Sudden failure without indication.	Give indication before failure by making more noise.
Application	Steam and gas turbines, large size electrical motors, IC engine crank shaft, concrete mixtures.	Automobile front and rear axle, machine tool spindle, small size electrical motors, gear boxes.

Designation of Antifriction Bearing :-

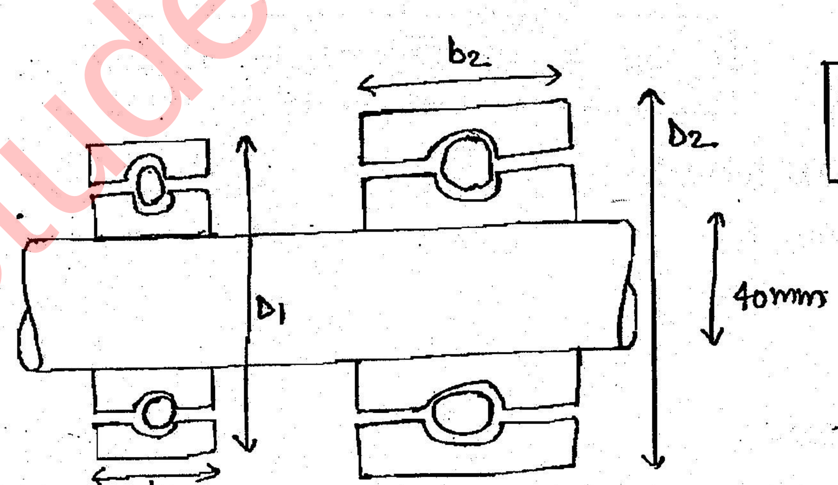


6 → DGBB
3 → Taper roller

Type of Series :-

Extra light series	light series	medium series	Heavy series	Extra heavy series
1	2	3	4	5
100 Series	200 series	300 series	400 series	500 series

Load capacity →
Dimensions →
Cost →



$b_2 > b_1$
$D_2 > D_1$

SKF 6308   SKF 6408

	Shaft Diameter
SKF 6300	10 mm
SKF 6301	12 mm
SKF 6302	15 mm
SKF 6303	17 mm

	By rule X 5
SKF 6304	20 mm
SKF 6305	25 mm

According Indian standard :-

IS (40) (BC) (03) → Type of Series

40 → Shaft Dia.

BC → Type of Bearing

BC → D.G.B.B

Different Terms used while selecting the Series for Antifriction Bearing :-

i) Equivalent Load :-

$$P_e \text{ or } P_m = S\,[X V F_r + Y F_a]$$

S → Service Factor or Shock factor

V → race rotation factor

F_r → Radial Load

F_a → Axial load.

X → Radial load factor

Y → Axial load Factor

S (Service factor):-

Steady load or No Shock S = 1

light shock → S = 1.5

Moderate shock → S = 2

Heavy Shock → S = 3

Extra heavy shock → S = 3.5

V (Race Rotation factor):-

If inner race rotate → V = 1

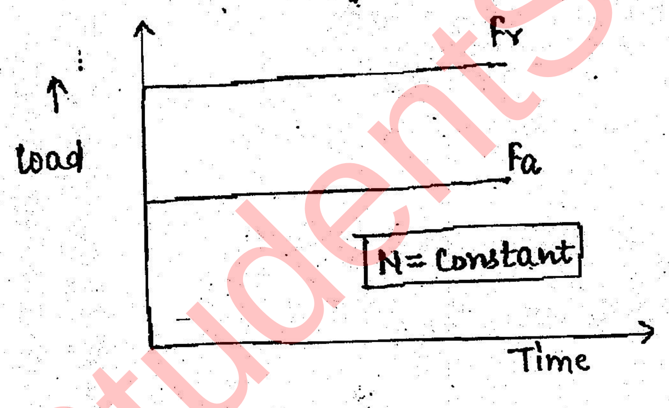
Outer race rotate → V = 1.2

X, Y :-

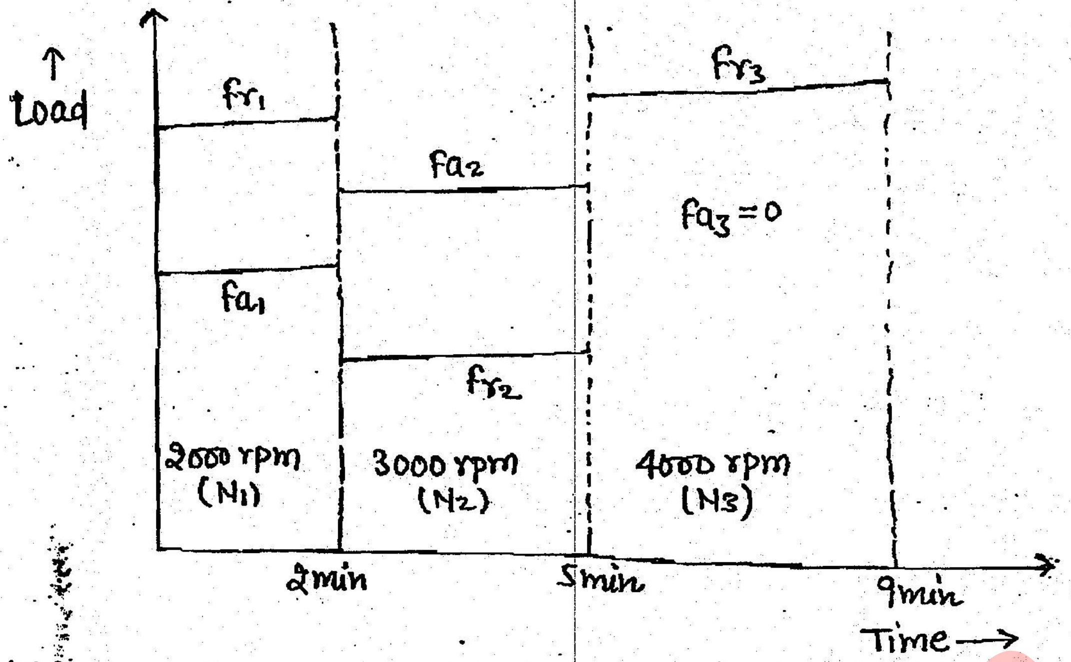
Thrust Ball Bearing → X = 0 , Y = 1

Cylindrical Roller Bearing → X = 1 , Y = 0

NOTE:-



The above Formula is valid only when load and speed remains constant with respect to time.



Now

$$Pe_1 = S_1 [X_1 V_1 Fr_1 + Y_1 fa_1]$$

$$Pe_2 = S_2 [X_2 V_2 Fr_2 + Y_2 Fa_2]$$

$$Pe_3 = S_3 [X_3 V_3 Fr_3]$$

Cubical Mean Load :-

$$P_e \text{ or } P_m = \sqrt[3]{\frac{n_1 Pe_1^3 + n_2 Pe_2^3 + n_3 Pe_3^3}{n_1 + n_2 + n_3}}$$

n_1, n_2, n_3 → No. of revolutions that a given bearing has undergone in respective region time T_1, T_2, T_3.

From above →
$n_1 = 4000$ rev
$n_2 = 9000$ rev
$n_3 = 16000$ rev

LIFE OF ANTIFRICTION BEARING :-

Life of antifriction Bearing is defined as the no. of revolution that a bearing has undergone before the evidence of first fatigue failure either in the races or in the rolling element.

Life of AFB = No. of revolution

Eg: Life ⇒ 1000 hr, 200 rpm

Life = 1000 × 60 × 200 rev

Nominal Life / Rated Life / L_{90} / L_{10} / Life / Life with 90% Reliability :-

- Always defined for group of identical Bearing.

Nominal life or Rated life for the group of identical Bearing is defined as the no. of revolution that 90% of group of identical Bearing can serve or exceed at a given speed without any failure.

eg: 1000 Bearings

$L_{90} = 400MR$ $\quad$ $L_{60} > L_{90}$

Relationship between L_{90} and L :-

$$\frac{L}{L_{90}} = \left[\frac{\ln\left(\frac{1}{R}\right)}{\ln\left(\frac{1}{R_{90}}\right)}\right]^{\frac{1}{1.17}}$$

$$\Rightarrow \frac{L_{60}}{L_{90}} = \left[\frac{\ln\left(\frac{1}{.6}\right)}{\ln\left(\frac{1}{.9}\right)}\right]^{\frac{1}{1.17}}$$

$$\boxed{L_{60} = 3.85\, L_{90}}$$

$$\Rightarrow \frac{L_{50}}{L_{90}} = \left[\frac{\ln\left(\frac{1}{.5}\right)}{\ln\left(\frac{1}{.9}\right)}\right]^{\frac{1}{1.17}}$$

Average life → $\boxed{L_{50} = 5\, L_{90}}$

: Average life 5 times greater than Nominal life.

$$\begin{bmatrix} L_{10} = L_{90} \\ L_{20} = L_{80} \\ L_{30} = L_{70} \\ L_{40} = L_{60} \\ L_{50} = L_{50} \end{bmatrix}$$

* Find the relation between L_{20} & L_{90}.

$$\frac{L_{20}}{L_{90}} = \left[\frac{\ln\left(\frac{1}{.2}\right)}{\ln\left(\frac{1}{.9}\right)}\right]^{\frac{1}{1.17}} \quad (X)$$

$$\frac{L_{20}}{L_{90}} = \frac{L_{80}}{L_{90}} = \left[\frac{\ln\left(\frac{1}{.8}\right)}{\ln\left(\frac{1}{.9}\right)}\right]^{\frac{1}{1.17}} \quad (\checkmark)$$

$$\frac{L_{20}}{L_{90}} = \frac{L_{20}}{L_{10}} = \left[\frac{\ln\left(\frac{1}{.2}\right)}{\ln\left(\frac{1}{.1}\right)}\right]^{\frac{1}{1.17}} \quad (X)$$

* GATE BOX

$$L_{90} = \left(\frac{C}{P_e}\right)^{K}$$

Dynamic Load Capacity [C]

OR

Dynamic Load Rating

It is Defined as the maximum load that 90% of the group of identical bearings can withstand for a minimum life of 1 million rev.

⇒ Dynamic load capacity remains constant for a given bearing

If $P_e = C \Rightarrow L_{90} = 1MR$

$P_e > C \Rightarrow L_{90} < 1MR$

$P_e < C \Rightarrow L_{90} > 1MR$

$$L_{90} = \left(\frac{C}{P_e}\right)^K$$

Put in MR

C → Dynamic load capacity

P_e → actual load

K = 3 → Ball Bearing

$K = \frac{10}{3}$ → Roller Bearing

P-214 Q-4.8

$L_{90} = 2000 \times 60 \times 600 = 72\ MR$

$C = 22\ KN$

$$L_{90} = \left(\frac{C}{P_e}\right)^K$$

We know that

$$P_e = \uparrow f_r\,\big|_{max} + \underbrace{F_a}_{=0}\downarrow_{min}$$

∴ $P_e = ?$

$$72 = \left(\frac{22}{P_e}\right)^3$$

$$\boxed{P_e = 5.29\ KN}$$

Ques A Single row deep groove ball bearing has a dynamic load capacity 40500 N and operating on following work cycle.

(1) Radial load of 15000 N at 500 rpm for 25% of the time.

(2). Radial load of 10000 N at 700 rpm for 50% of the time.

(3) Radial load of 7000 N at 400 rpm for the remaining time.

What is the expected life of bearing in hours.

Solⁿ

Load	Speed	Time	no. of rev.
(P_{e1}) $F_r = 15000$ N	500 rpm	.25 t	125 t = n_1
(P_{e2}) $F_r = 10000$ N	700 rpm	.5 t	350 t = n_2
(P_{e3}) $F_r = 7000$ N	400 rpm	.25 t	100 t = n_3

$$P_e = \sqrt[3]{\frac{125t(15000)^3 + (350t)(10000)^3 + (100t)(7000)^3}{125t + 350t + 100t}}$$

$$\boxed{P_e = 11192.32 \text{ N}}$$

$$L_{90} = \left(\frac{C}{P_e}\right)^3$$

$$= \left(\frac{40500}{11192.32}\right)^3$$

$$L_{90} = 47.38 \text{ MR}$$

$$N_{av} = \frac{\text{Total no. of rev.}}{\text{Total time}}$$

$$= \frac{575t}{t}$$

$$\boxed{N_{av} = 575 \text{ rpm}}$$

$$L_h \times 60 \times N_{av} = L_{90}$$

$$L_h \times 60 \times 575 = 47.38 \times 10^6$$

$$\boxed{L_h = 1373.3 \text{ hr}}$$

A BALL Bearing is anticipated to have a life of 400MR under a equivalent radial load 10KN with 80% Reliability.

Find out -

(1) If load is doubled find out the L_{30}.

(2) Find out the life with 60% reliability under the load of 15KN.

$P_e = 10KN$

$L_{80} = 400MR$

$$\frac{L_{80}}{L_{90}} = \left[\frac{\ln\left(\frac{1}{.8}\right)}{\ln\left(\frac{1}{.9}\right)}\right]^{\frac{1}{1.17}}$$

$L_{90} = 210.62MR$

Case 1:- $P_e' = 20KN$

$$L_{90} \propto \frac{1}{P_e^3}$$

$$L_{90}' = \frac{L_{90}}{8} = \frac{210.62}{8}$$

$L_{90}' = 26.32MR$

Now

$$\frac{L_{30}'}{L_{90}'} = \frac{L_{70}'}{L_{90}'} = \left[\frac{\ln\left(\frac{1}{.7}\right)}{\ln\left(\frac{1}{.9}\right)}\right]^{\frac{1}{1.17}}$$

$L_{30}' = 74.65MR$

Case 2 → $P_e'' = 15KN$

$$L_{90}'' = \frac{L_{90}}{(1.5)^3}$$

$L_{90}'' = 62.41MR$

$$\frac{L_{60}''}{L_{90}''} = \left[\frac{\ln\left(\frac{1}{.6}\right)}{\ln\left(\frac{1}{.9}\right)}\right]^{\frac{1}{1.17}}$$

$L_{60}'' = 240.55MR$