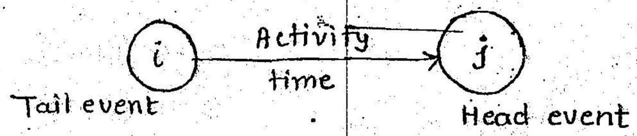
PERT and CPM

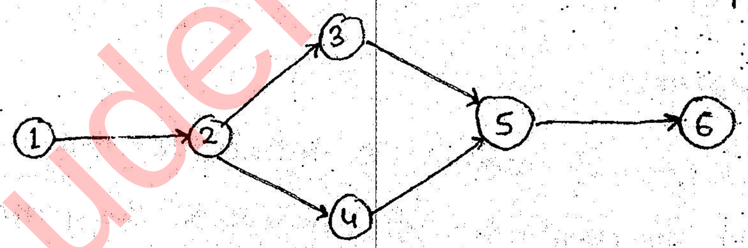


Project :- It is a group or combination of inter-related activities that must be executed in a certain fixed order before the entire task is completed. Activities are inter-related in a logical sequence in the sense that some activities can only be started when all the activities earlier to it are completed.

Event :- It denotes the point of time or accomplishment occurring at the moment and is used to denote the starting and the end point of an activity. Event neither consumes any time nor resources for its completion.

Activity :- It is the recognisible part of the project which consumes time and resources for its completion and it may involve physical or mental work.

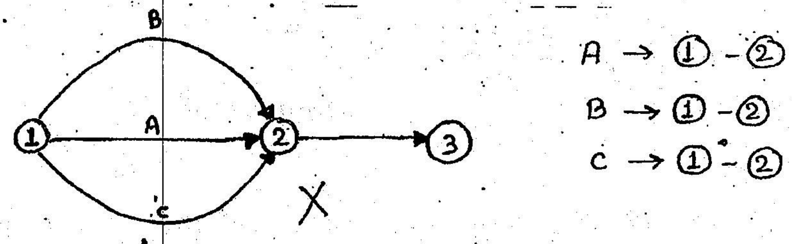
Network Diagram :- It is a graphical representation of the logical sequence in which different activities are inter-related to each other while completing a project.



Rule for network construction

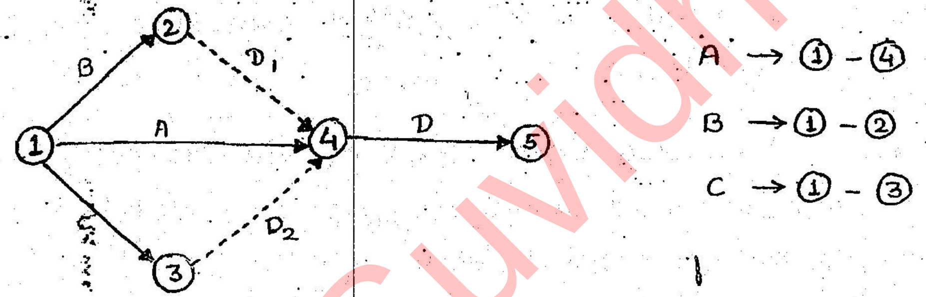
i) An activity can only be started when all activities earlier to it are completed.

ii) No two or more activities may have the same head and the tail events.



In these condition to represent the same logic we need to use dummy activity.

<u>Dummy Activity</u> :- An activity which is used to show the logic, dependancy or relationship of one activity over the other but does not consume any time or resources for its completion. It is represented by dotted line arrow.

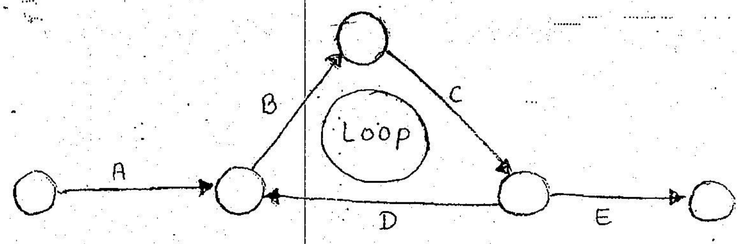


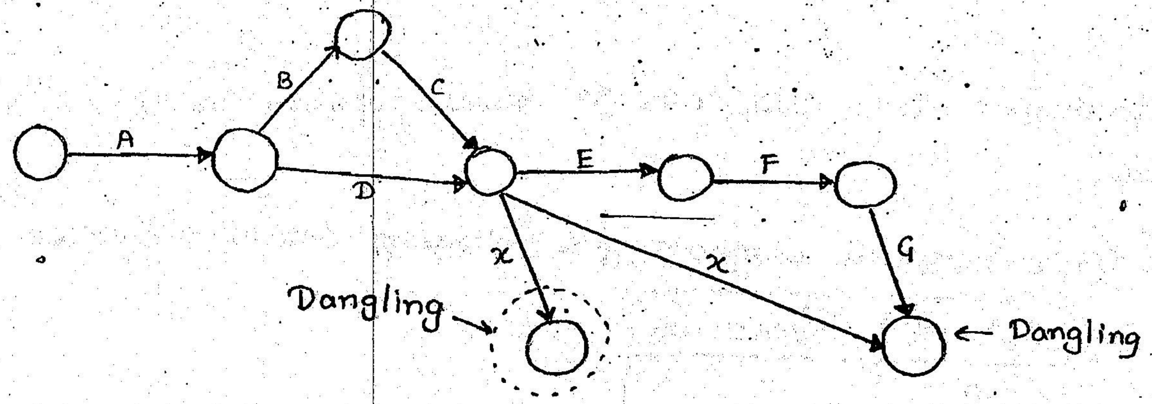
iii) Dummy activity should only be used when it is very necessary but there is no restriction on the no. of dummy activity used.

~~iv) In the network diagram~~

iv) The length and direction of arrow is indicative only and time flow from left to right on the network diagram

v) There should be no looping and dangling on the network diagram.

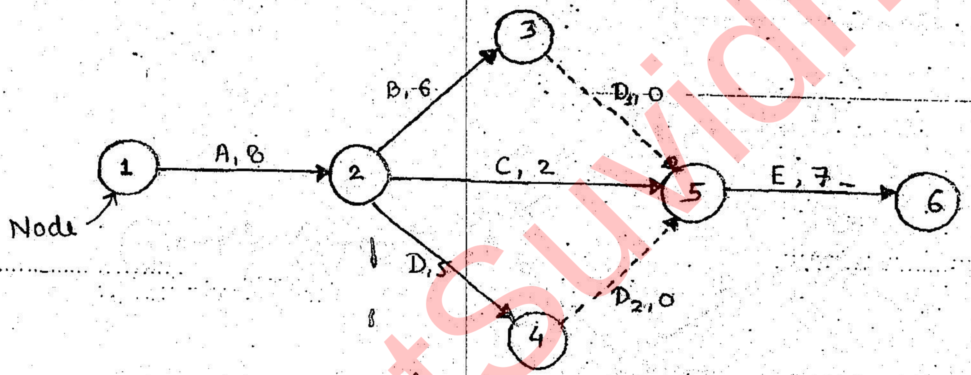




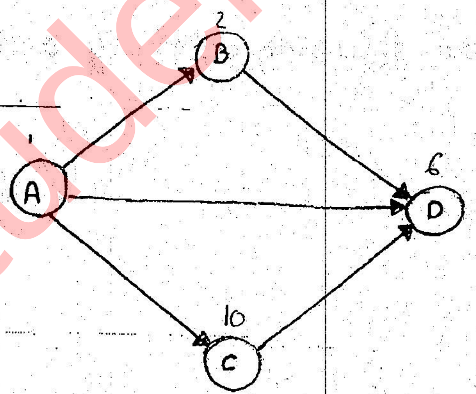
Dangling :- when activity other than final activity does not have any successor activity then the situation is called as dangling. Such activity should be connected directly to the last event of the network.

Types of Network Diagram :-

1) Event on Node (EON) or Activity on Arrow (AOA) :-



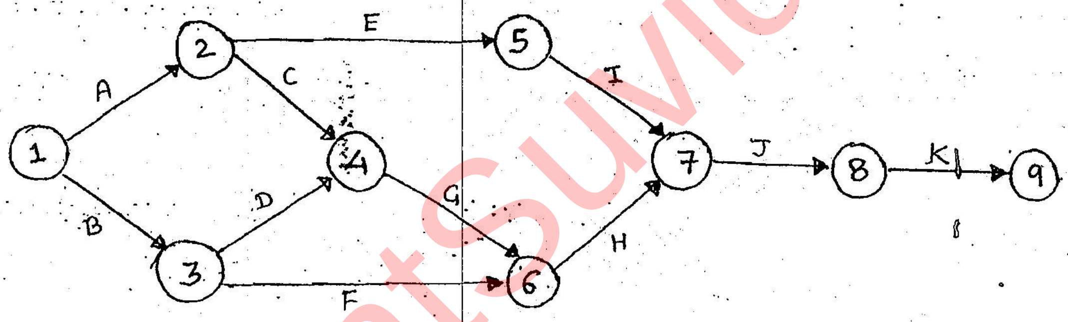
2) Activity on Node (AON) :-



Activity on Node does not require dummy activity and it is considered to be simple and easy, irrespective of these advantages. EON diagram is more popular in PERT and CPM.

Q → Draw network diagram for following set of activities.

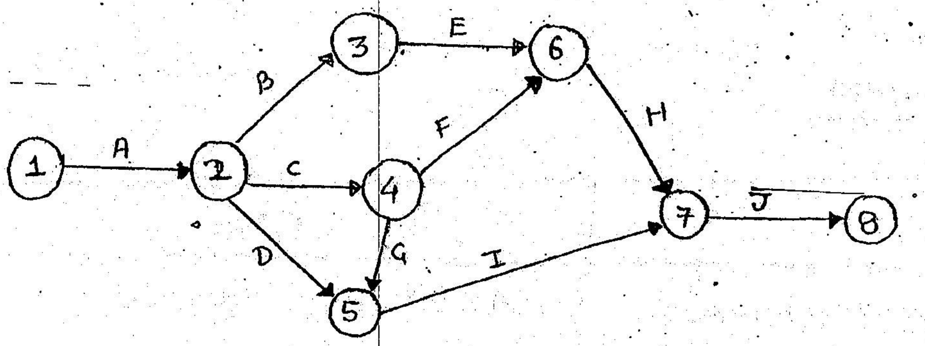
Activity	Precedence
A	—
B	—
C	A
D	B
E	A
F	B
G	C, D
H	G, F
I	E
J	H, I
K	J

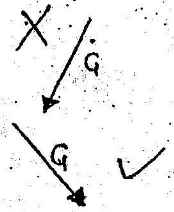


Fulkerson's Rule → Numbering of events.

Q → Draw the network diagram for following set of activities –

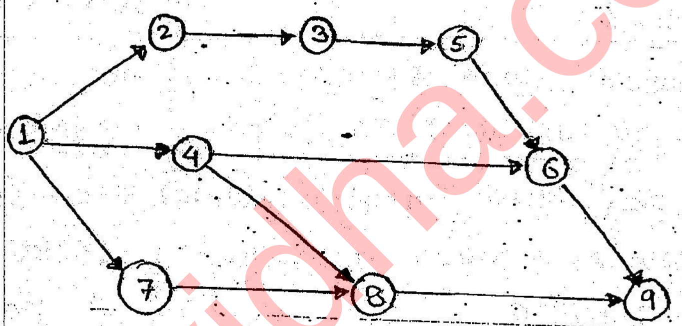
Activity	Precedence
A	—
B	A
C	A
D	A
E	B
F	C
G	C
H	E, F
I	D, G
J	H, I





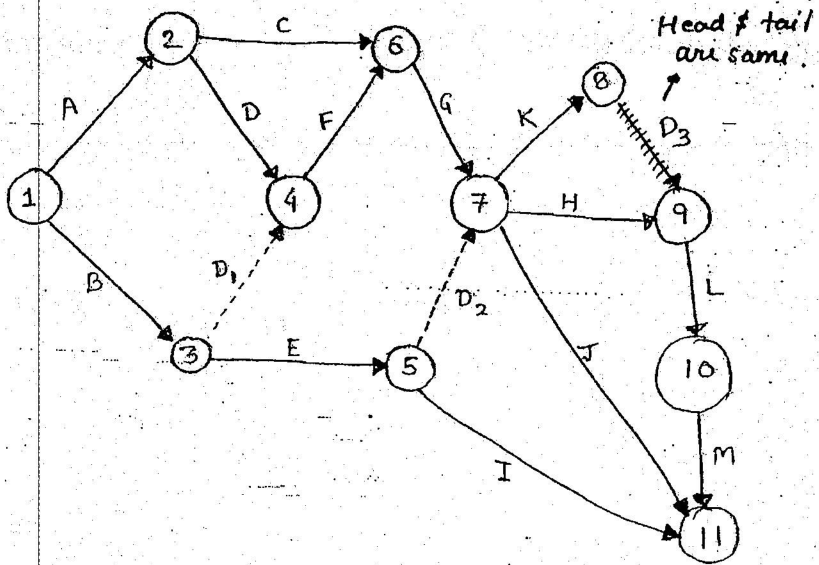
Q → Draw Network Diagram for following set of Activities —

Activity
1 – 2
1 – 4
1 – 7
2 – 3
3 – 5
4 – 6
4 – 8
5 – 6
6 – 9
7 – 8
8 – 9



Q –

Activity	Precedence
A	–
B	–
C	A
D	A
E	B (Dummy)
F	B D (Dummy)
G	C F
H	G E
I	E (Dummy)
J	G E (Dummy)
K	G E
L	H, K
M	L



PERT v/s CPM :-

PERT	CPM
• Program (Project) Evaluation and Review Technique	Critical Path Method
• It is event oriented	It is activity oriented
• It is associated with probablistic activities	associated with deterministic activities
• It is based upon 3-time estimate to complete an activity.	based upon single time to complete an activity
• It is used where time required to complete various activities is not certain	Used for repetative jobs where one has prior experience of handling similar project
• It usually does not consider cost analysis	It gives importance to cost analysis and crashing is done to minimise the cost of CPM project.
• It is used mainly for research and development project	Used mainly for construction project.

PERT :- It is used for uncertain project and is based on three time estimation to complete an activity.

There are :-

1) **Optimistic time** (t_o or a) :-

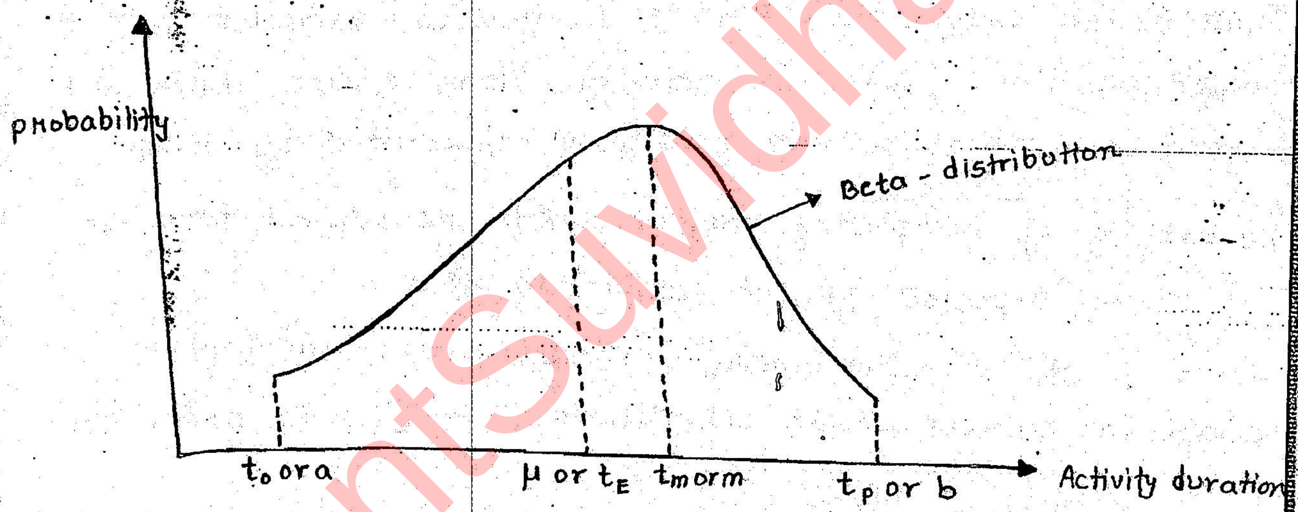
It is the minimum time required to complete an activity when everything goes according to the plan.

2) **Pessimistic time** (t_p or b) :-

It is the maximum time required to complete an activity when everything goes against the plan.

3) **Most likely time** (t_m or m) :-

It is the time require to complete an activity when executed under normal working condition.



The fundamental assumption in PERT is that the three time estimate form the end point of the distribution curve and the activity is assumed to follow Beta - distribution. It is also assumed that the prob. of completing activity in time a and b is equal. And the prob. of completing activity in time m is 4 times of either a or b.

The average or expected time to complete an activity is given by

$$\mu \text{ or } t_E = \frac{a + 4m + b}{6} = \frac{t_o + 4t_m + t_p}{6}$$

S.D → $\sigma = \left(\frac{b-a}{6}\right)$, = [redacted]

Variance $= \sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$

Note : Variance give the measure of uncertainity of activity completion. Higher the value of variance larger the uncertainity will be.

CPM :-

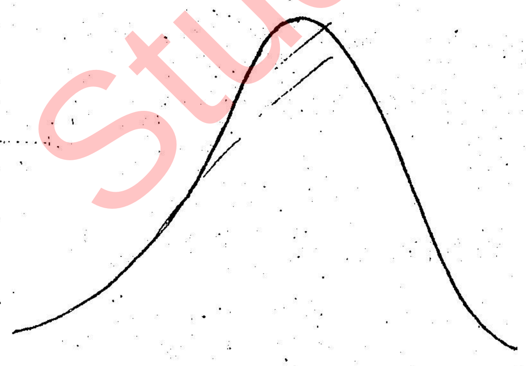
Critical path is the maximum time consuming path from the first event to the last event in a network diagram. Time taken along the cpm is termed as expected project completion time (T_E). The activity along critical path are termed as critical activity and are represented by ⟹

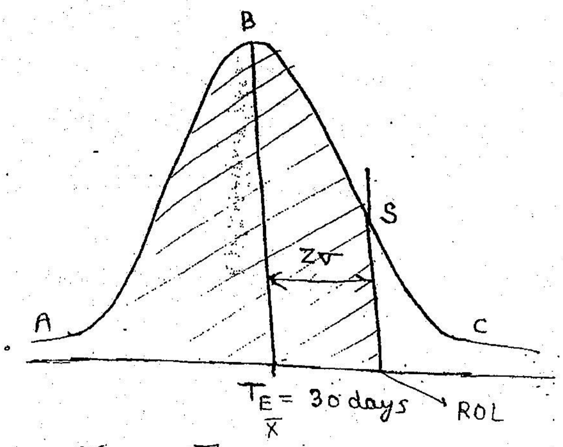
Probability of completing project within scheduled time :-

→ If T_E is expected project completion time, σ is std deviation along critical path, then prob of completing project within scheduled time (T_s) is given by -

$$Z = \frac{T_S - T_E}{\sigma}$$

Z → std normal variate





$T_E \rightarrow \bar{X}$

$T_S \rightarrow$ ROL

prob. of comp. task before 35 days –

$T_s = 35$ days.

$$P(T_s) = \frac{\text{Area } ABS}{\text{Area } ABC}$$

$ROL = \bar{x} + z\sigma$

$T_s \doteq T_E + z\sigma$

$$Z = \frac{T_s - T_E}{\sigma}$$ if $Z = 1.645$, Prob = 95%

$\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8$

CP

$\sigma = \sqrt{\text{sum of variance along critical Path}}$

* $\sigma = \sqrt{\sigma_1^2 + \sigma_3^2 + \sigma_5^2 + \sigma_7^2}$.

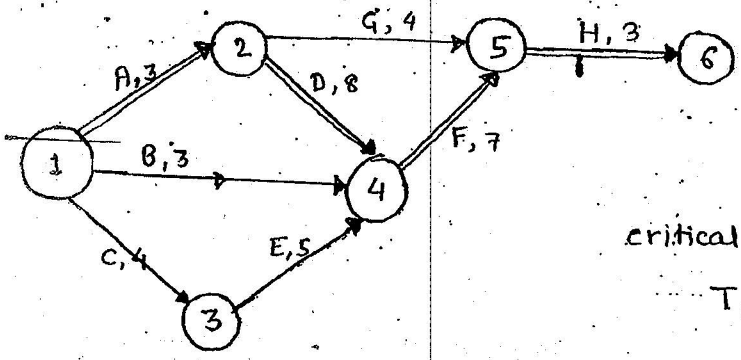
Q – For the following set of activities draw the network diagram and determine –

i) critical path and expected project completion time.

ii) the probability of completing project in 20 days

iii) if a company makes an aggrement to complete project in 22 days failing which they would pay Rs 10,000 / day as fine. Find the prob. that the fine may be paid but not exceeding Rs 20,000.

Activity	Precedence	Time a	Time m	Time b	$t_E = \frac{a+4m+b}{6}$	$\sigma = \frac{b-a}{6}$
A	–	1	3	5	3	4/6
B	–	2	3	4	3	
C	–	3	4	5	4	
D	A	2	9	10	8	8/6
E	C	4	5	6	5	
F	B, D, E	5	6	13	7	8/6
G	A	2	4	6	4	
H	G F	0	3	6	3	6/6



critical path = A D F H

T_E = 21 days

1) 1 - 2 - 5 - 6 = 10
2) 1 - 2 - 4 - 5 - 6 = 21
3) 1 - 4 - 5 - 6 = 13
4) 1 - 3 - 4 - 5 - 6 = 19

b) $Z = \frac{T_S - T_E}{\sigma}$

$$\sigma = \sqrt{\left(\frac{4}{6}\right)^2 + \left(\frac{8}{6}\right)^2 + \left(\frac{8}{6}\right)^2 + \left(\frac{6}{6}\right)^2}$$

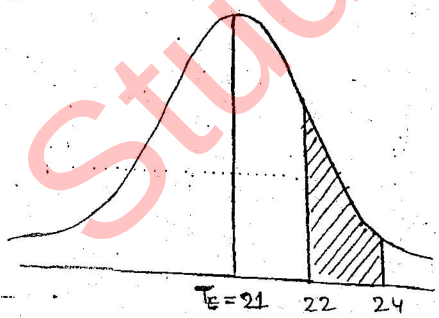
$$\rightarrow \frac{1}{6}\sqrt{16 + 64 + 64 + 36} = \frac{\sqrt{180}}{6} \doteq 2.236 \text{ days}$$

$$Z = \frac{T_S - T_E}{2.236}$$

$$= \frac{20 - 21}{2.236} = -0.4472 \xrightarrow{Prob} 32.7\%$$

MS → SD → Sh+3 → P(1)
↗ Prob

c)



$$Z_1 = \frac{24 - 21}{2.236} = 1.34 \xrightarrow{Prob} 91.06\%$$

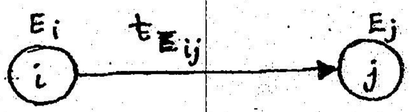
$$Z_2 = \frac{22 - 21}{2.236} = 0.447 \xrightarrow{Prob} 67.26\%$$

$$Z = 23.7\%$$

Critical Path :-

The procedure for finding critical path is similar to both PERT and CPM and consists of 2 phases –

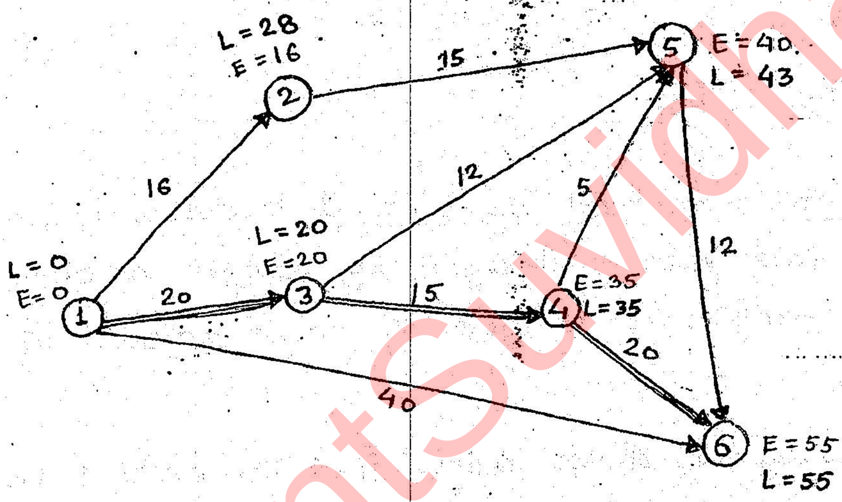
i) **Forward Pass computation** :- In this we compute the time at which an event is expected to be completed at the earliest.



where, E_i → earliest expected time for event i

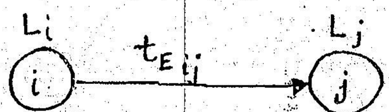
E_j → earliest expected time for event j

$t_{E_{ij}}$ → expected time for activity i, j



Event	Earliest time
1	0
2	16
3	20
4	35
5	max [31, 32, 40] = 40
6	max [52, 55, 40] = 55

ii) **Backward Pass computation** :- In this we compute the time by which an event must be completed at the latest.



where, L_i → latest allowable time for event i

L_j → latest allowable time for event j

$t_{E_{ij}}$ → expected time for activity ij

Event	Latest time
6	55
5	43
4	Min [38, 35] → 35
3	min [31, 20] → 20
2	28
1	min [12, 0, 15] → 0

* For any activity to be critical the following 3 conditions must be satisfied —

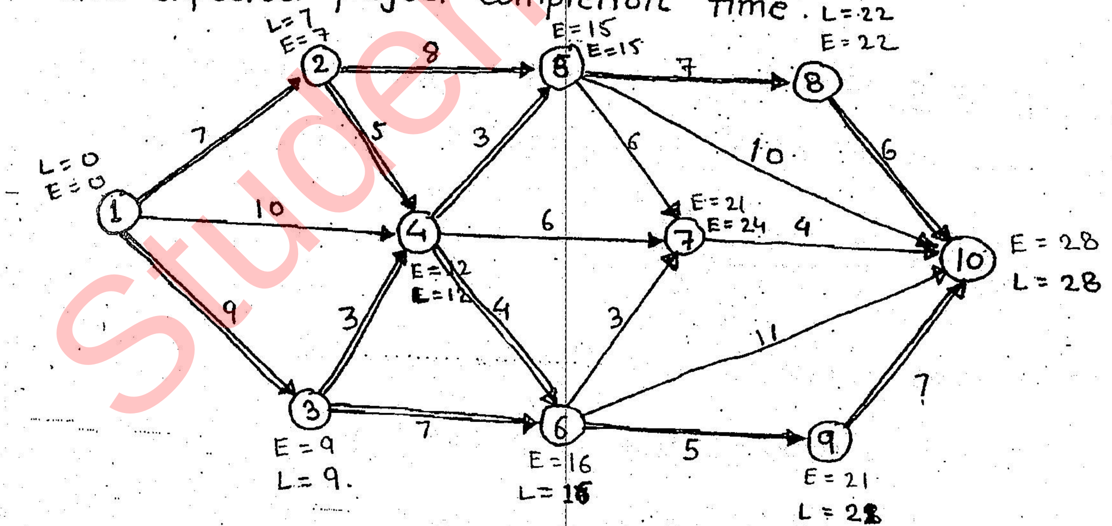
i) Head event slack = 0 ; $L_j - E_j = 0$

ii) Tail event slack = 0 ; $L_i - E_i = 0$

iii) $L_j - L_i = E_j - E_i = t_{E_{ij}}$

Note :- Critical path is termed as critical because if any activity on this path is delayed by certain amount of time, the whole project is delayed by same amount of time.

Q → For the network diagram shown below find the critical path and expected project completion time.



1) 1 - 2 - 5 - 8 - 10

2) 1 - 2 - 4 - 5 - 8 - 10

3) 1 - 2 - 4 - 6 - 9 - 10

4) 1 - 3 - 6 - 9 - 10

5) 1 - 3 - 4 - 6 - 9 - 10

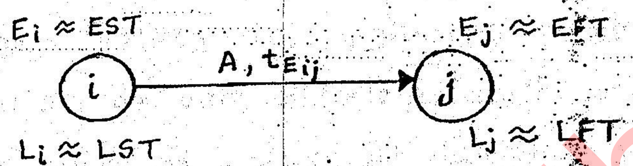
6) 1 - 3 - 4 - 5 - 8 - 10

NOTE – In case of PERT if there are more than 1 critical path, then in order to determine probability we select the path having max standard deviation. Due to that probability will be less but we will be on the safer side as project must be completed by that time.

$$\downarrow z = \frac{T_S - T_E}{\sigma \uparrow}$$

$\sigma_1 = 4.2 \downarrow$ 78% $\sigma_2 = 3.6 \downarrow$ 85% $\sigma_3 = 5.1 \downarrow$ 67% safe

Slack and Float :-



$$E_i + t_{E_{ij}} = E_j$$

$$EST + t_{E_{ij}} = EFT$$

The terms like earliest expected time, Latest allowable time and slack corresponds to event in PERT while the terms like earliest start time, earliest finish time, Latest start time, latest finish time and float corresponds to activity in CPM.

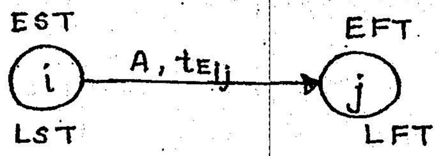
i) Slack or Event Float :-

$S_i = L_i - E_i$ $S_j = L_j - E_i$

Slack = denotes the amount of time by which a perticular event can be delayed without delaying the project schedule.

2) **Float** :- It is of 3 types -

a) **Total Float (TF)** :-



$$TF = LFT - EFT = LST - EST$$

It denotes the amount of time by which an activity can be delayed without delaying the project completion date. It is the extra time available for an activity without delaying the project schedule. If total float value is

i) +ve, resources are surplus and can be alocated for other activities.

ii) zero, resources are just sufficient to complete activity on time

iii) -ve, resources are not sufficient and activity may not complete on time.

b) **Free Float (FF)** :-

It is that part of total float which can be used without affecting the float of suceeding activity. It is the extra time by which an activity can be delayed so that the suceeding activity can be started at their earliest start time.

$$FF = TF - \text{Head event slack}$$

c) Independent Float :-

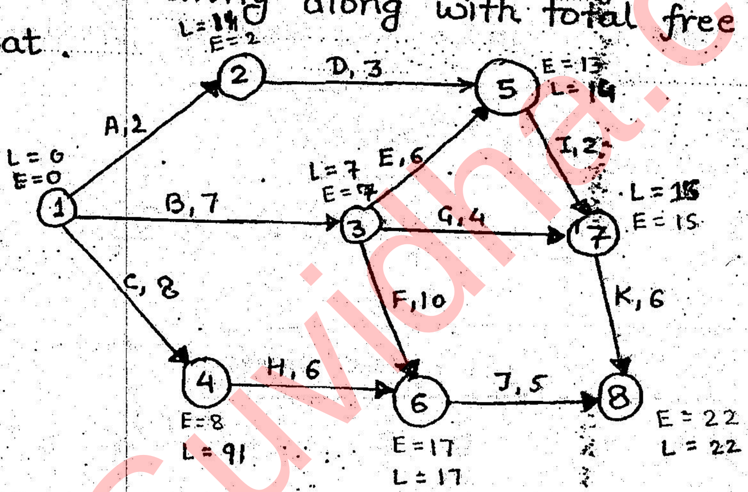
It is the amount of time which can be used without effecting either the head or the tail events.

$$IF = FF - \text{Tail event slack}$$

$$TF \geq FF \geq IF$$

for critical path -
$TF = FF = IF = 0$

Q → For the network diag shown below find critical path and expected project completion time. Draw the table showing the details for each activity along with total free and independent float.



Activity	$t_{E_{ij}}$	Earliest		Latest		Float		
		EST	EFT	LST	LFT	Total	FF	I.F
A , 1-2	2	0	2	9	11	9	0	0
B , 1-3	7	0	7	0	7	0	0	0
C , 1-4	8	0	8	3	11	3	0	0
D , 2-5	3	2	5	11	14	9	0	0
E , 3-5	6	7	13	8	14	1	8	-1* → 0 (obj)
F , 3-6	10	7	17	7	17	0	0	0
G , 3-7	4	7	11	12	16	5	4	4
H , 4-6	6	8	14	11	17	3	3	0
I , 5-7	2	13	15	14	16	1	0	-1* → 0 (obj)
J , 6-8	5	17	22	17	22	0	0	0
K , 7-8	6	15	21	16	22	1	1	0

Imp (*) For any activity i-j -

$$TF = L_j - (E_i + t_{E_{ij}})$$

← Network diagram

for D = 14 - (2+3) = 9

for A →
~~to~~ TF = 9
If we increase the time of A by 9, then it become critical → (job comp. time)

* $$FF = TF - \text{Head event slack}$$

D → FF = 9 - 1 = 8

FF tells how much time the activity should be delayed so that it would not disturb the comming forward activity.

$$FF = E_j - (E_i + t_{E_{ij}})$$

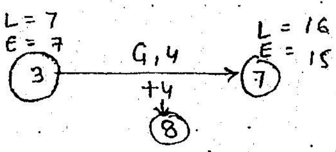
for D = FF = 13 - (2+3) = 8

* $$IF = FF - \text{Tail event slack}$$

for D = IF = 8 - 9 = -1 (conv) → 0 (objective)

IF tells how much time the activity should be delayed so that head event and tail event does not changes.

Forward → zada → chalega
Backward → kam → chalega



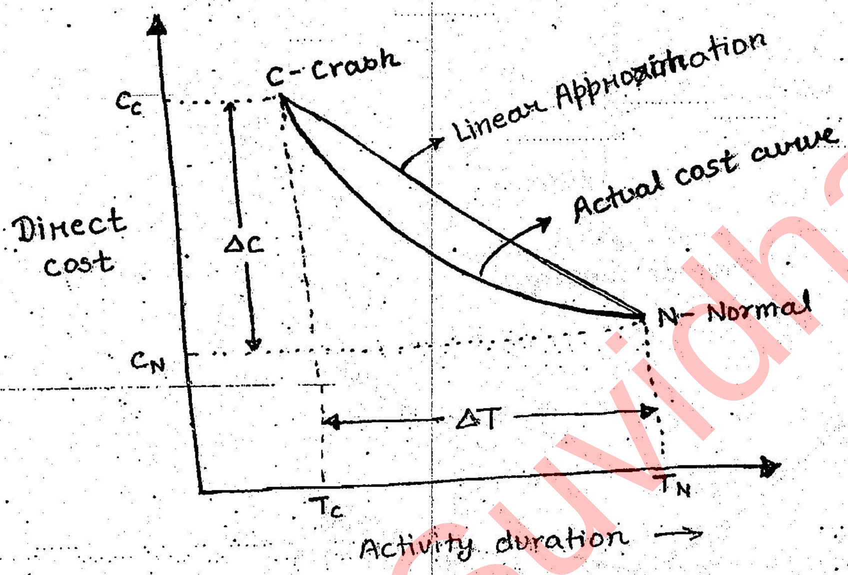
IF = 4

$$IF = E_j - (L_i + t_{E_{ij}})$$

Crashing or Time - cost

It is an extension of critical path method that consider the compromise b/w the time, and the cost required to complete the project. The total cost of any project consist of direct and indirect cost involved in its completion.

i) Direct Cost : It is the cost directly involved in the completion of an activity. It include direct material, direct labor, cost of m/c, equipment etc.,



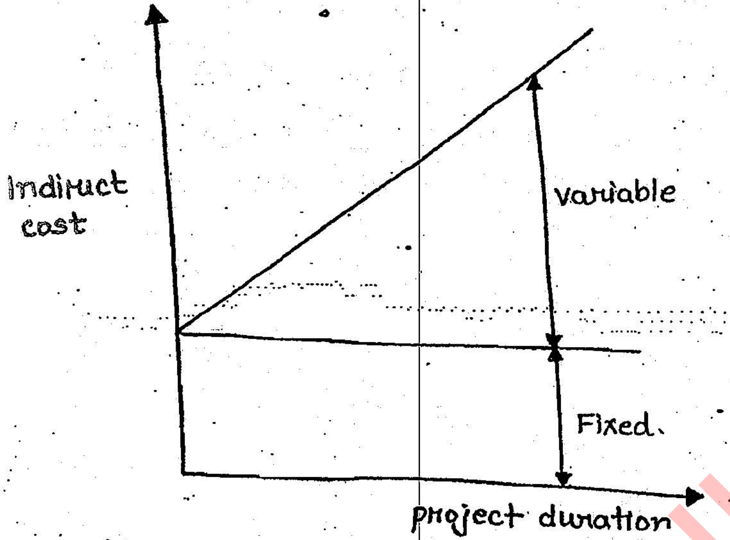
Crash time is the minimum activity duration to which an activity can be compressed by increasing the resources and hence by increasing direct cost. The slope of the line give the amount of increase the direct cost per unit time for crashing an activity.

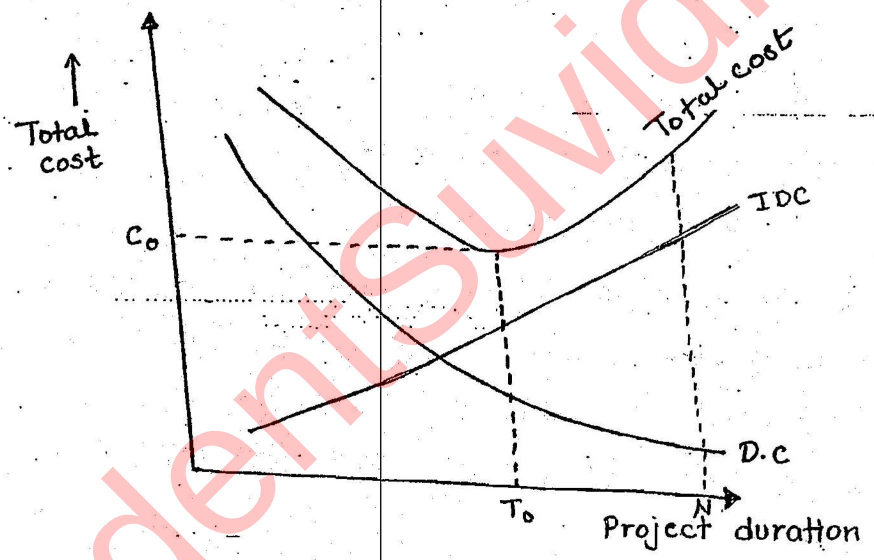
$$\text{cost time Slope} = \frac{\Delta C}{\Delta T} = \frac{C_c - C_N}{T_N - T_c}$$

eg - Normal: 10 days Rs 8000 Crash: 6 days Rs 14000

$$\frac{\Delta C}{\Delta T} = \frac{6000}{4} = \text{Rs } 1500/\text{day}$$

ii) Indirect cost :- It is the cost not directly involved in the completion of an activity but it is compulsory req. for safe and timely completion of project.





The objective of crashing a network is to determine optimum project duration corresponding to min. cost of project and the steps involved are -

i). In the critical path select the critical activity having the min cost slope.

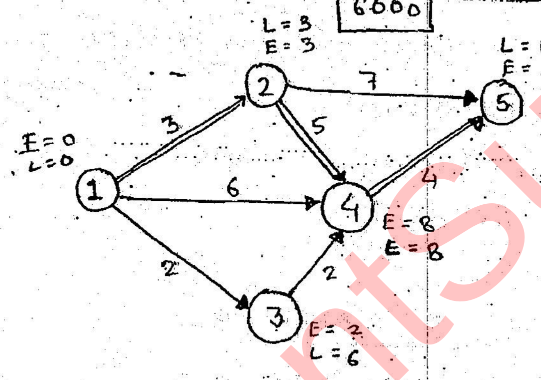
ii) Reduce the duration of this activity by 1 time period.

iii) Revise the network diagram by adjusting the time and the cost of crashed activity. Again find critical path, Project duration and TC of project

iv) If the optimum project duration is obtained then stop, otherwise repeat from step 1.

Q → Draw the Network diagram and crash the network to optimum project duration corresponding to min. cost of project. It is given that I.C is Rs 900 / day.

Activity i-j	Normal time (days)	Normal cost (Rs)	Crash time (days)	Crash cost (Rs)	ΔC/ΔT
1 - 2	3	500	2	1000	500
1 - 3	2	750	1	1500	750
1 - 4	6	1400	4	2600	600
2 - 4	5	1000	3	1800	400
2 - 5	7	1150	6	1450	300
3 - 4	2	800	2	800	-
4 - 5	4	1000	2	2400	700
		6600			



TE = 12 days

TC = DC + IDC

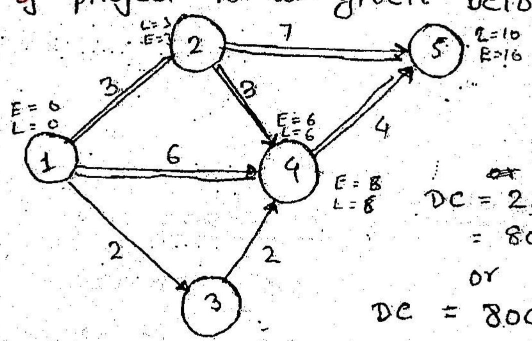
DC = 6600

IDC = 900 X 12 = Rs 10800

TC = 6600 + 10800

TC = Rs 17400

Now crashing the min cost of activity along the critical path. It is activity 2-4 so crashing it by 2 days. The revised network diagram and the cost of project is as given below –



DC = 2 X 400 = 800 + –

or

DC = 800 + 6600 = 7400

TE = 10 days

~~DC = 6600~~

IDC = 900 X 10 = 9000

TC = DC + IDC

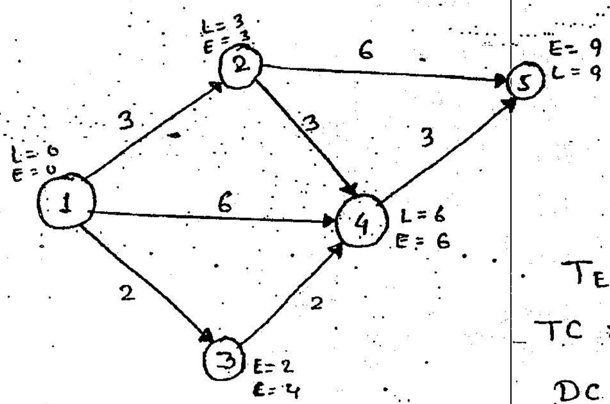
= ~~6600~~ 7400 + 9000 = ~~15600 Rs~~

= 16400 Rs

PTO

Now the network has 3 critical path and by crashing any one activity project duration does not change. so we need to crash atleast 2 activity simultaneously which gives 3 options. Out of these we select the option for which the summation of cost slope is min.

1) 1-2 and 1-4 → 500 + 600 = 1100
2) 2-5 and 4-5 → 300 + 700 = 1000 → min.
3) 1-2 and 4-5 → 500 + 700 = 1200



we can ~~increase~~ crash the duration of 4-5 by 2 days but its of no use as DC is increased 700 further. (2×700) -

T_E = 9 days

TC = DC + IDC

DC = Rs 6600 + (400 × 2) + (100.0)   (1st crashing, 2nd crashing)

= Rs 8400

IDC = 9 × 900 = Rs 8100

TC = Rs 16500

<u>objective</u> -

1st crash →	2nd crash →
DC ↑ Rs 800	DC ↑ Rs 1000
IDC ↓ Rs 1800	IDC ↓ Rs 900
TC ↓ Rs 1000	TC ↑ Rs 100